Video Traffic Modeling Using Seasonal ARIMA Models

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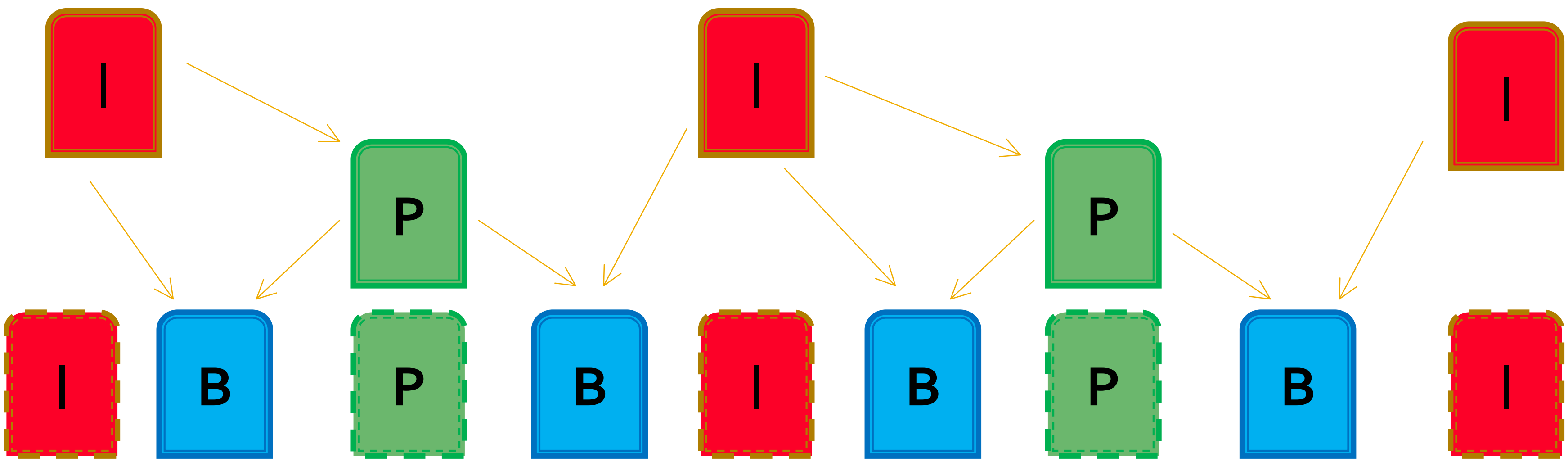
These slides are available on-line at:
http://www.cse.wustl.edu/~jain/wimax/video82.htm

1. Video Compression overview
2. Seasonal ARIMA Model
3. Model for all frames combined
4. Models for I, P, B frames

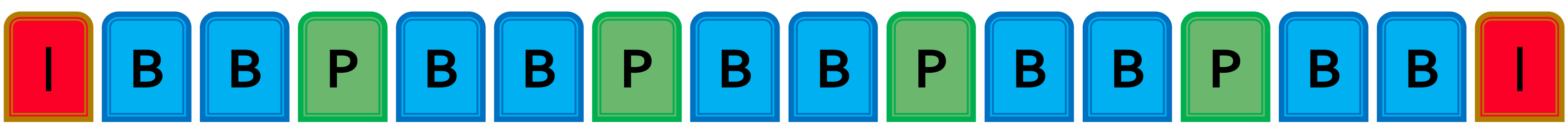
Goals

- One approach to model video traffic is to just determine the distribution of the frame sizes
- This assumes that the traffic is white noise and there is no predictability
- If there is predictability, it can be used for resource management
- Predictability $\Rightarrow$ Correlation in frame sizes
- In video traffic, there is significant correlation. If a particular scene takes place in a complex background $\Rightarrow$ Large frame sizes through out the scene
- Similarly, if the scene is simple, the frame sizes will remain low through out the scene.

Group of Pictures



A Typical Group Of Pictures (GOP)



Transmission Order of a GOP

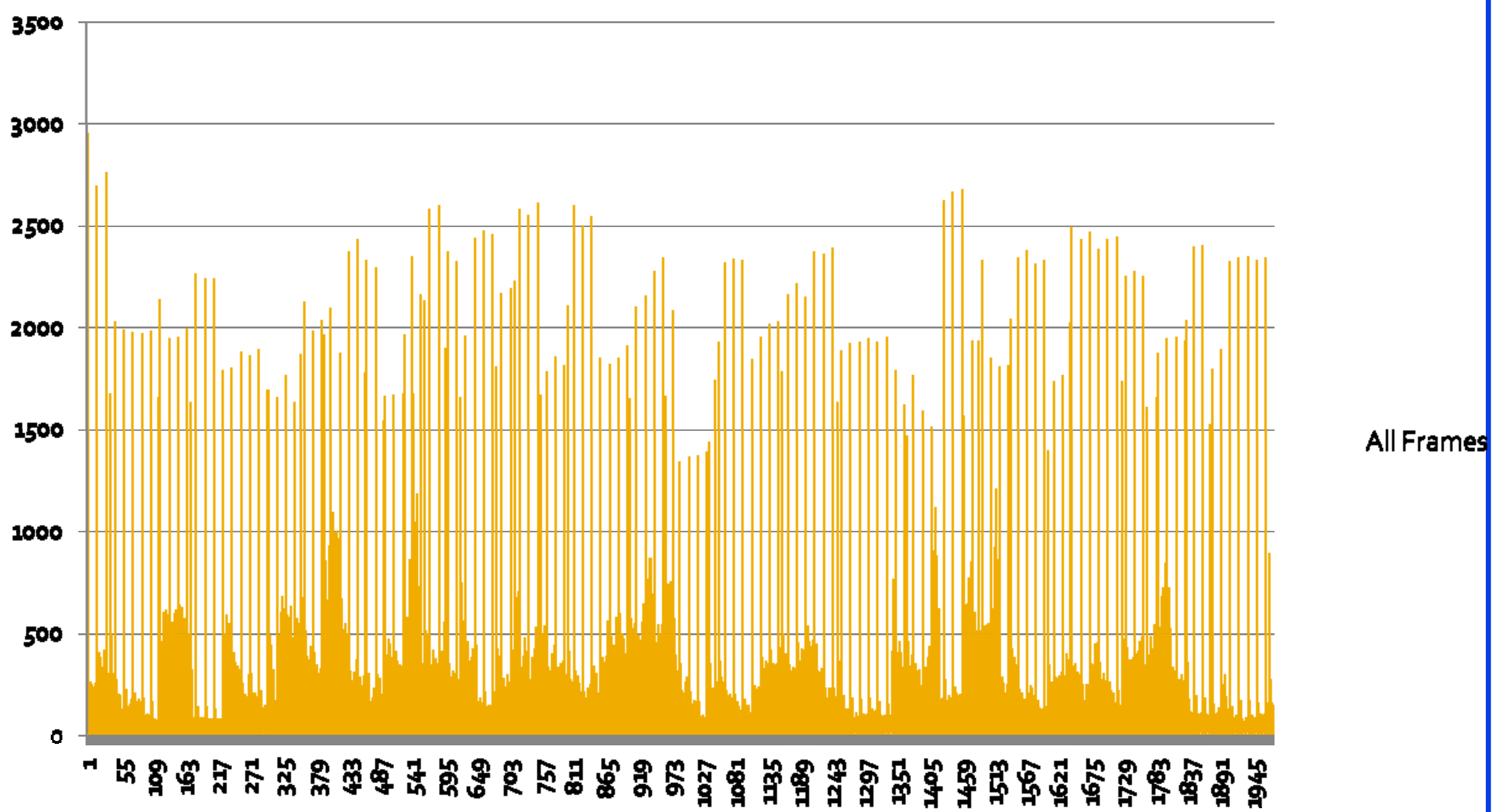
MPEG Encoding

- Video Scene is divided into group of pictures (GOP)
- The default GOP size for NTSC: 15
- GOP consists of:
  - **I (Intracoded) frames**: Each frame of video is still discrete and can be accessed as a unique point
  - **P (Predicted) frames**: P frames are predicted based on prior I or P frames
  - **B (bi-directional predicted) frames**: B frames are coded based on a forward prediction from a previous I or P frame, as well as a backward prediction from a succeeding I or P frame

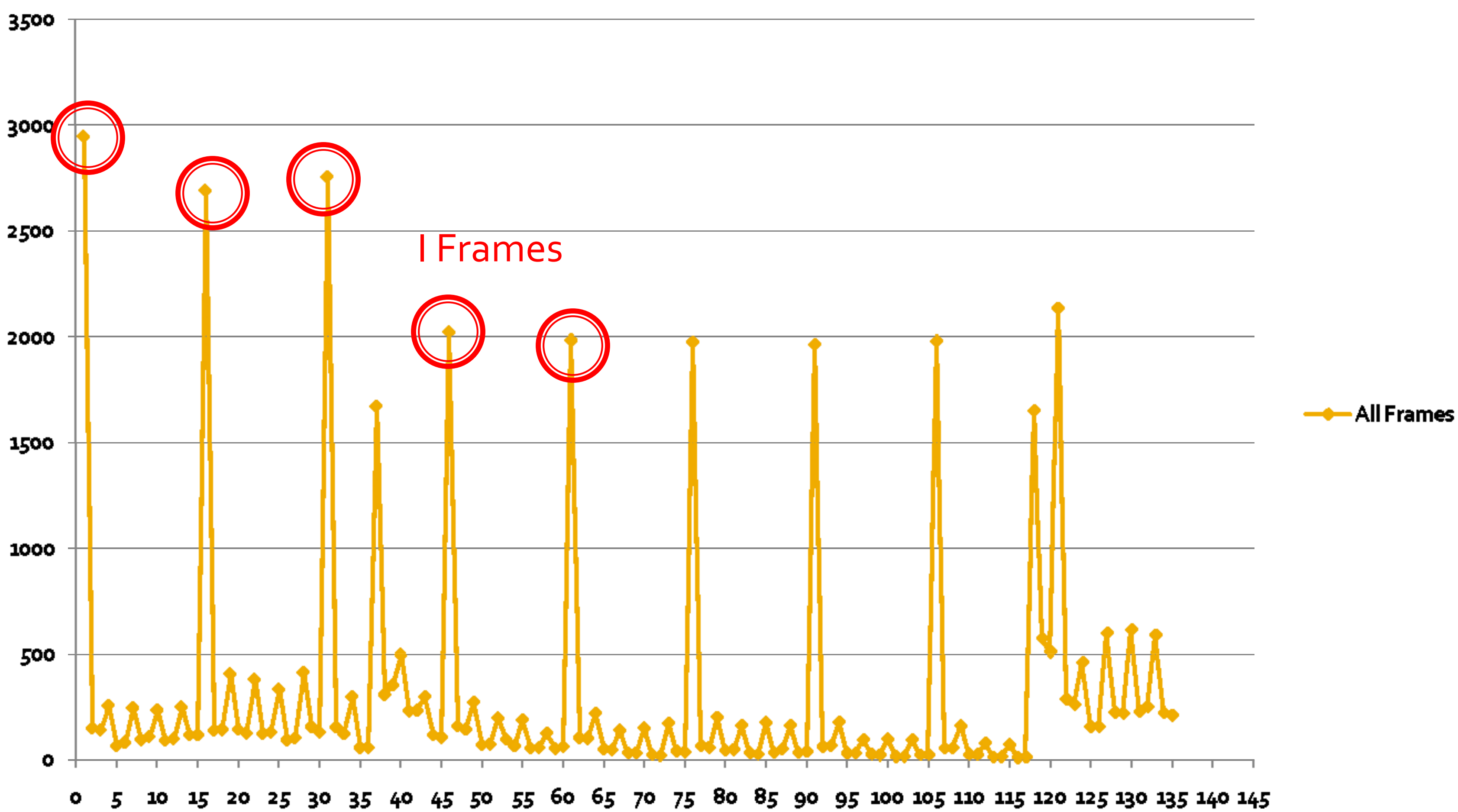
Video Trace

- Using ffmpeg library, we encoded 1:24 minutes movie clip
  - At 350kbps
  - QVGA: 320x240
  - GOP size of 15
- GOP sequence is :
  - IBBPBBPBBPBBPBB

Video Frames

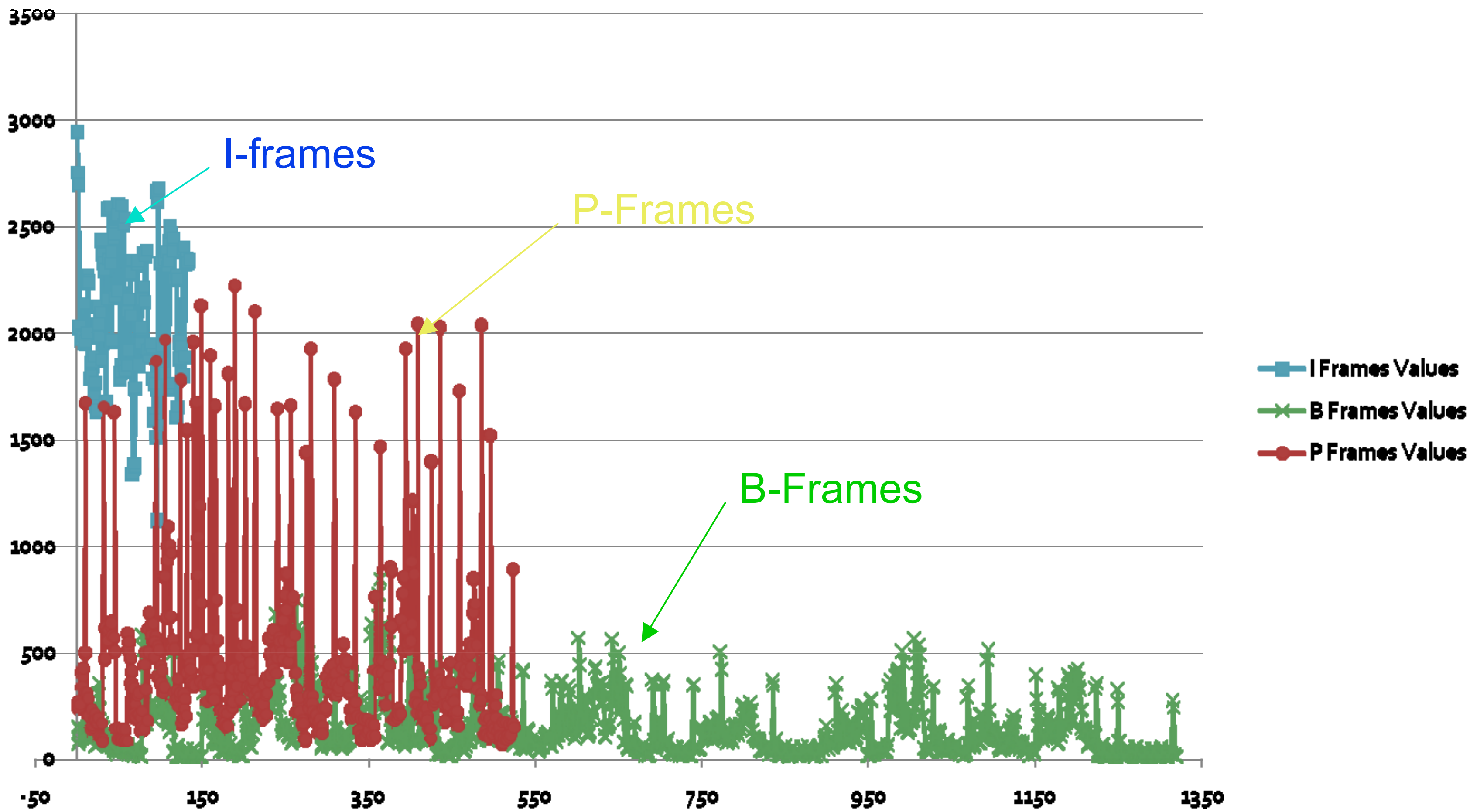


Video Frames: A Closer Look



- Observation: Every 15th frame is a large (I) frame.

