

# Introduction to Wireless Coding and Modulation



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Audio/Video recordings of this class lecture are available at:

<http://www.cse.wustl.edu/~jain/cse574-14/>

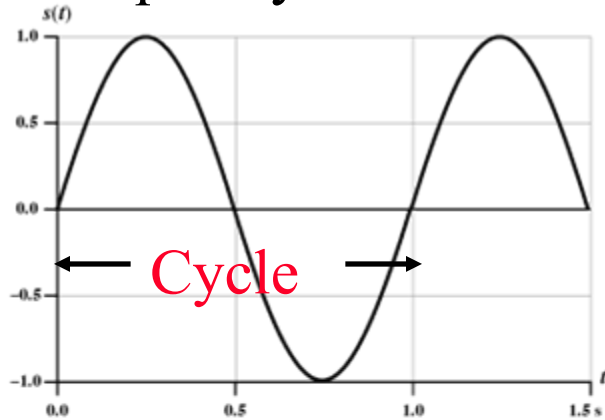


1. Frequency, Wavelength, and Phase
2. Electromagnetic Spectrum
3. Coding and modulation
4. Shannon's Theorem
5. Hamming Distance
6. Multiple Access Methods: CDMA
7. Doppler Shift

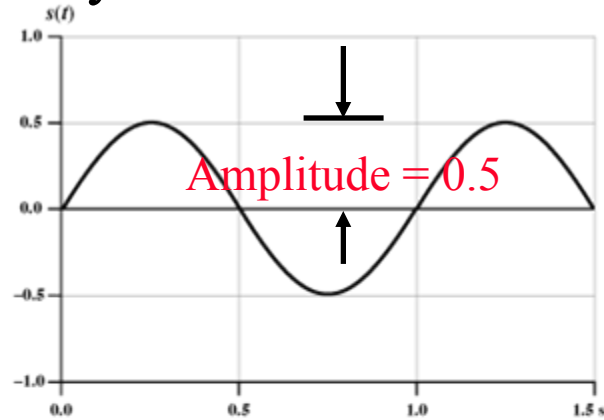
Note: This is the 1<sup>st</sup> in a series of 2 lectures on wireless physical layer. Signal Propagation, OFDM, and MIMO are covered in the next lecture.

# Frequency, Period, and Phase

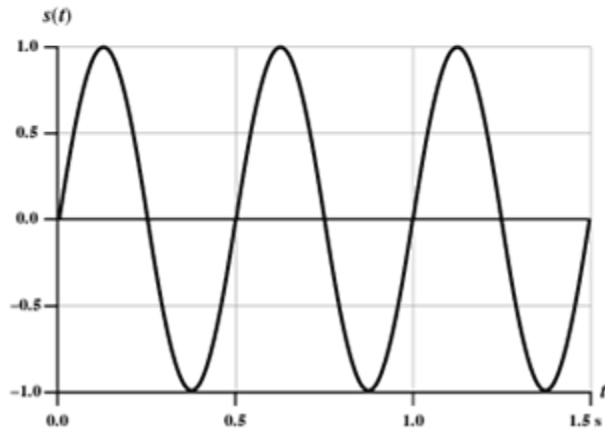
- $A \sin(2\pi ft + \theta)$ ,  $A$  = Amplitude,  $f$  = Frequency,  $\theta$  = Phase, Period  $T = 1/f$ , Frequency is measured in Cycles/sec or **Hertz**