Video Frames: I, P, B Size Distribution



- Observation: I, P, B frames have very different levels.

Auto-Regressive Models

- AR(1) Model:

$$y(t) = a(1)\, y(t-1) + w(t)$$

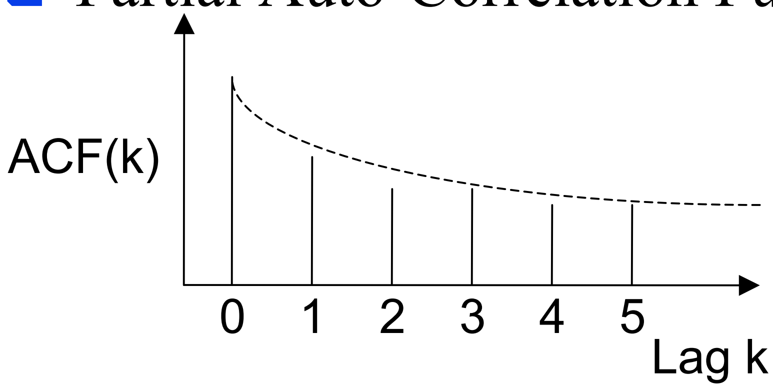
- AR(p) Model:

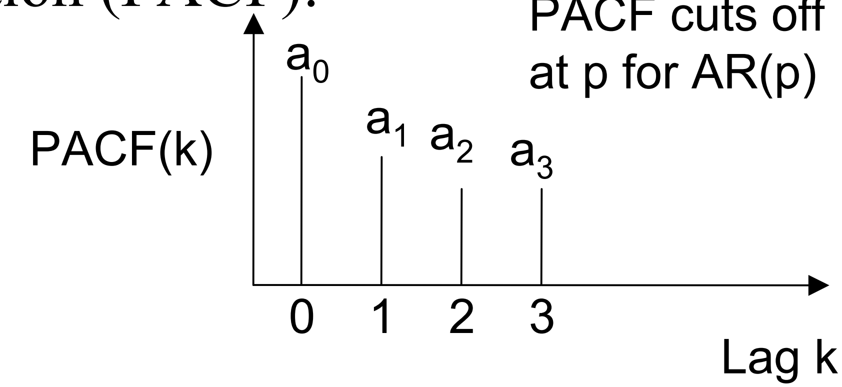
$$y(t) = a(1)\, y(t-1) + a(2)y(t-2)+\ldots+a(p)y(t-p)+w(t)$$

- AR(0) Model: $y(t)=w(t) \Rightarrow$ White noise
- Auto-Correlation Function (ACF):

$$AC(k)=cov(Y(t),y(t-k))/Var(y(t))$$

- Partial Auto-Correlation Function (PACF):





Moving Average Models

- MA(1) Model:

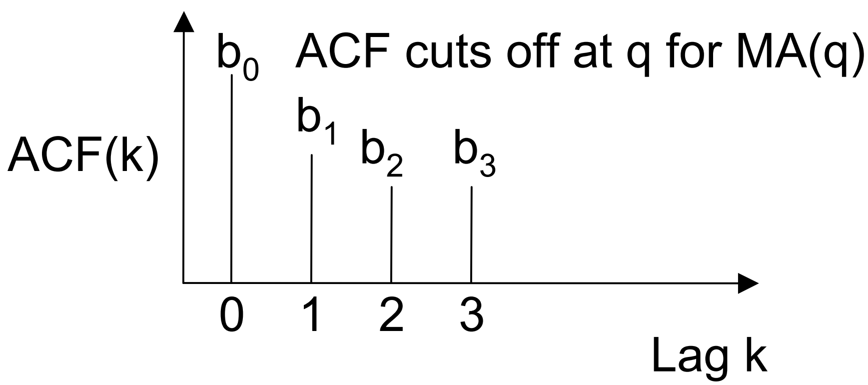
$$y(t) = w(t)+b(1)\, w(t-1)$$

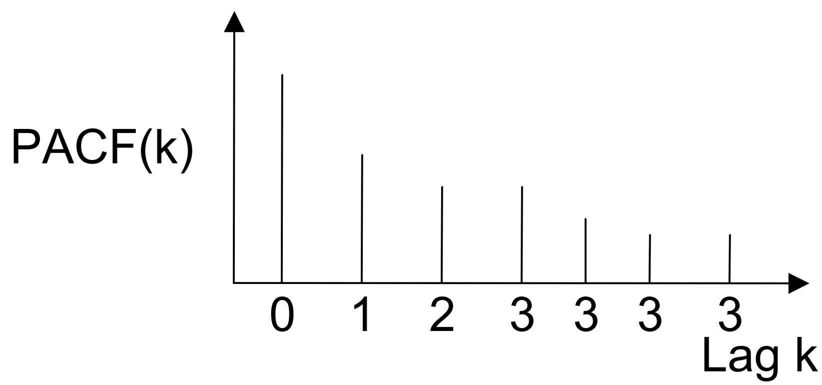
- MA(q) Model:

$$y(t) = w(t)+b(1)\, w(t-1) + b(2)w(t-2)+\ldots+b(q)w(t-q)$$

- MA(0) Model:

$$Y(t)=w(t) \Rightarrow \text{White noise}$$





ARIMA Models

- ARMA(p,q) Model:

$$y(t) + a(1)y(t-1)+\ldots+a(p)y(p) = w(t)+b(1)\,w(t-1)+\ldots+b(q)w(t-q)$$

- Using Backward operator: $Dy(t)= y(t-1)$

$$\{1+a(1)D+a(2)D^2+\ldots+a(p)D^p\}y(p)=\{1+b(1)D+\ldots+b(q)D^q\}w(t)$$

$$A(D)y(p) = B(D)w(t)$$

- Auto-regressive Integrated Moving Average Model: ARIMA(p,d,q)

$$A(D)(1-D)^d y(p) = B(D)w(t)$$

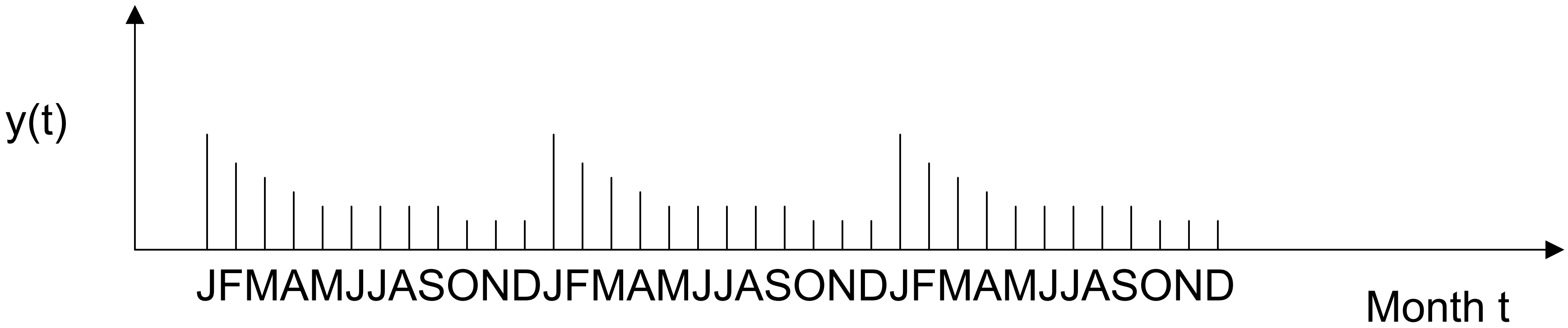
- Example: ARIMA(1,1,1) Model

$$(1+a_1D)(1-D)y(t)=(1+b_1D)w(t)$$

$$(1+a_1D-D-a_1D^2)y(t) = (1+b_1D)w(t)$$

$$y(t)-(1-a_1)y(t-1)-a_1y(t-2)=w(t)+b_1w(t-1)$$

Seasonal ARIMA Model



- Period of 12 $\Rightarrow$ Model y(t)-y(t-12) as a ARIMA(p,d,q) series
- ACF will show spikes at seasonal period s
- Seasonal ARIMA(p,d,q) (P,R,Q)s model:

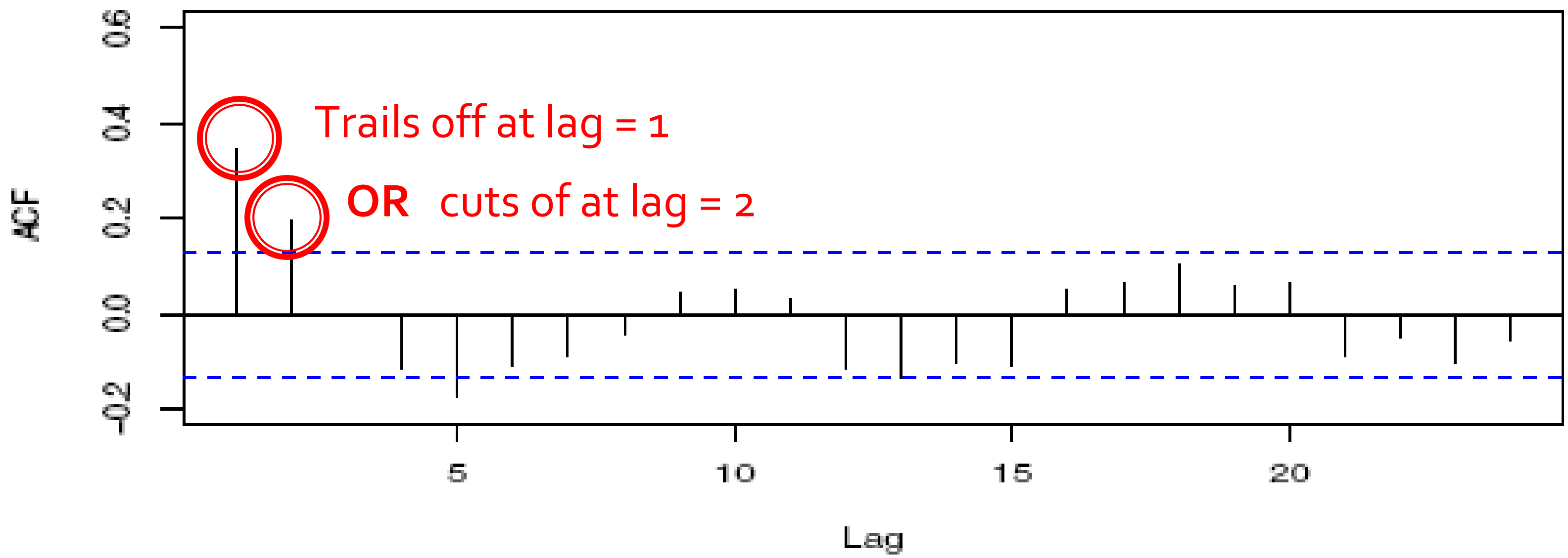
$$A(D)F(D^s)(1-D^s)Ry(t)=B(D)\ G(D^s)w(t)$$

- Example: $(1,0,0)x(0,1,0)_{12}$ Model

$$(1+a_1 D)(1-D^{12})y(t)=w(t)$$

$$y(t)-y(t-12)-a(1)\{y(t-1)-y(t-13)\}=w(t)$$

Interpreting ACF and PACF

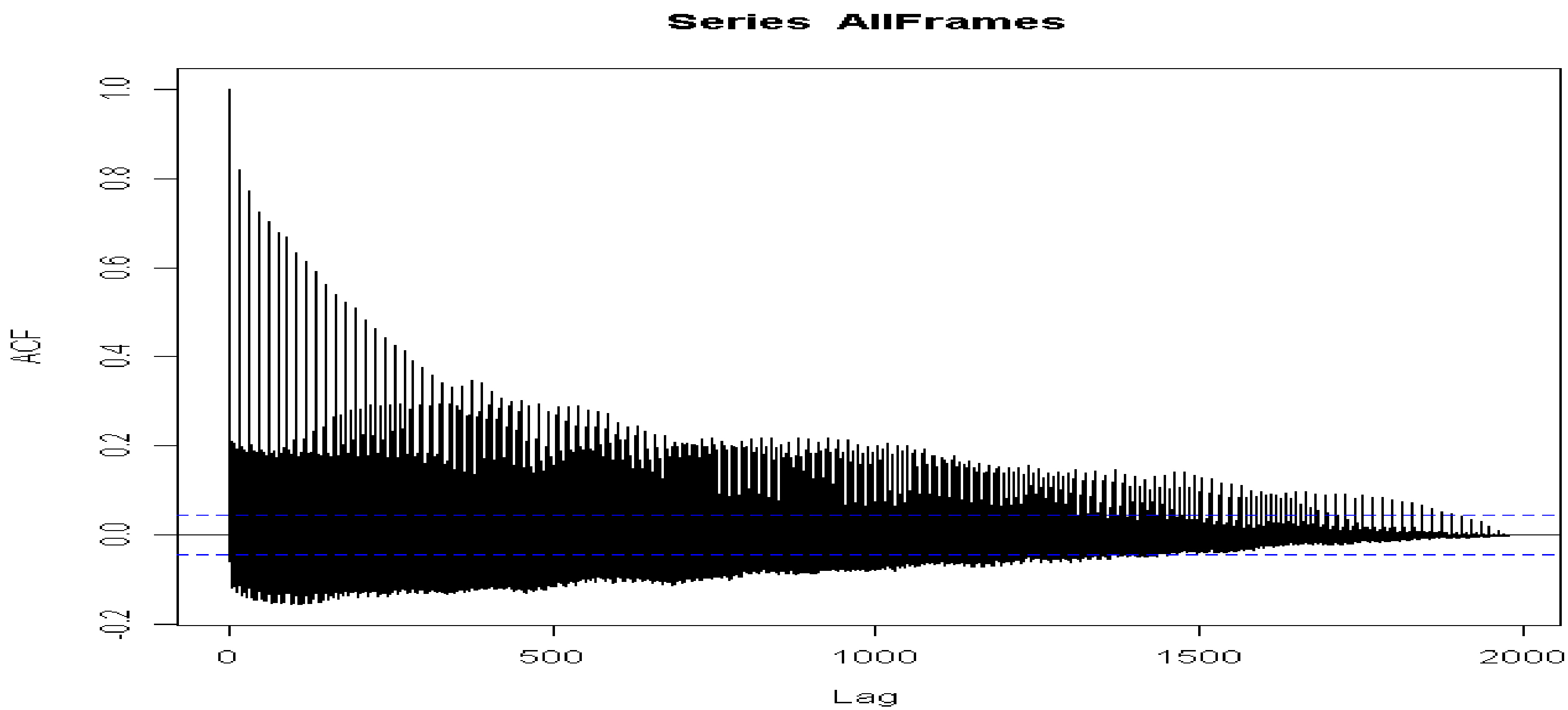


	AR(p)	MA(q)	ARIMA(p,q)
ACF	Trails off	Cuts off at lag q	Trails off
PACF	Cuts off at lag p	Trails off	Trails off [1]

] : Time Series Analysis and Its Applications: With R Examples

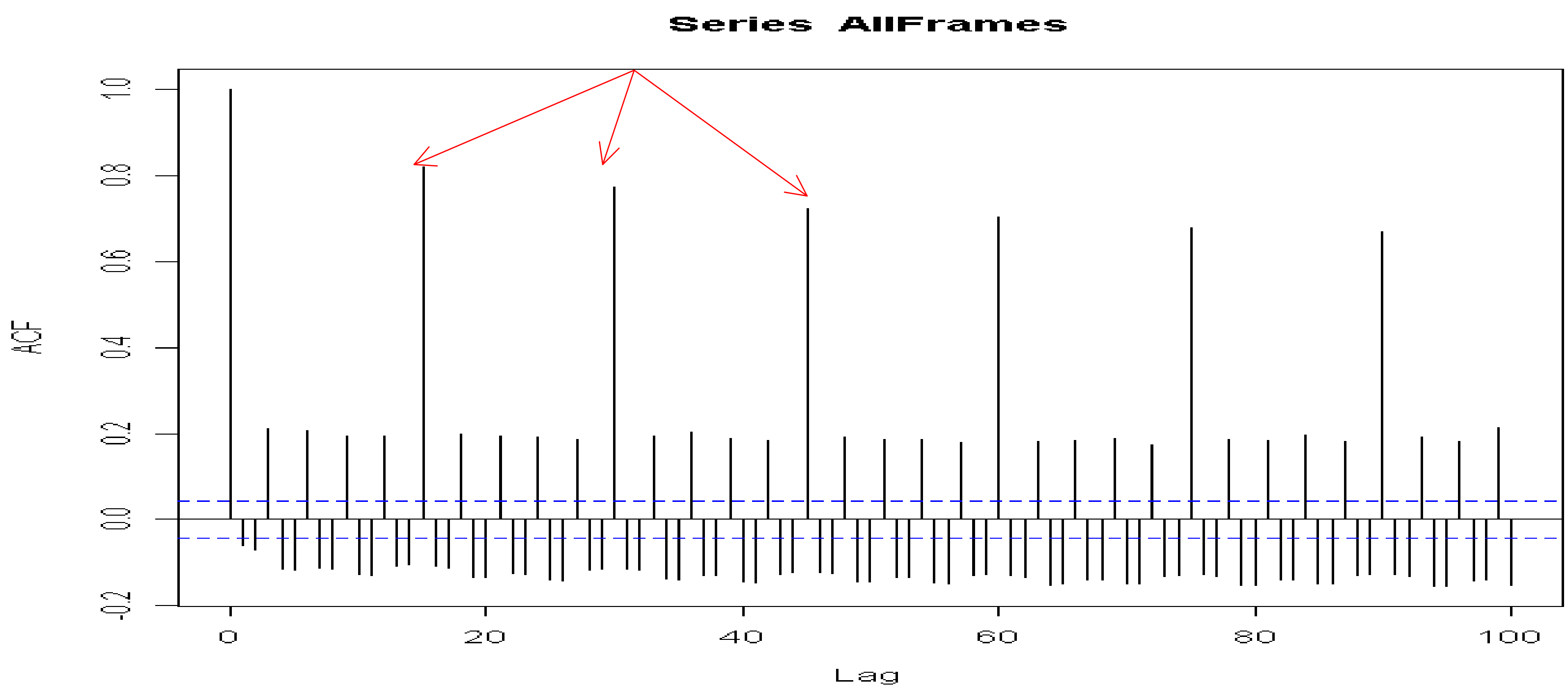
Traffic Modeling – All Frames 1

- ACF (Auto-Correlation Function) graph shows many trends and show how much video frames are correlated



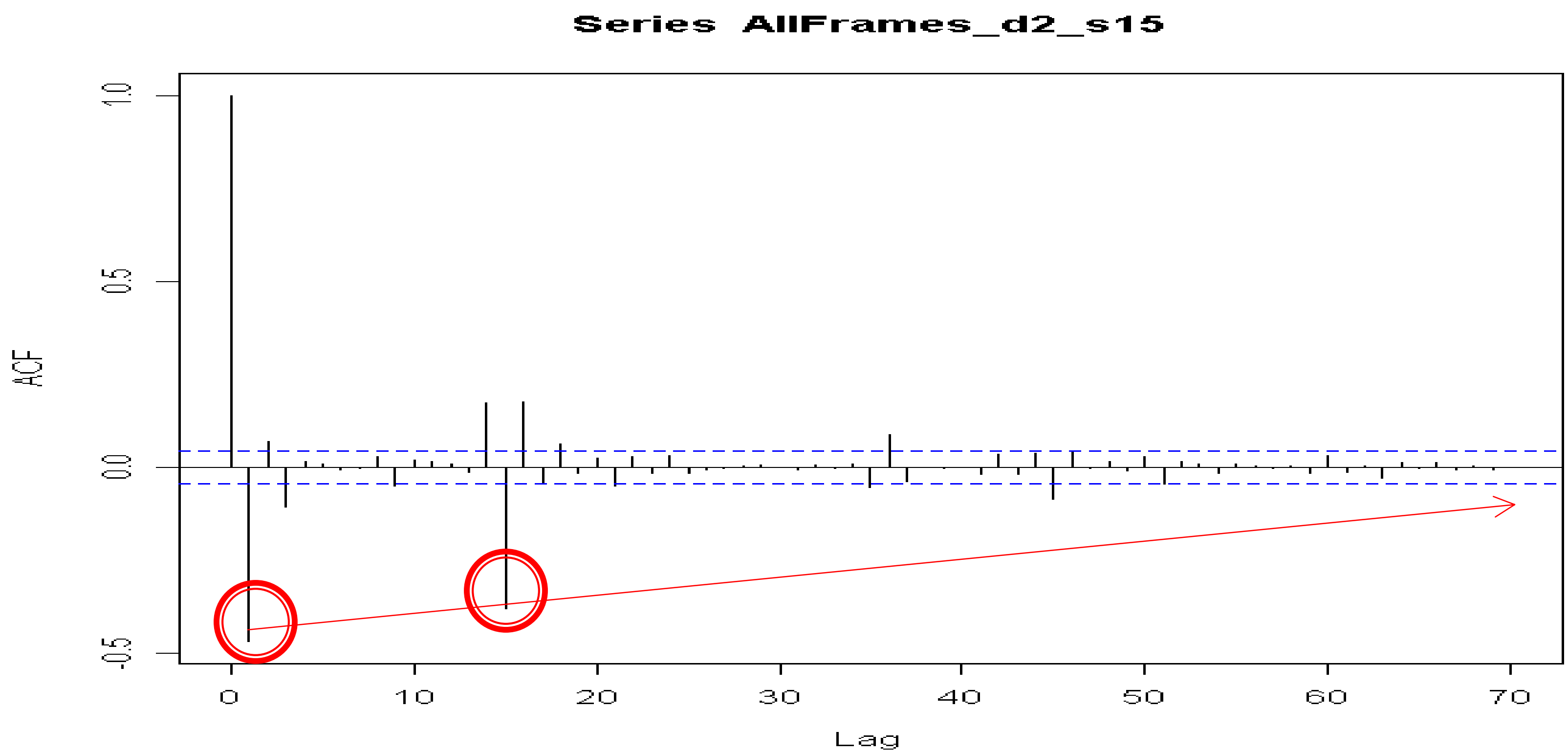
Traffic Modeling – All Frames 2

- A closer look at the ACF graph shows a strong continual correlation every 15 lag ➔ GOP size



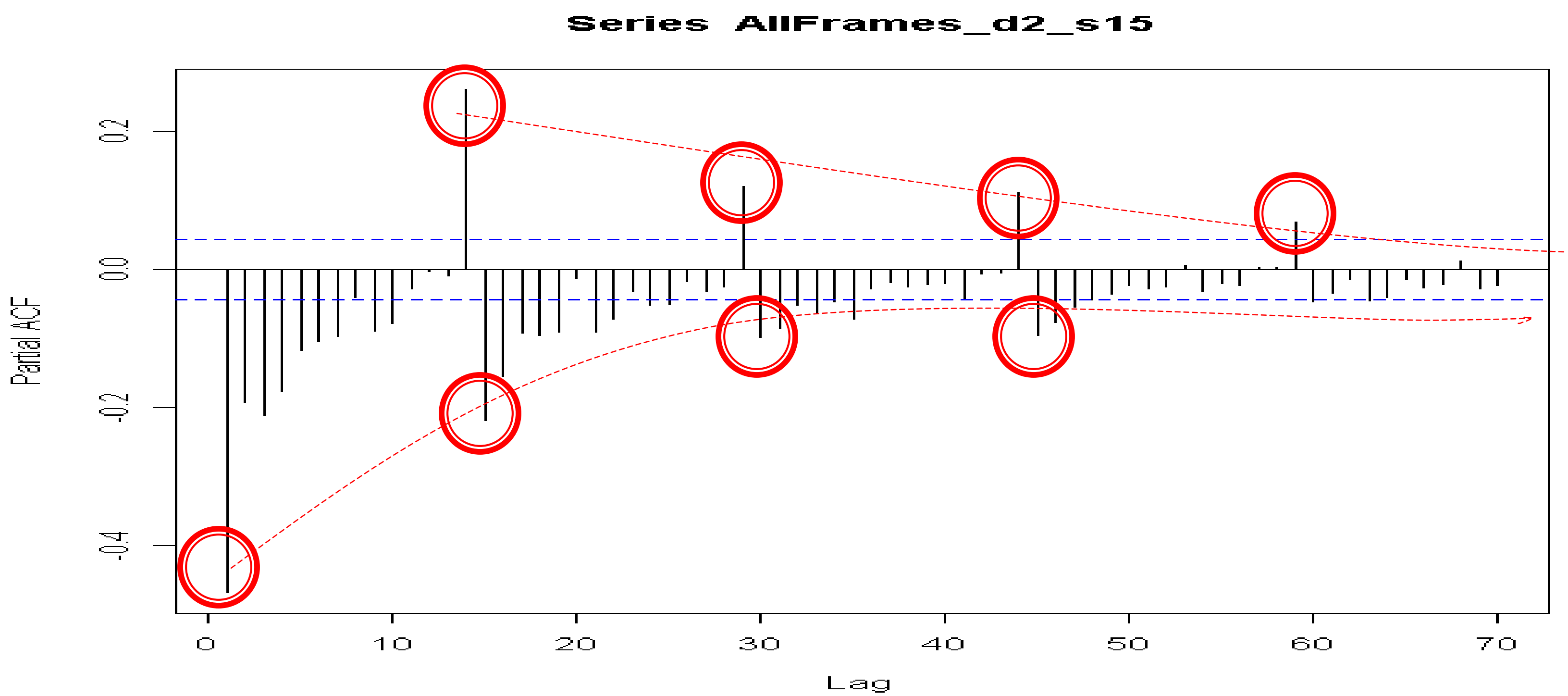
Traffic Modeling – All Frames 3

- ACF graph after second level of differentiation with seasonal part of 15

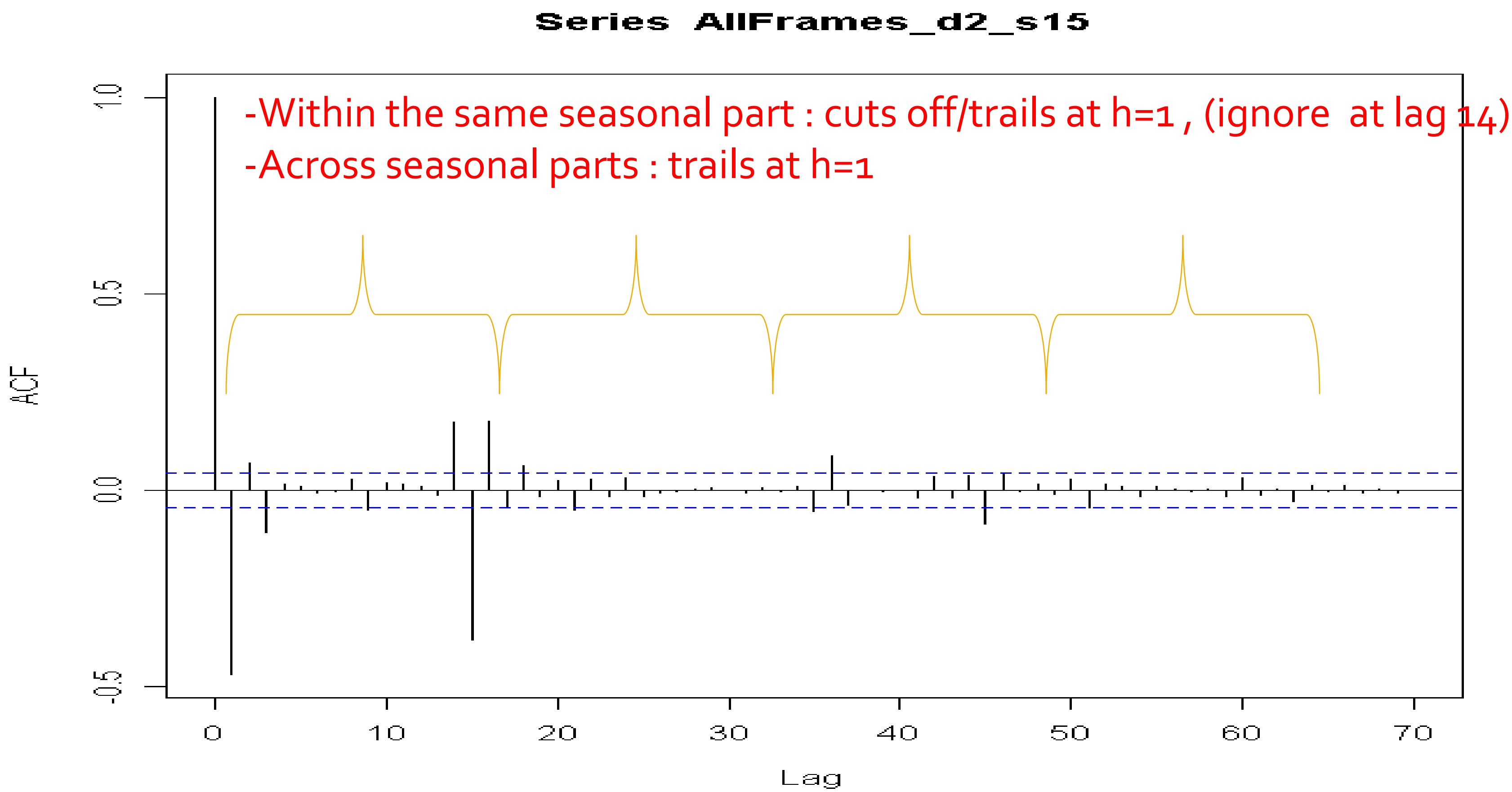


Traffic Modeling – All Frames 4

- PACF (Partial Auto-Correlation Function) shows another decaying trend around lags 15,30,45 . Also around 14,29,44