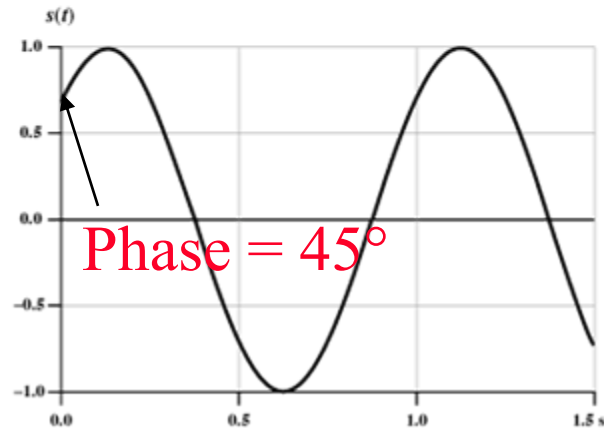
(a)  $A = 1, f = 1, \phi = 0$



(b)  $A = 0.5, f = 1, \phi = 0$



(c)  $A = 1, f = 2, \phi = 0$



(d)  $A = 1, f = 1, \phi = \pi/4$

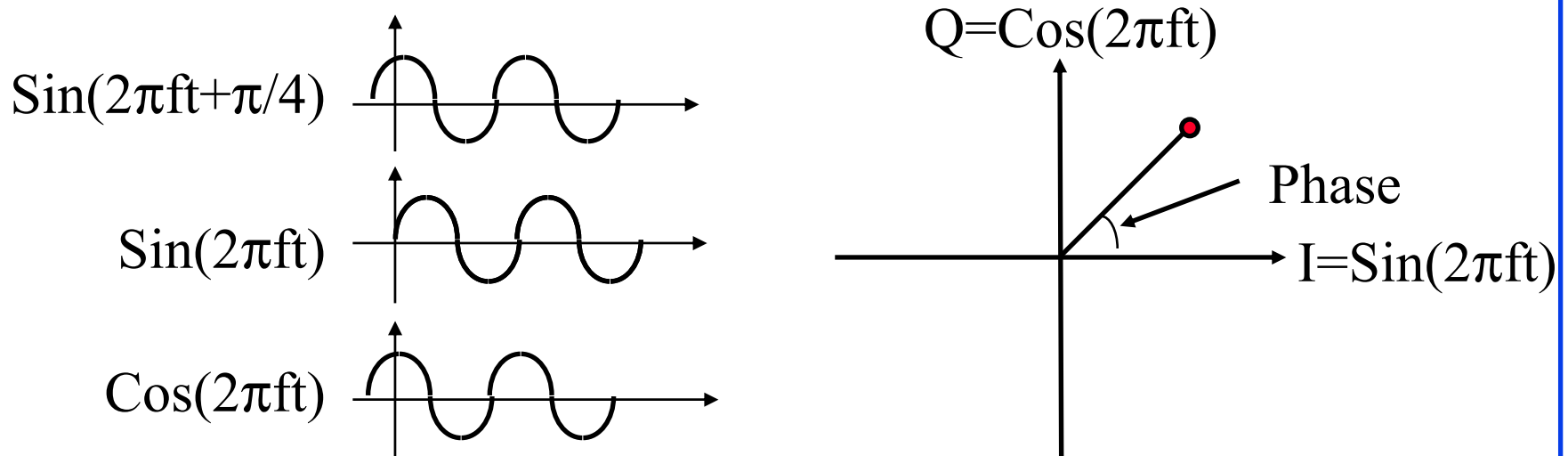


# Phase

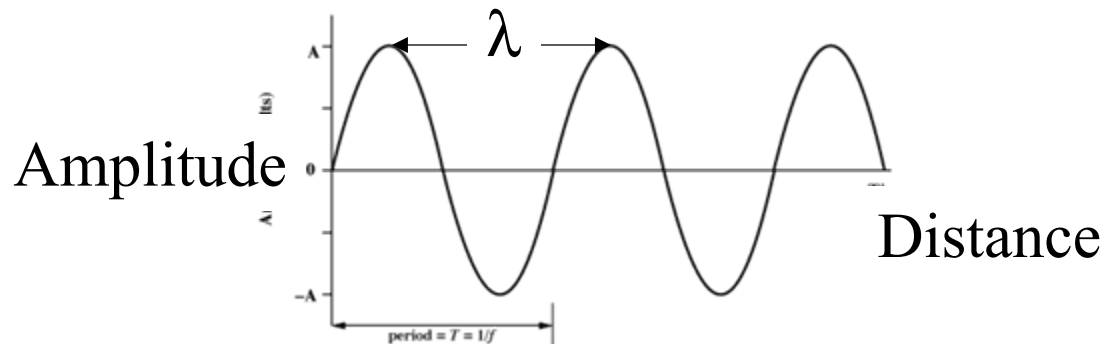
## □ Sine wave with a phase of $45^\circ$

$$\begin{aligned}\sin(2\pi ft + \frac{\pi}{4}) &= \sin(2\pi ft) \cos(\frac{\pi}{4}) + \cos(2\pi ft) \sin(\frac{\pi}{4}) \\ &= \frac{1}{\sqrt{2}} \sin(2\pi ft) + \frac{1}{\sqrt{2}} \cos(2\pi ft)\end{aligned}$$