Traffic Modeling – All Frames 5



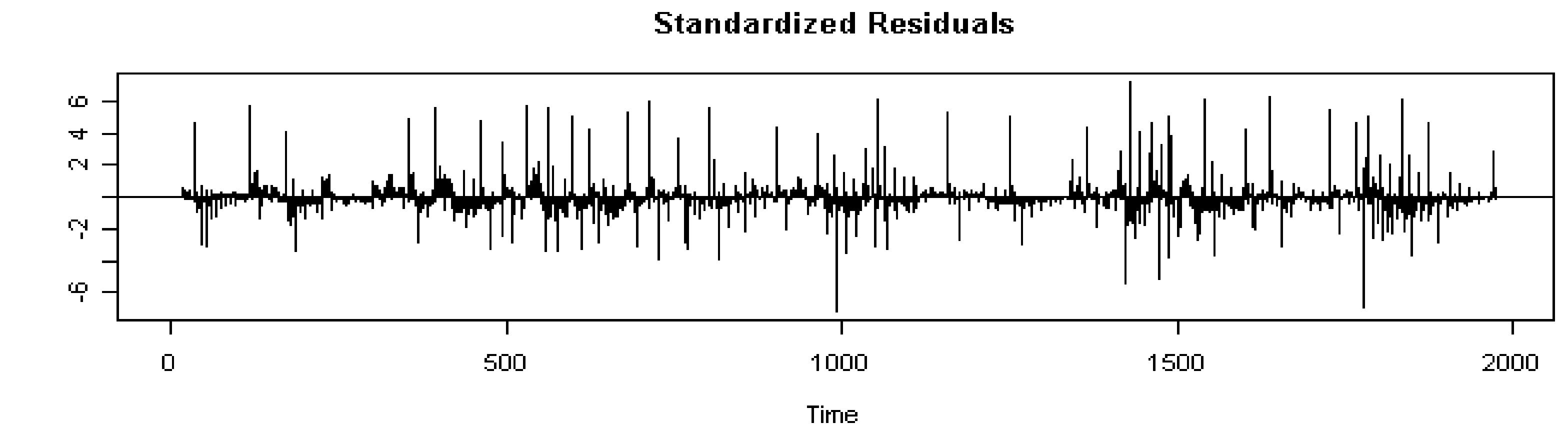
Traffic Modeling – All Frames 6

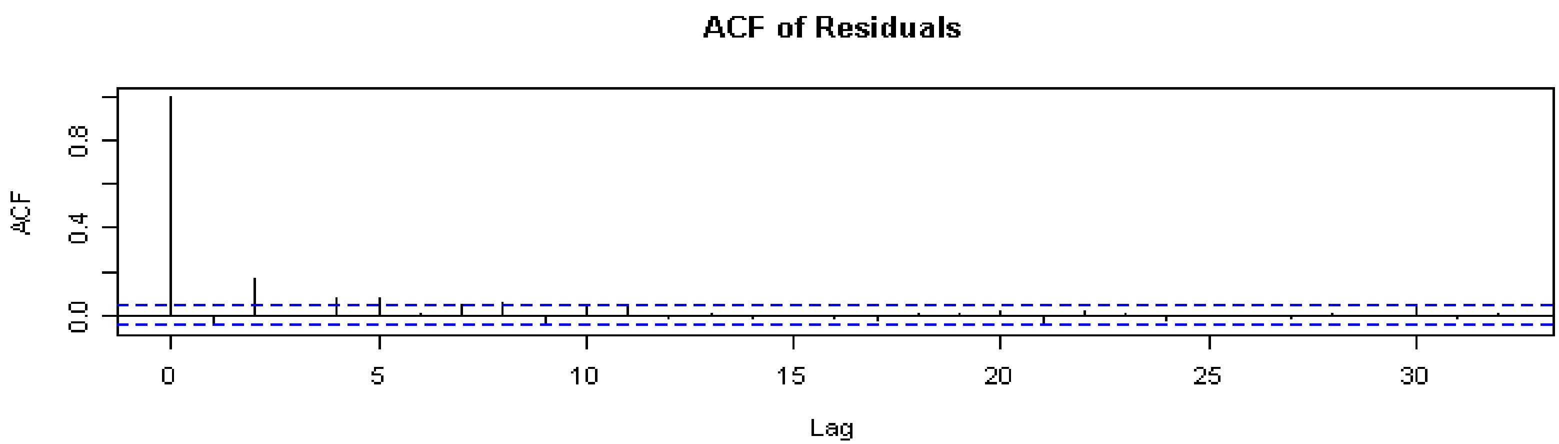
Traffic Modeling – All Frames 7

- The chosen seasonal ARIMA model is:
  - $(1,1,1) \times (1,1,1)_{15}$
- Coefficients :

	ar (1)	ma(1)	sar(1)	sma(1)
	0.2675	-1.000	0.1615	-0.6723
se	0.0218	0.0018	0.0392	0.0298

Results 1

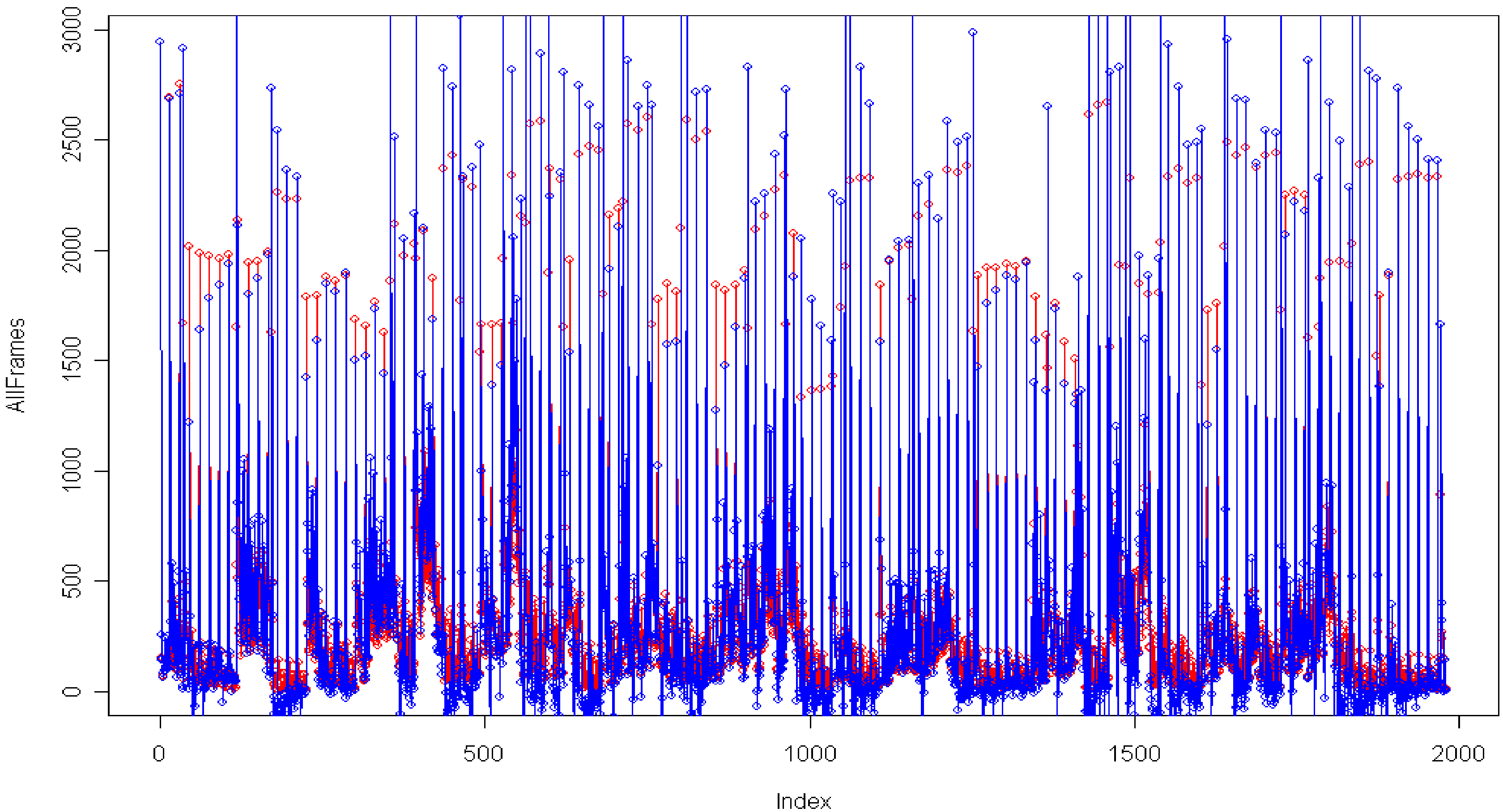

Standardized Residuals
-6
-2
2
4
6
0
500
1000
1500
2000
Time


ACF of Residuals
ACF
0.0
0.4
0.8
0
5
10
15
20
25
30
Lag

Results 2

Blue : Fitted Model
Red : Original data

$R^2 = 0.88$



Modeling All Frames: Seasonal ARIMA Model

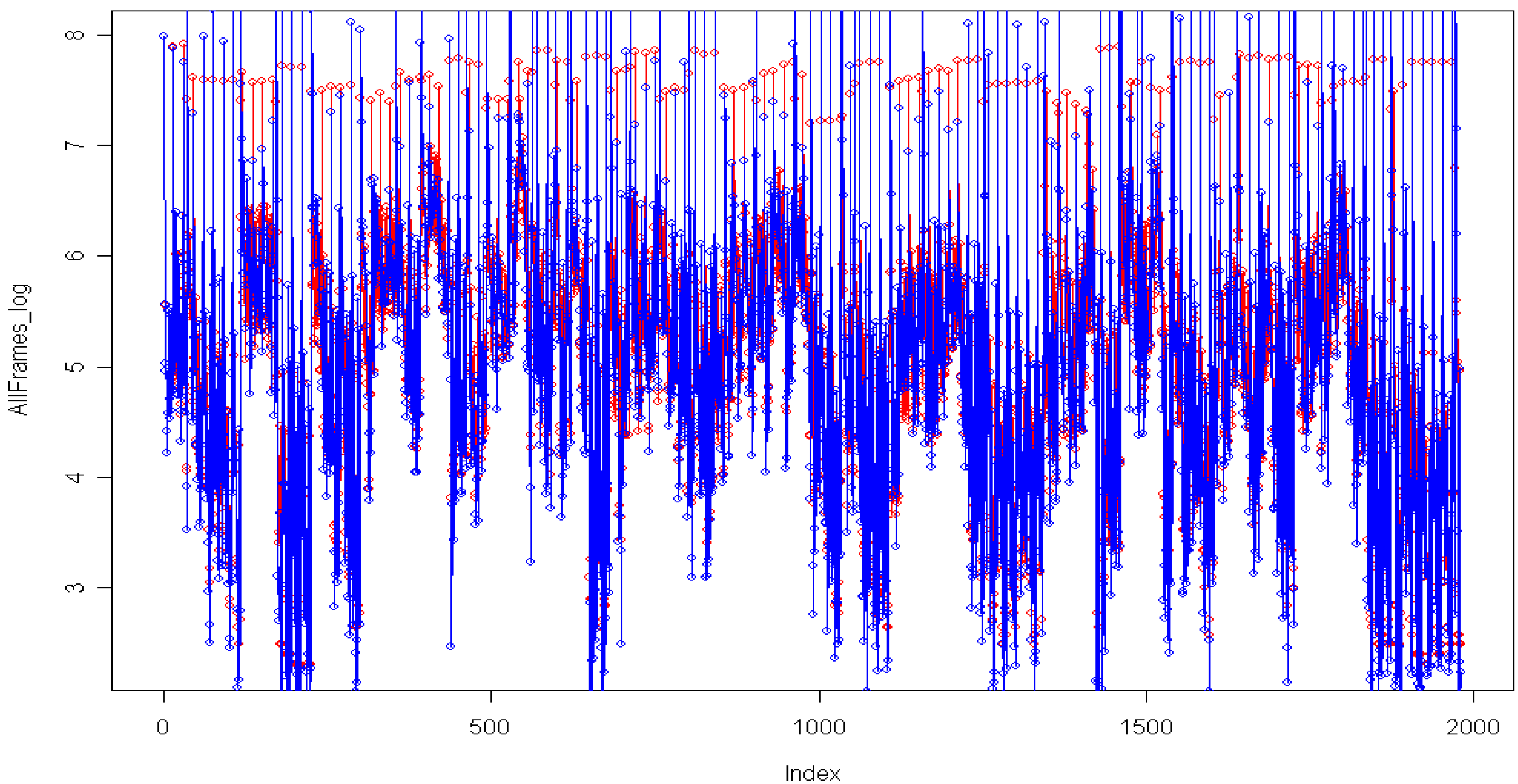
- The chosen seasonal ARIMA model is:
  - $(1,1,1) \times (1,1,1)_{15}$
- Coefficients :

	ar (1)	ma(1)	sar(1)	sma(1)
	0.5554	-0.8761	0.2256	-0.802
se	0.0508	0.0344	0.0358	0.023

Modeling All Frames: Log Seasonal Model

Blue : Fitted Model
Red : Original data

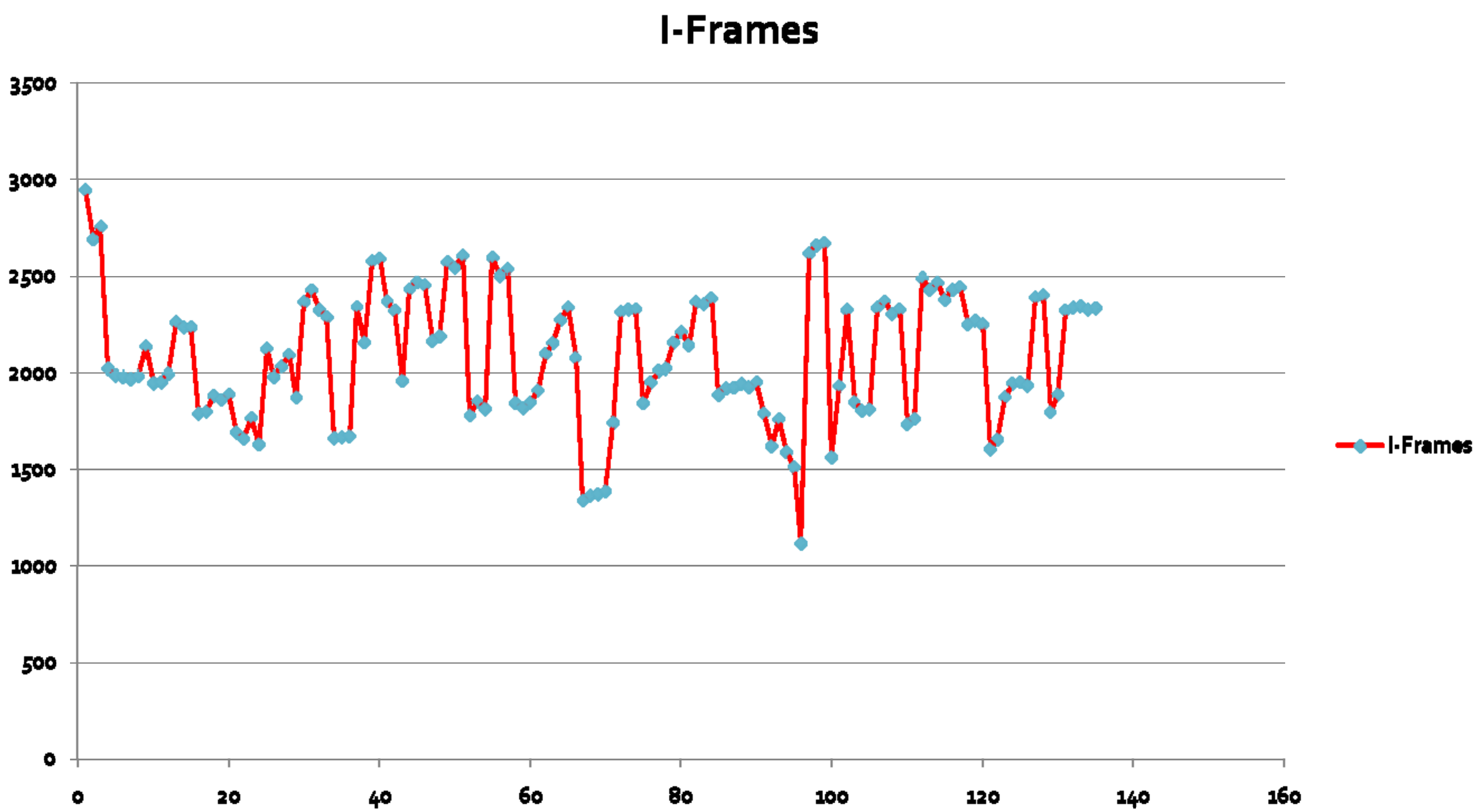
$R^2 = 0.99$



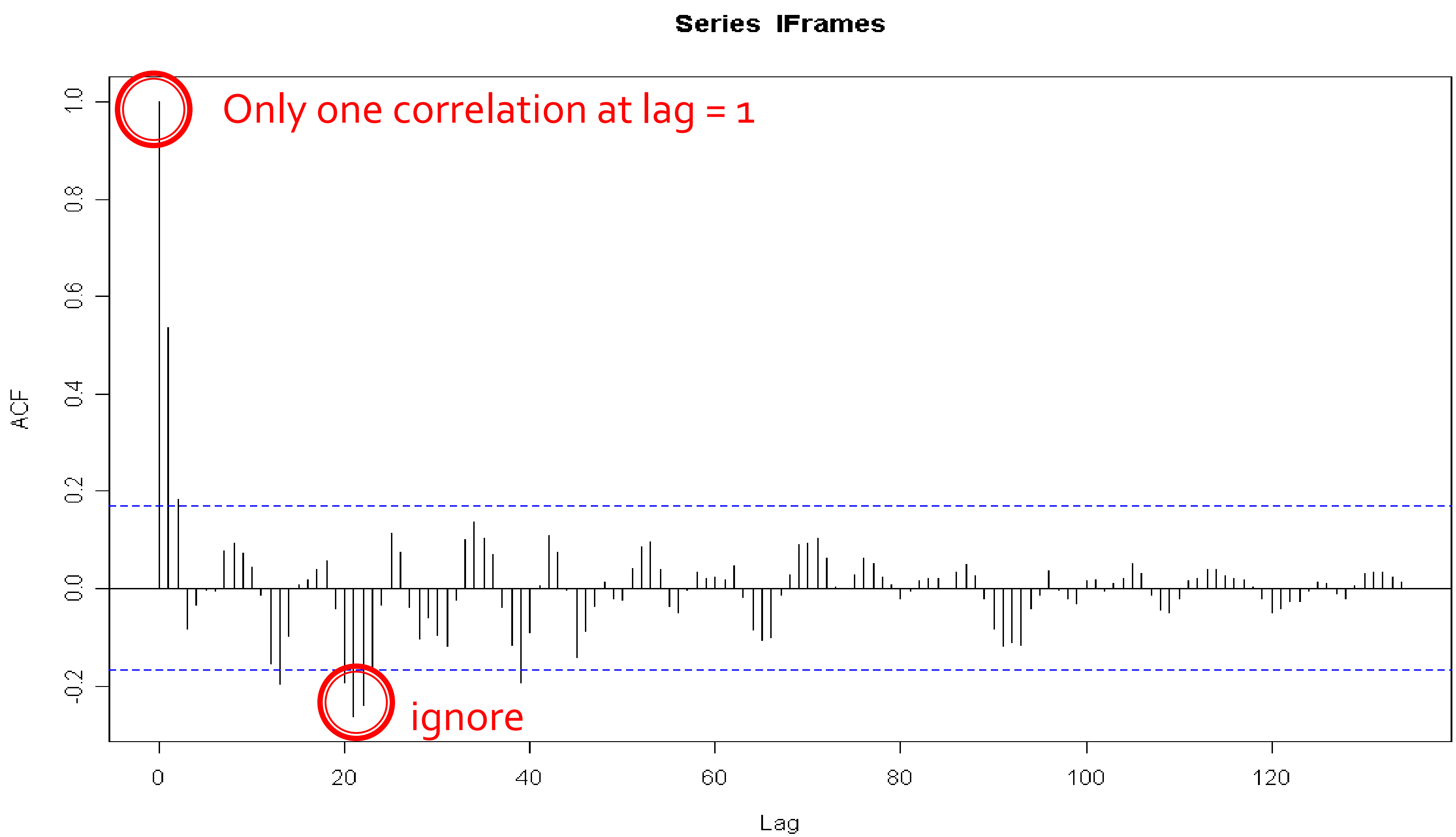
Modeling I, P, B Frames Separately

- Another approach to consider while modeling video traffic is to separate the three video frame types as individual models
  - Create separate models for I-Frames, P-Frames and B-Frames
  - Compare the final results with the one obtained from the previous approach

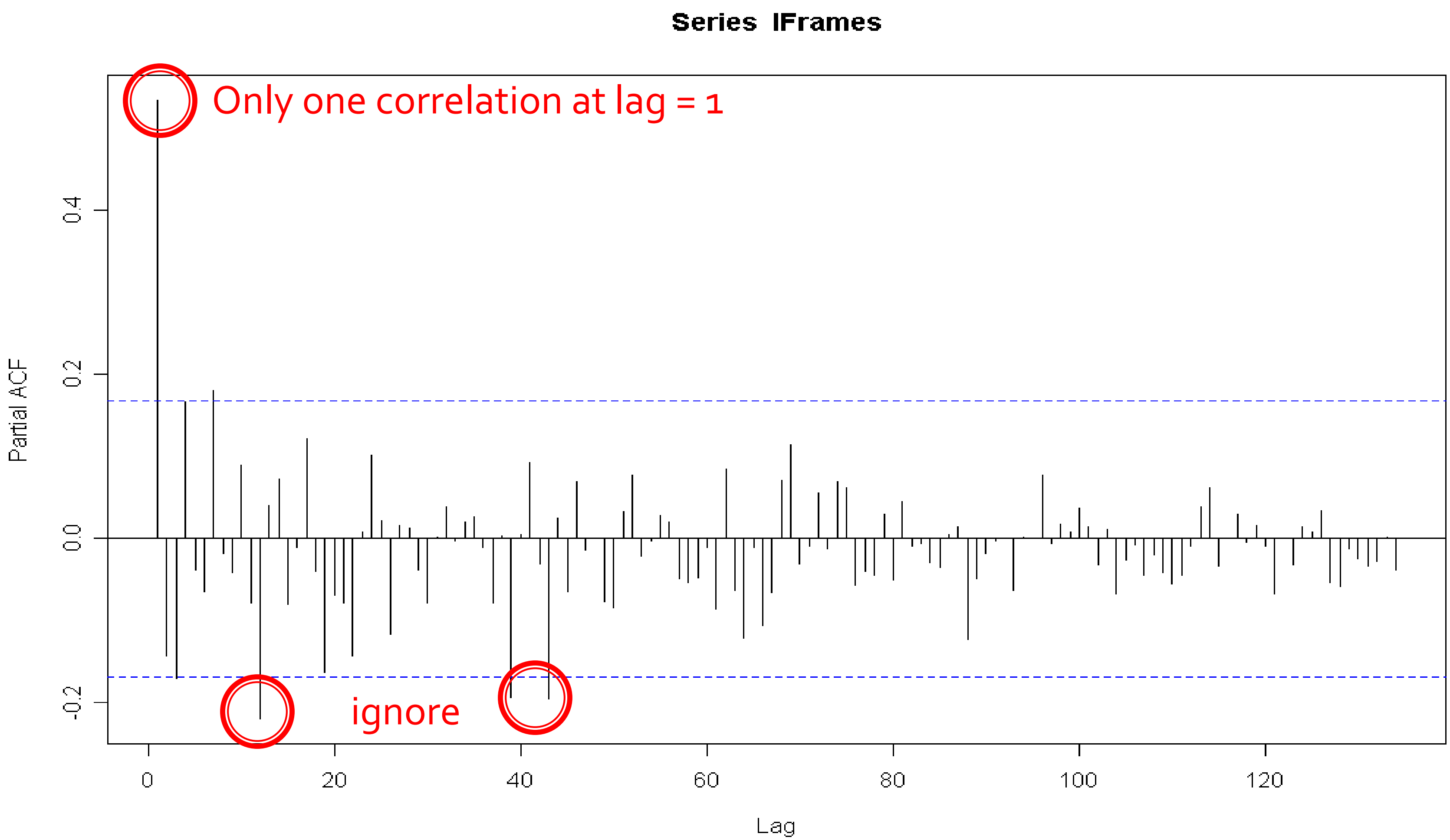
Modeling I Frames 1



Modeling I Frames 2



Modeling I Frames 3



Akaike's Information Criterion (AIC)

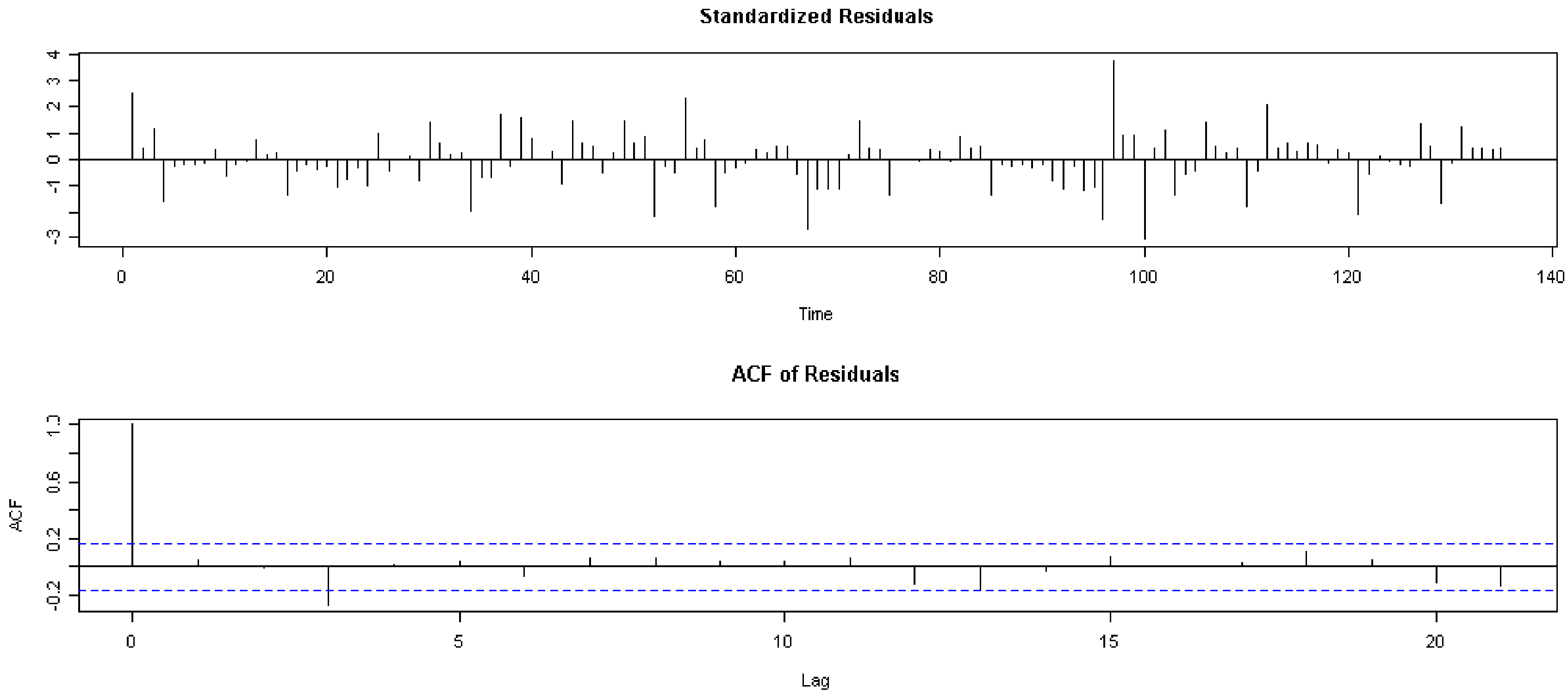
- The Proposed models are: ARIMA(0,0,1), ARIMA(1,0,0), ARIMA(1,0,1)
- Determine the best model based on **AIC (Akaike's Information Criterion)**
- AIC = 2k –2 ln L
  = 2k + n[ln(2π RSS/n) + 1]
  Here k is the number of parameters and L is the "goodness of the model"
- RSS = Residual sum of squares
- Lower AIC ⇒ Smaller number of parameters and lower errors

[1] : http://en.wikipedia.org/wiki/Akaike_information_criterion

Modeling I Frames 4

- Proposed models : ARIMA(0,0,1), ARIMA(1,0,0), ARIMA(1,0,1)
- Now we determine the best model based on **AIC** index (*lower is better*)
- ARIMA(0,0,1) [AIC] = 1922.87
- ARIMA(1,0,0) [AIC] = 1913.42 √
- ARIMA(1,0,1) [AIC] = 1913.82

Modeling I Frames 5



Modeling I Frames 6

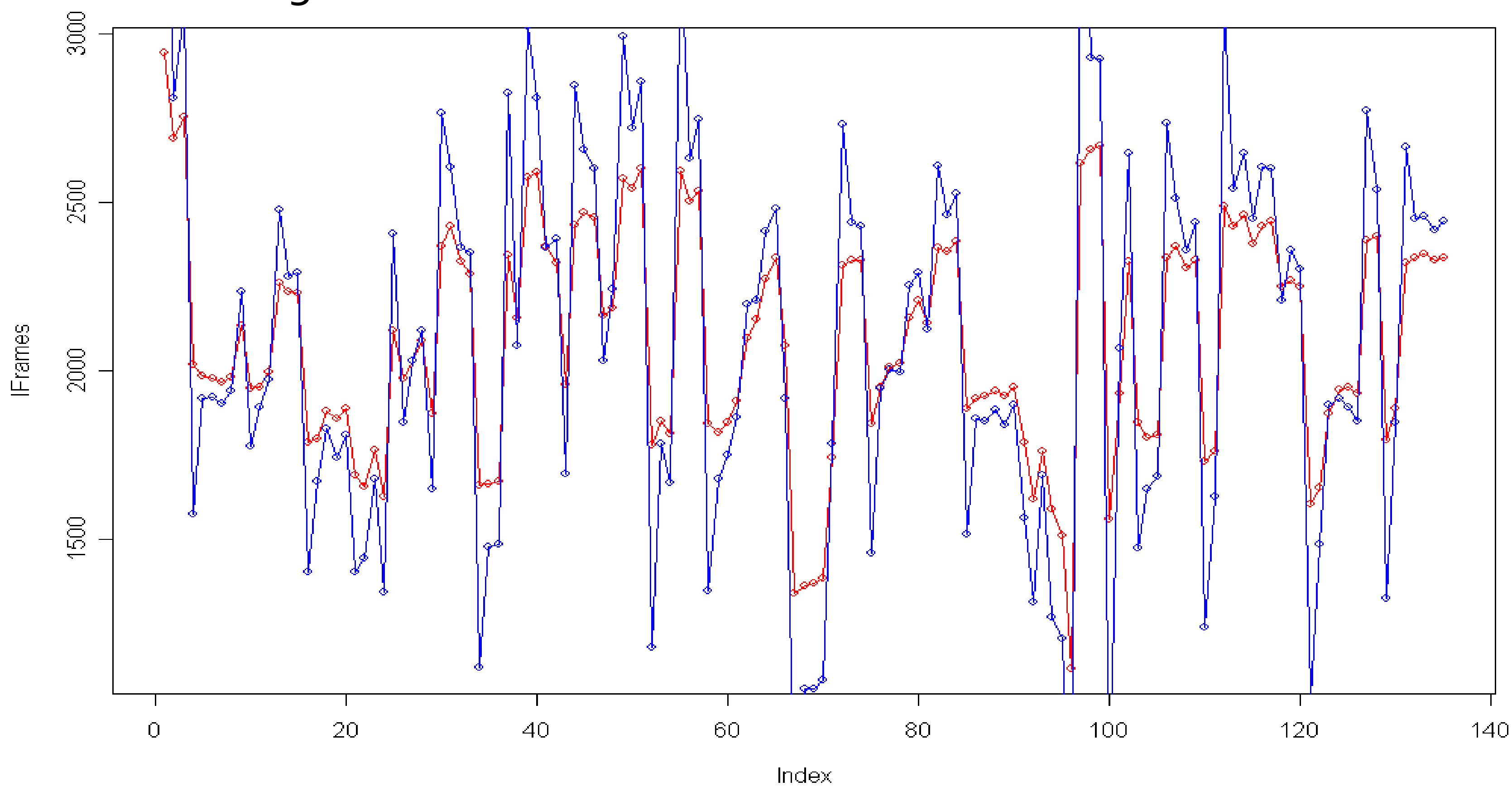
- Coefficients:

	ar (1)	intercepts
	0.5599	2097.352
se	0.0733	54.839

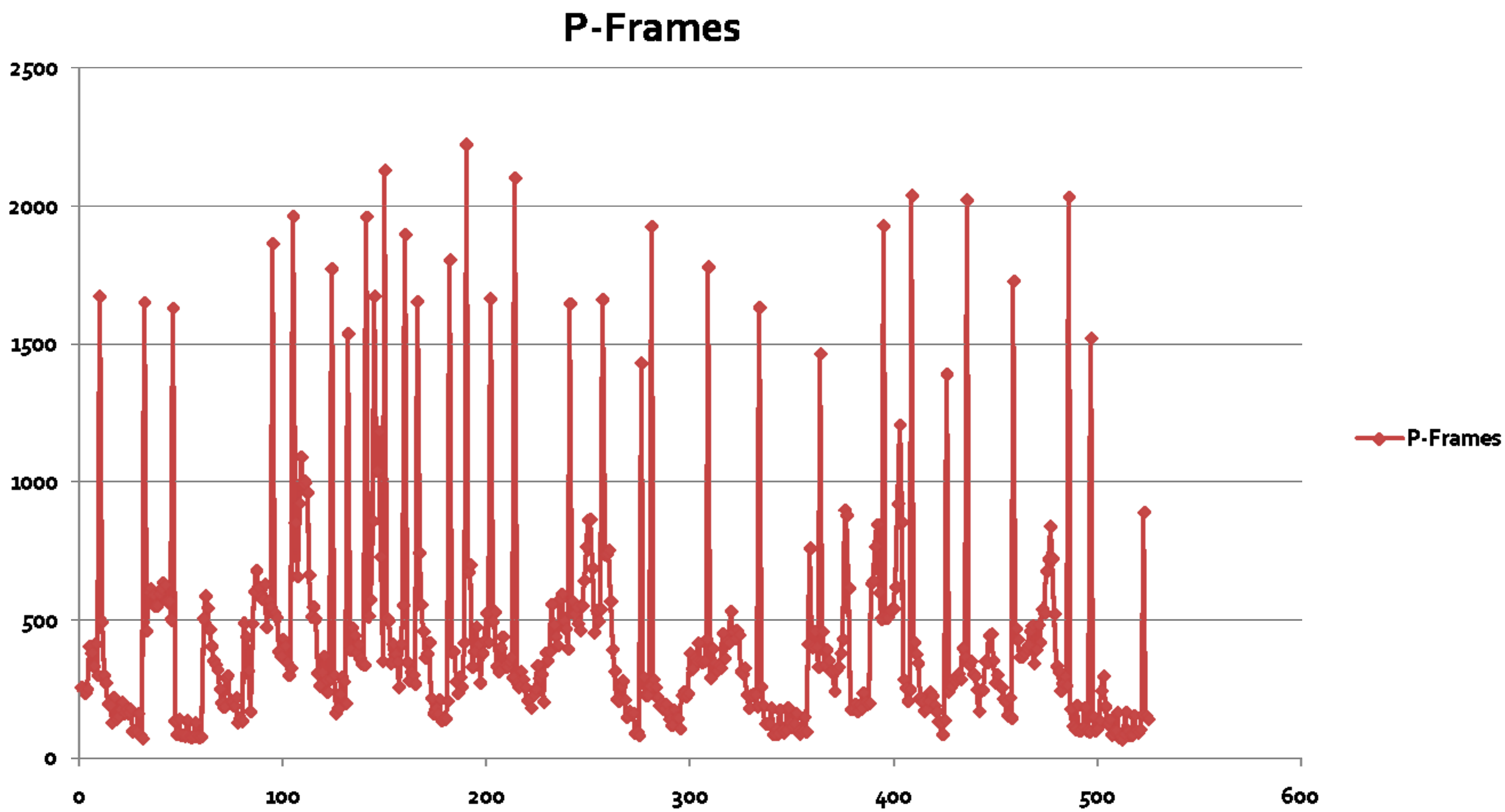
Modeling I Frames: Results

Blue : Fitted Model
Red : Original data

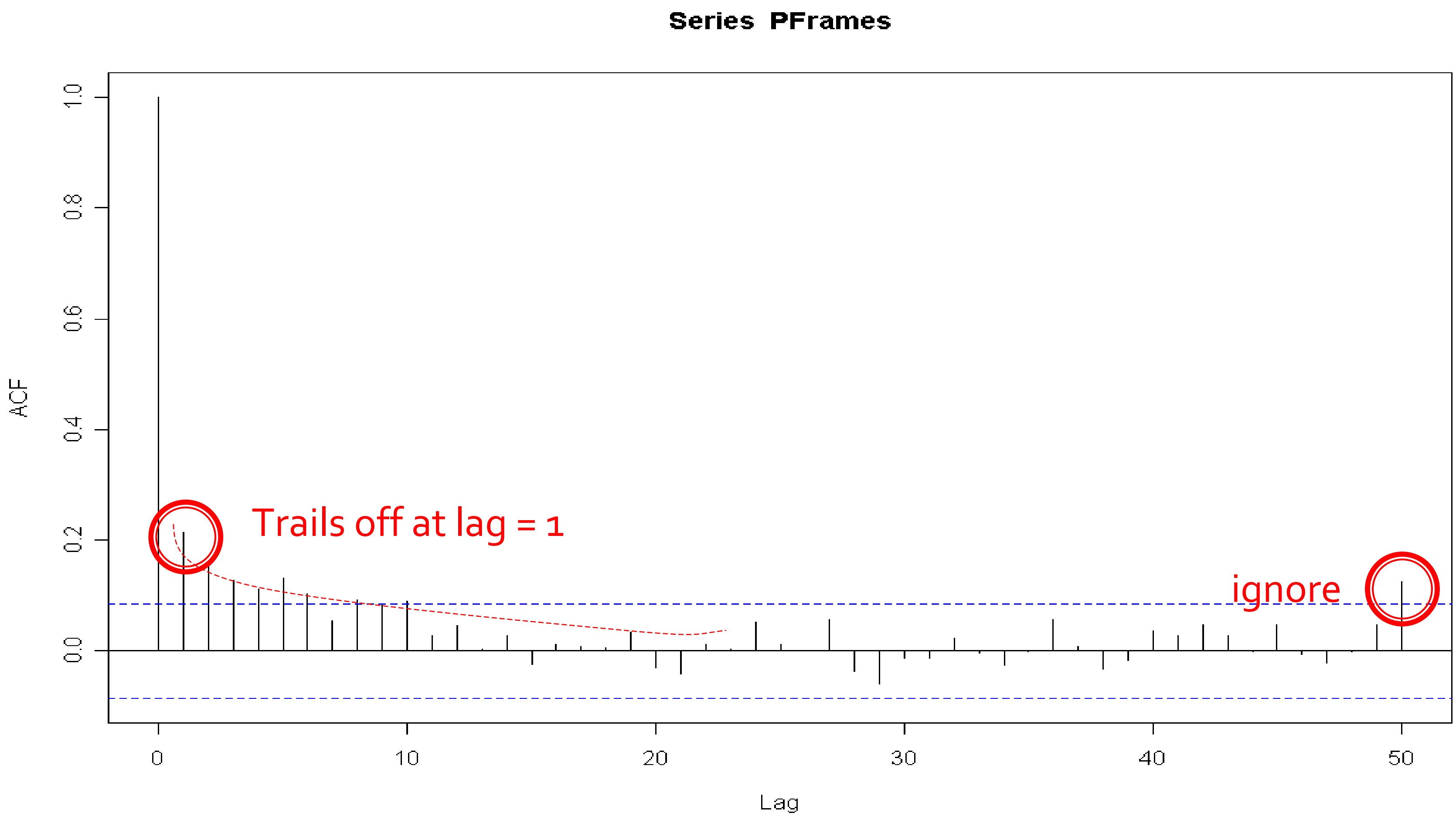
$R^2 = 0.98$



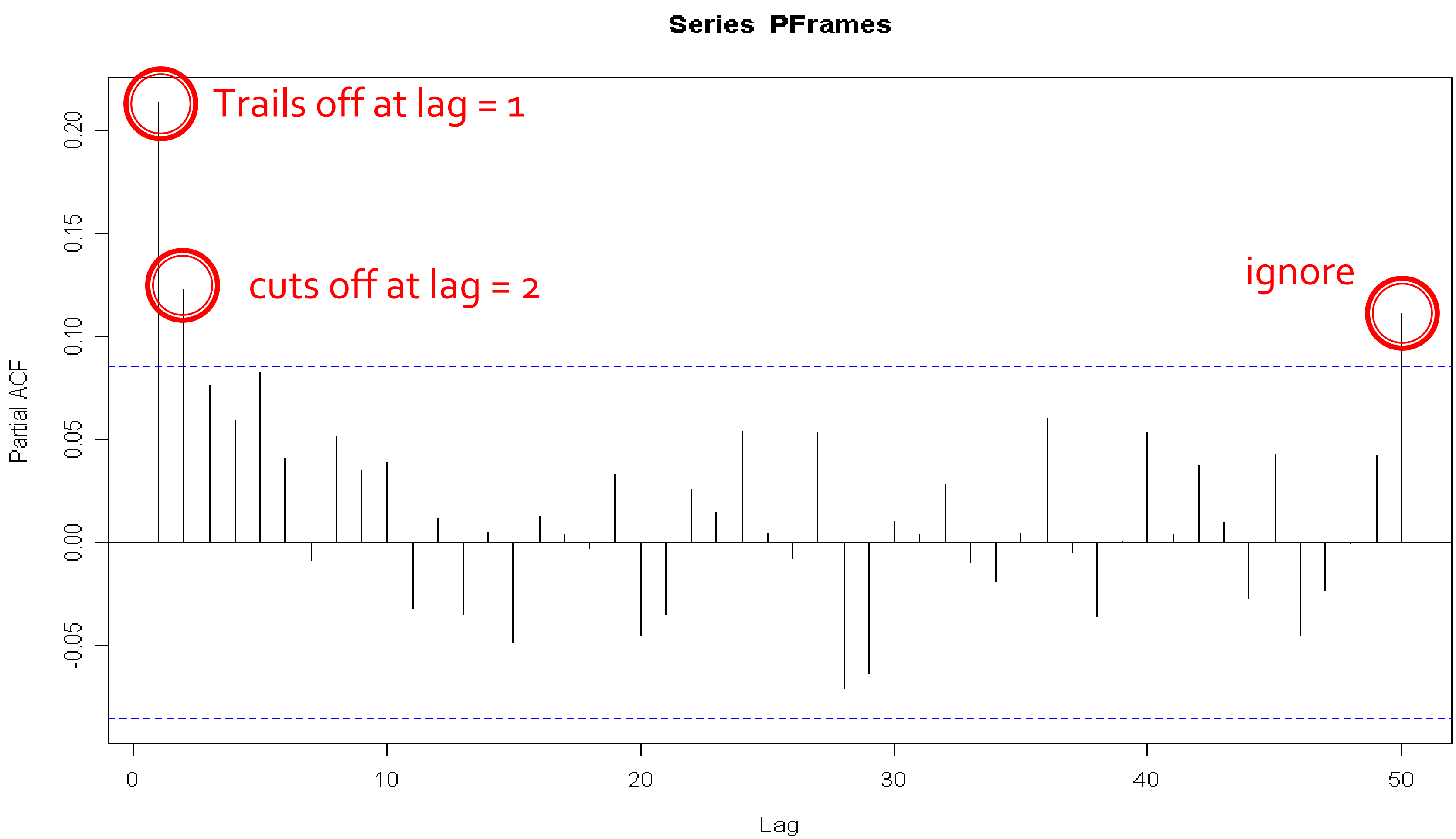
Modeling P Frames 1



Modeling P Frames 2



Modeling P Frames 3

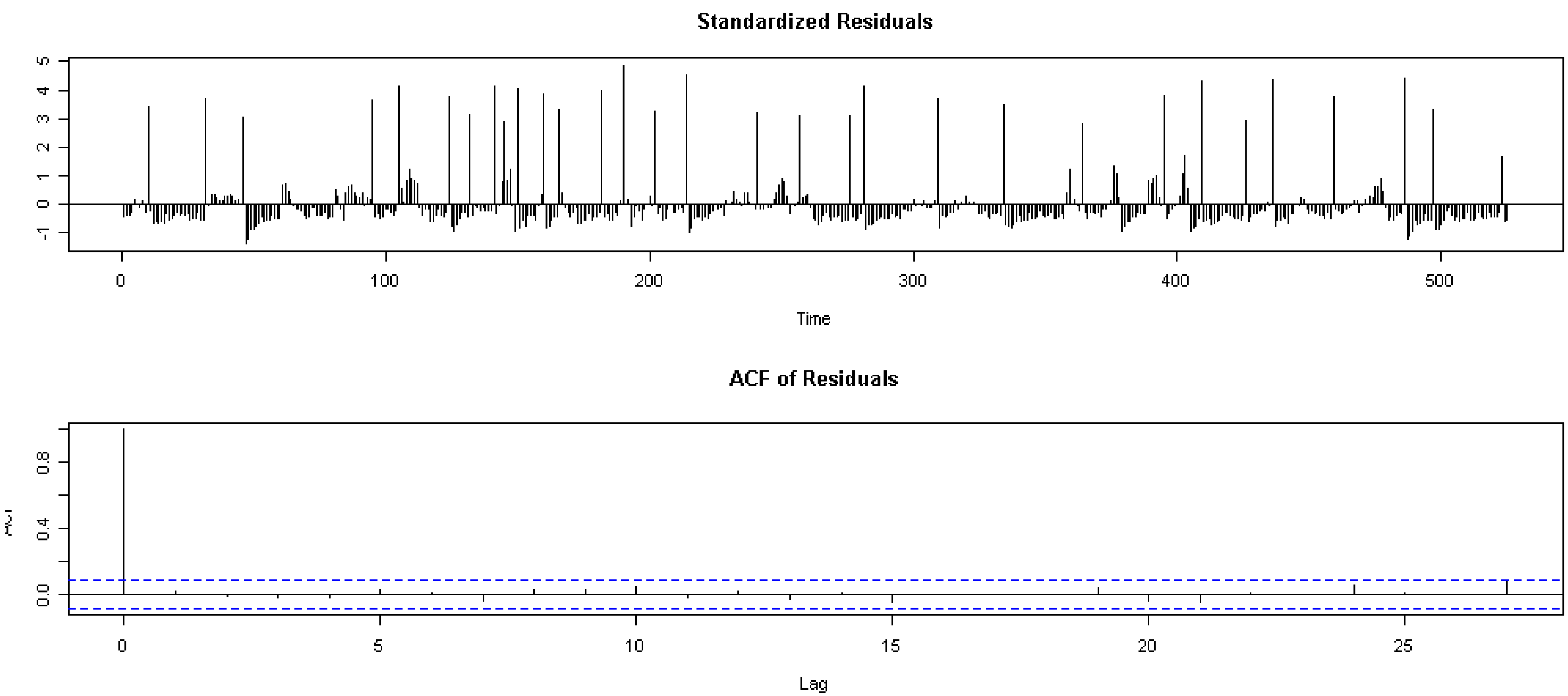


Modeling P Frames 4

- ACF : Trails off at lag =1
- PACF : Trails off at lag =1, cuts off at lag =2
- ARIMA models
  - ARIMA(1,0,1) ➔ AIC = 7724.41 √
  - ARIMA(2,0,0) ➔ AIC = 7733.21
- Model coefficients:

	ar (1)	ma(1)	intercepts
	0.4607	0.1479	2095.2637
se	0.1150	0.1126	51.1665

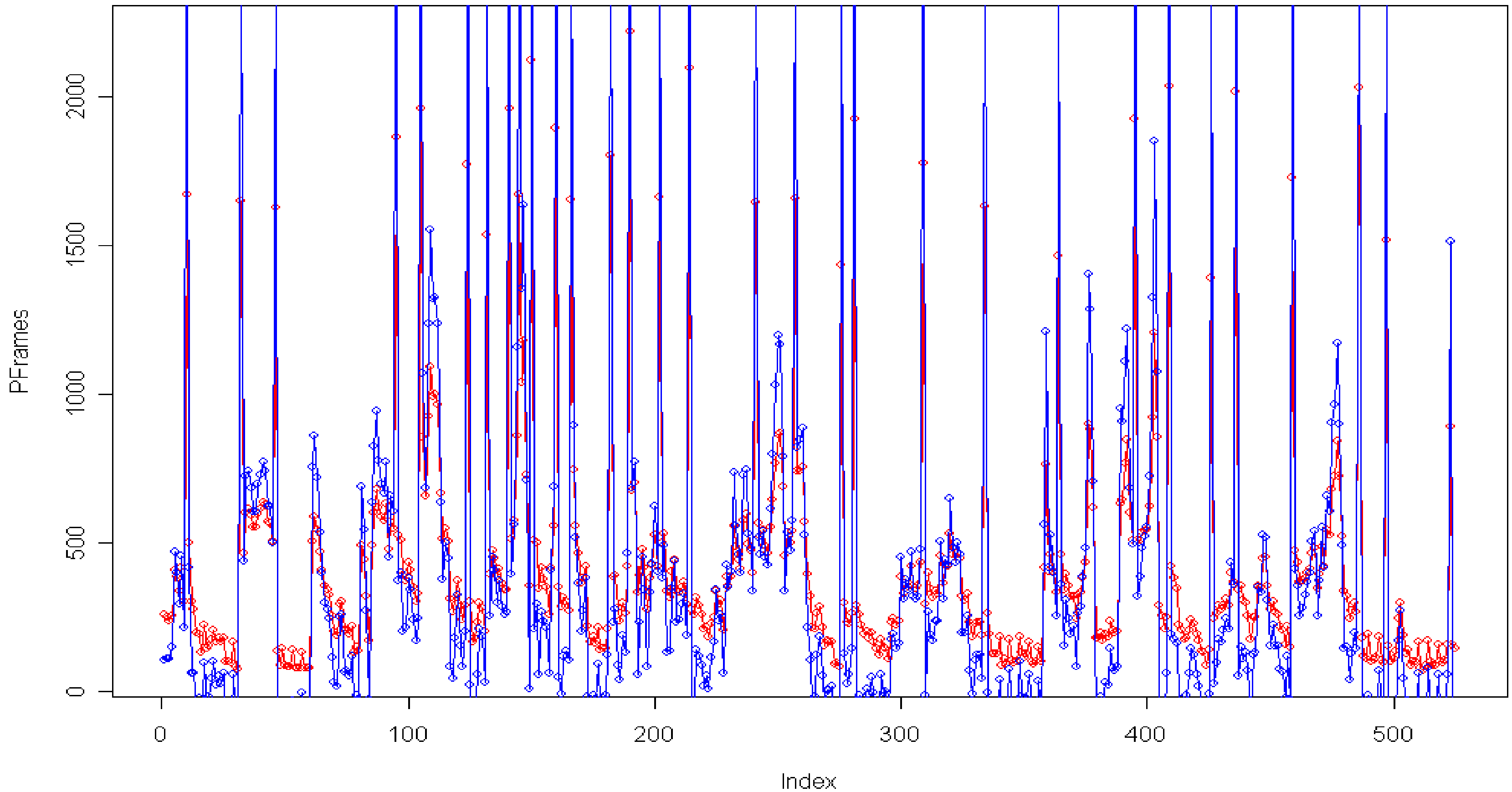
Modeling P Frames 5



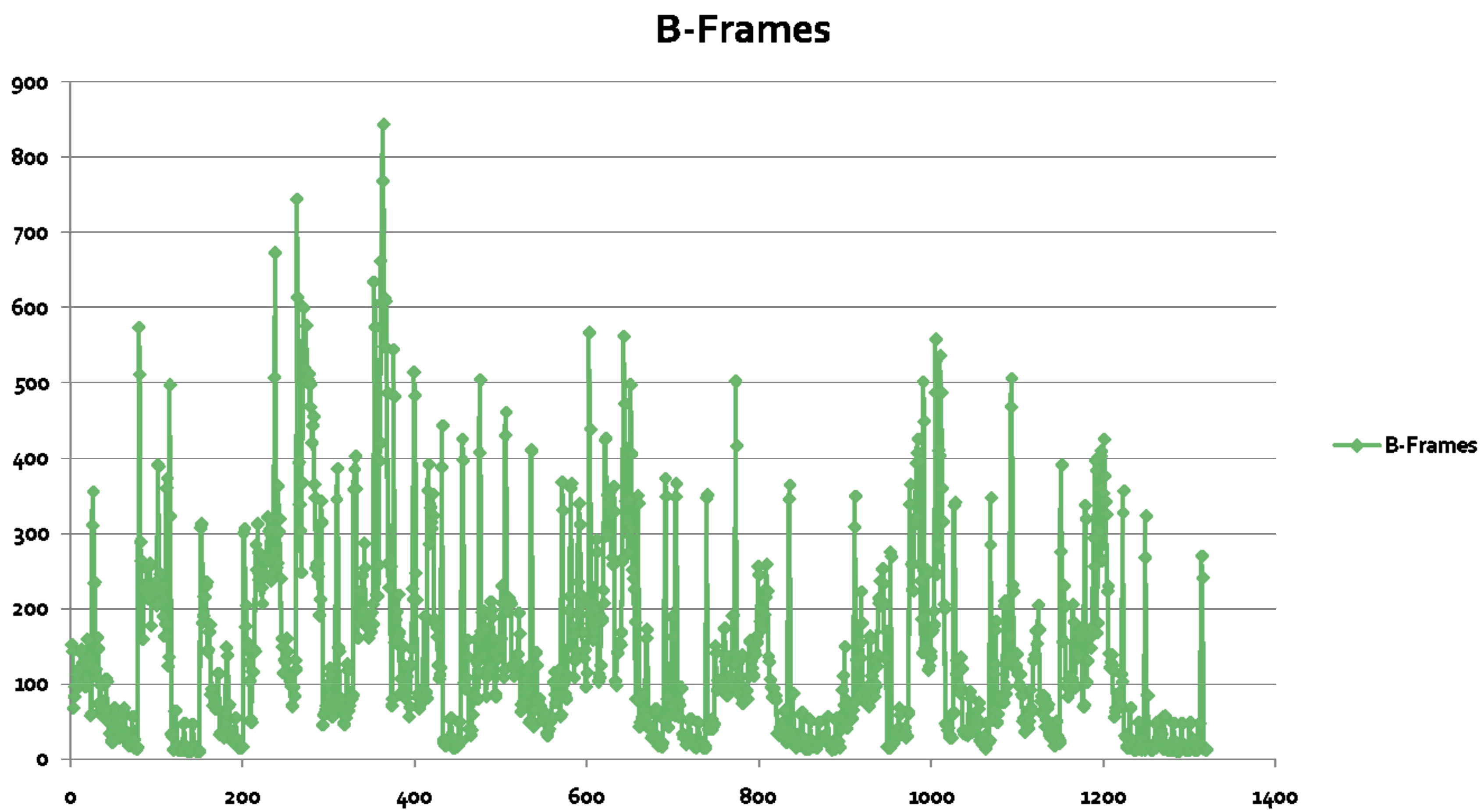
Modeling P Frames 6

Blue : Fitted Model
Red : Original data

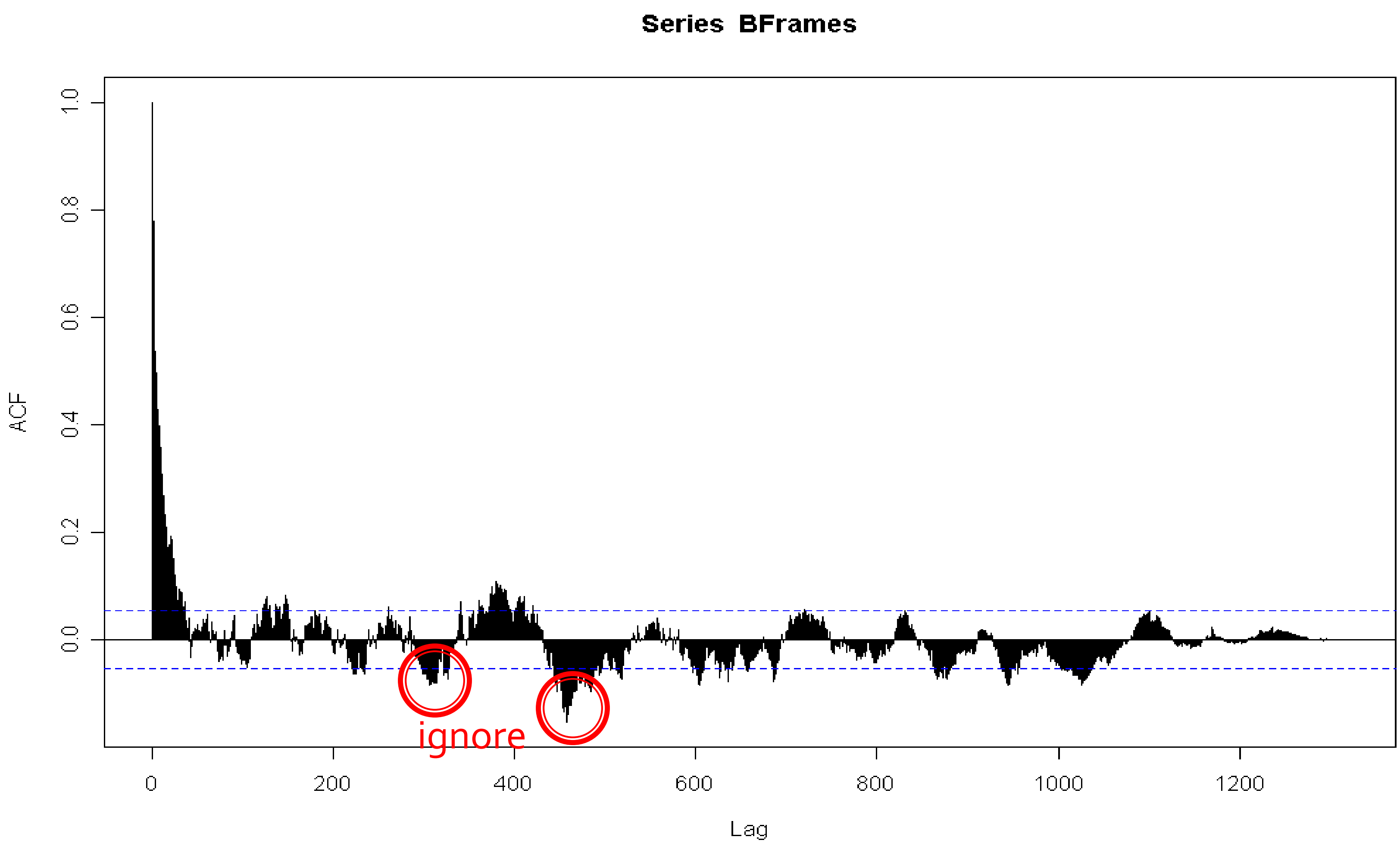
$R^2 = 0.81$



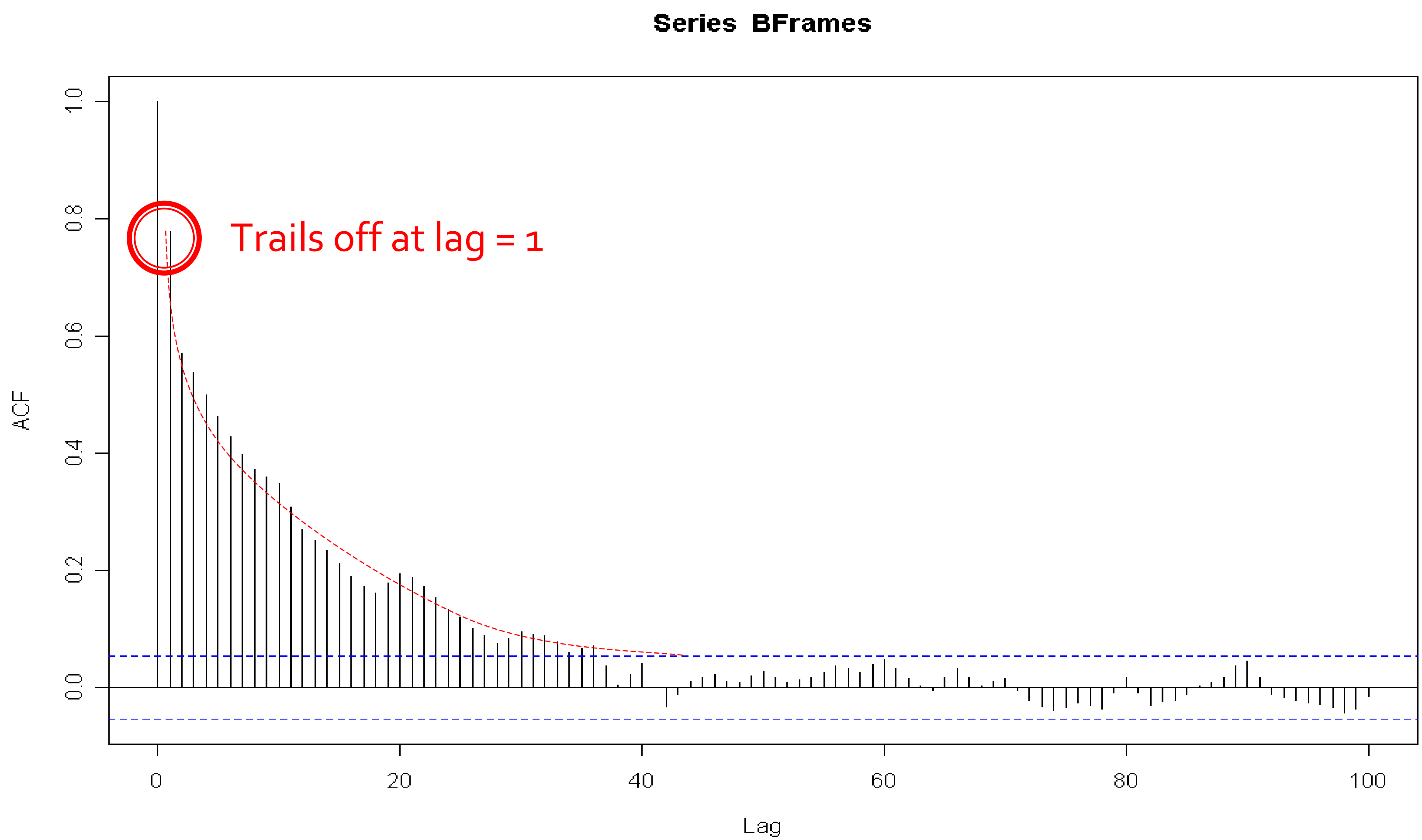
Modeling B Frames 1



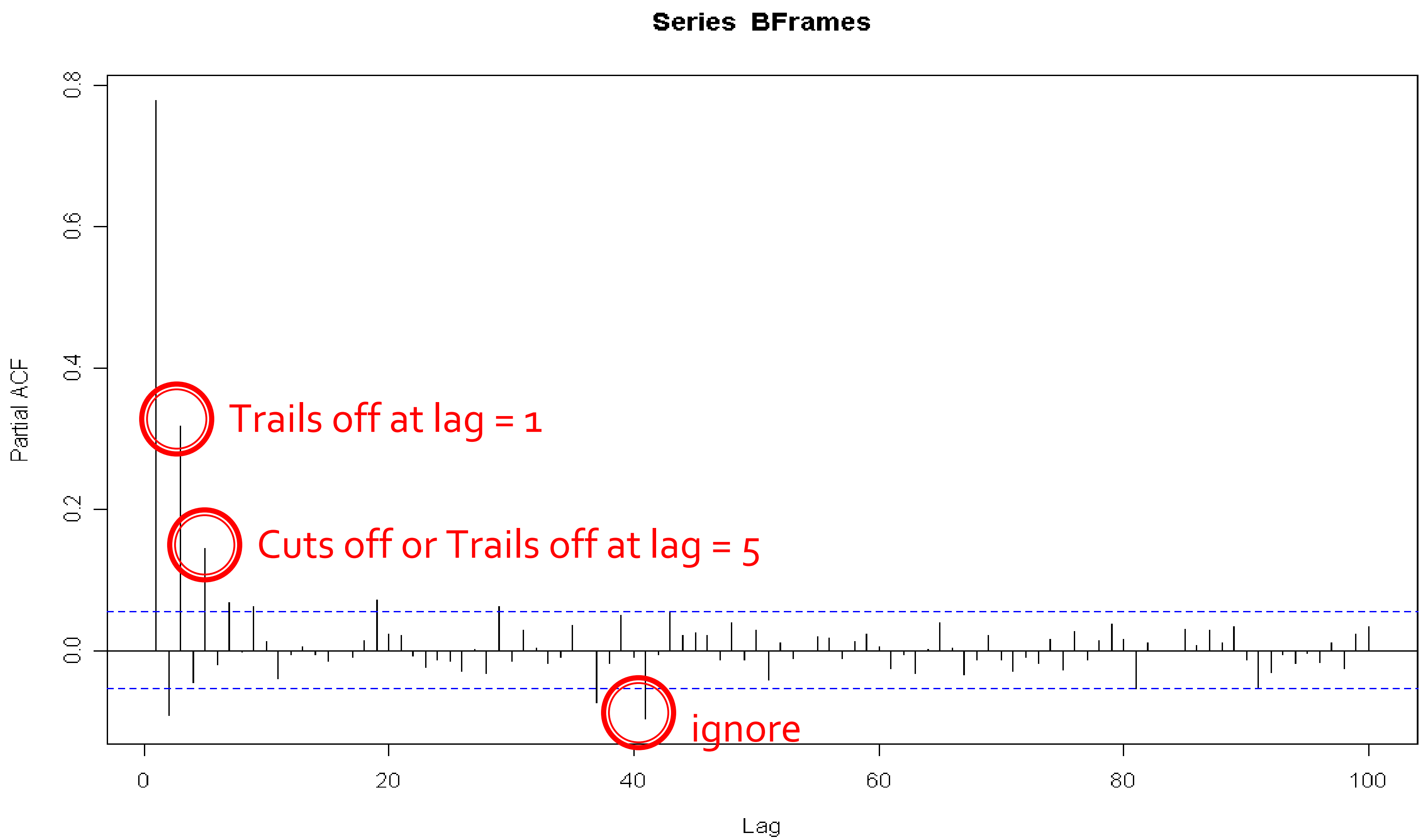
Modeling B Frames 2



Modeling B Frames 2



Modeling B Frames 3

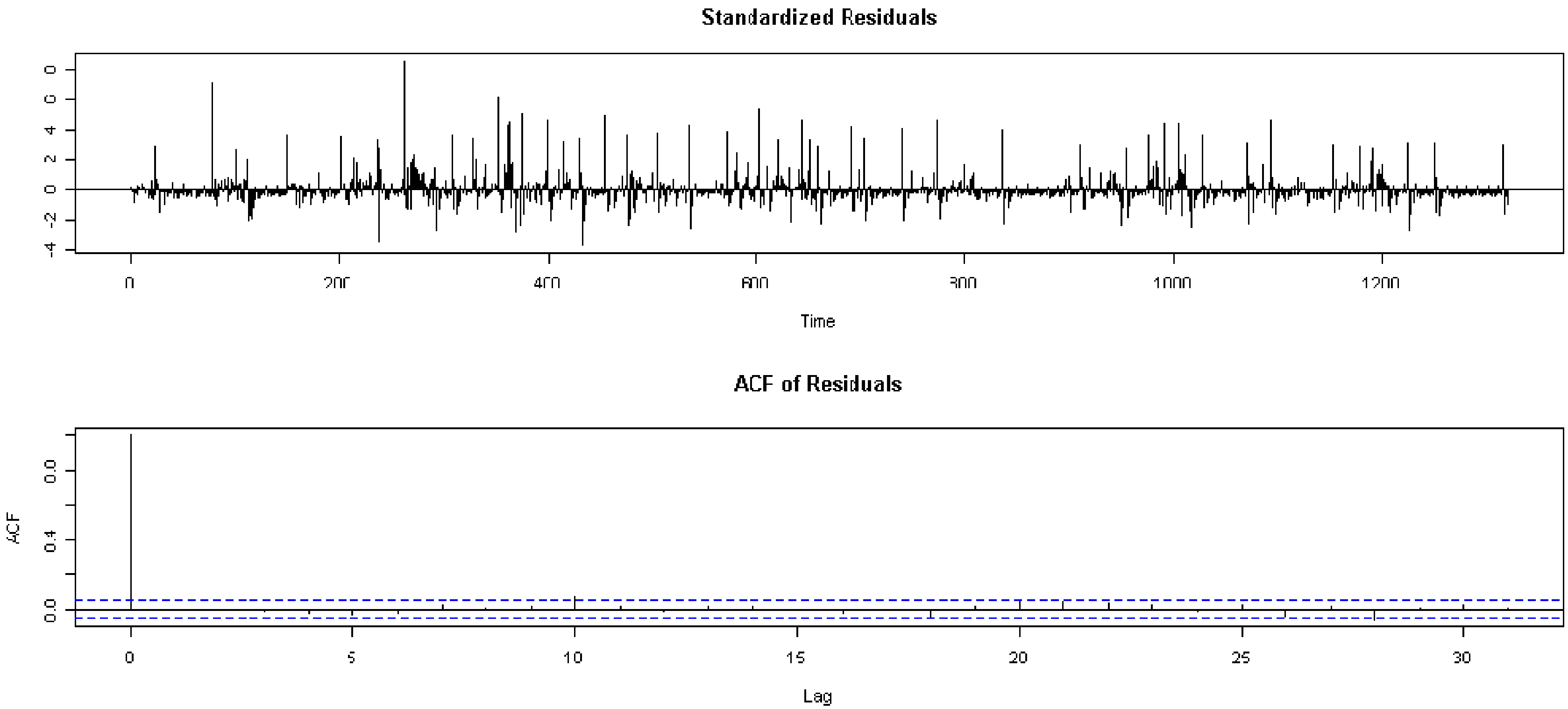


Modeling B Frames 4

- ARIMA Models
  - ARIMA(1,0,1) → AIC = 15315.06
  - ARIMA(5,0,0) → AIC = 15187.51
  - ARIMA(5,0,1) → AIC = 15186.66 √

	ar (1)	ar (2)	ar (3)	ar (4)	ar (5)	ma(1)	intercepts
	0.4318	-0.0072	0.2299	0.0046	0.1111	0.4791	137.6056
se	0.1768	0.1635	0.0770	0.0806	0.0351	0.1781	13.3287

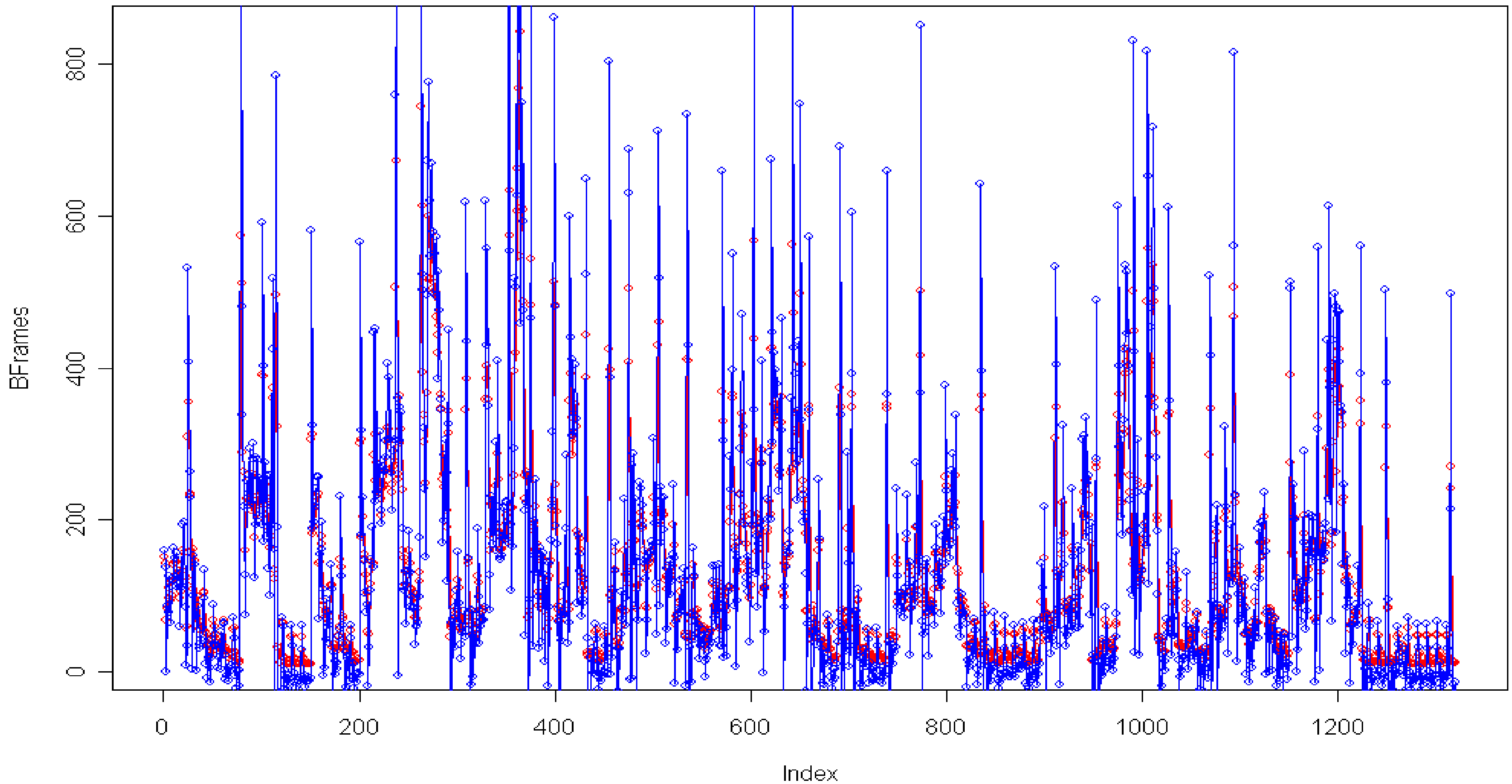
Modeling B Frames 5



Modeling B Frames 6

Blue : Fitted Model
Red : Original data

$R^2 = 0.89$



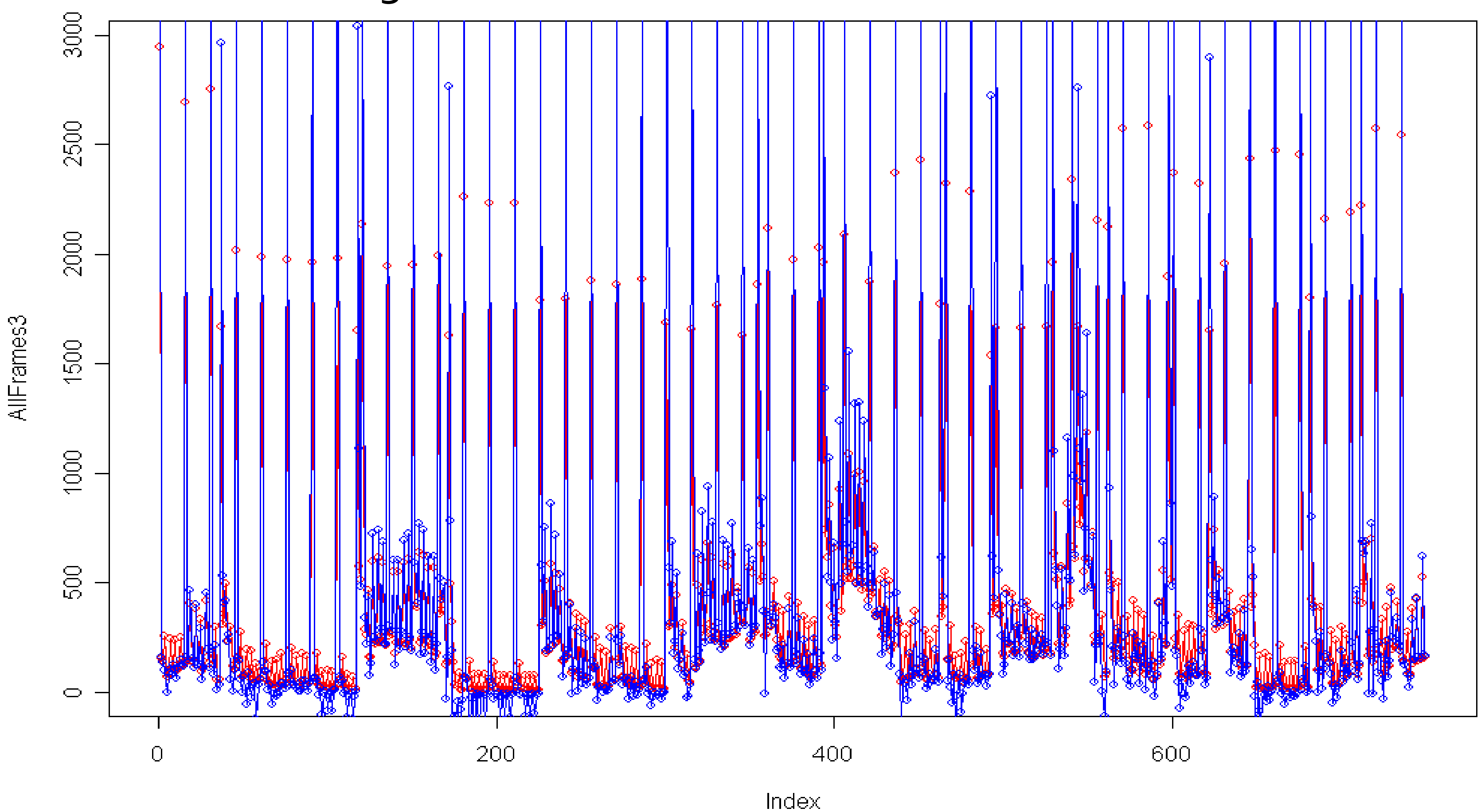
Combining I, P, B Models 1

- To get a full model for all frames, a combination of the model will be presented.
- Points to consider:
  - The three models represented for I-Frames, P-Frames and B-Frames did not show the seasonal trend
  - Models are easier to implement than the All-Frames models

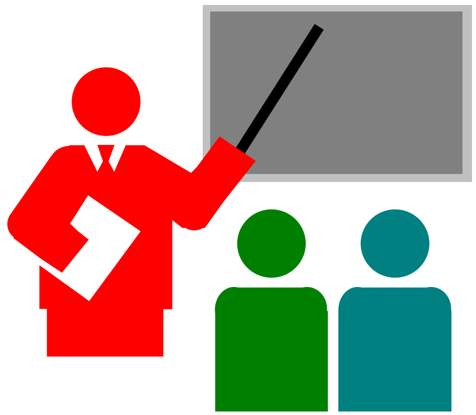
Combining I, P, B Models 2

Blue : Fitted Model
Red : Original data

$R^2 = 0.82$



Summary



- In this presentation we showed that a good model for video traffic is achievable using ARIMA model
- Seasonal ARIMA is a better approach to get a better video traffic model
- Creating individual models for each frame type is not necessary a better approach

Future Work

- Study and evaluate more video traces considering different encoding settings, GOP sizes, and video scene nature
- Present a traffic model the represents the variety of video streaming traffic
- Implement the model in NS2
- Present simulation results