In-phase component I + Quadrature component Q



# Wavelength



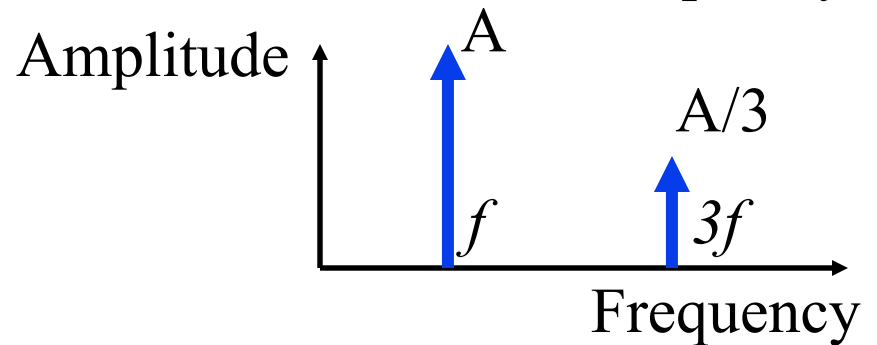
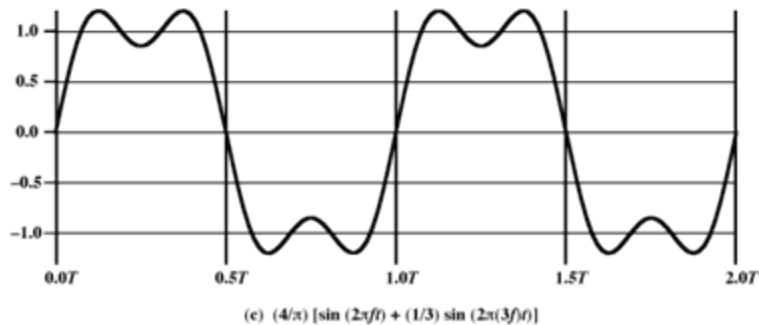
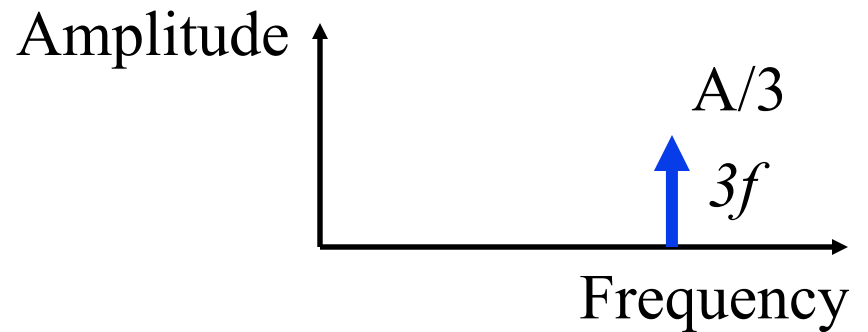
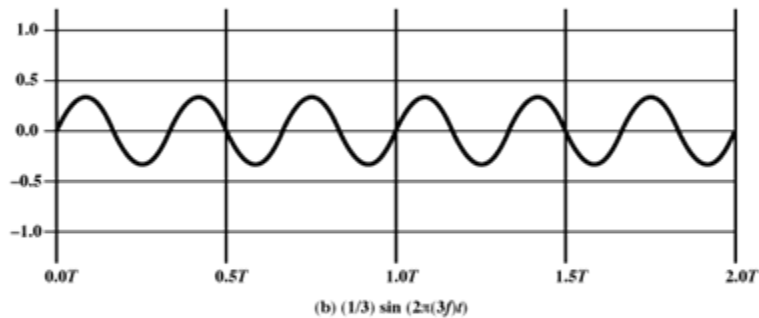
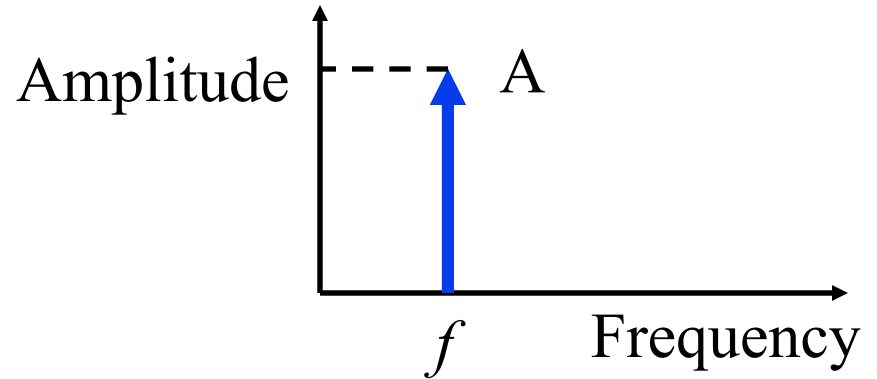
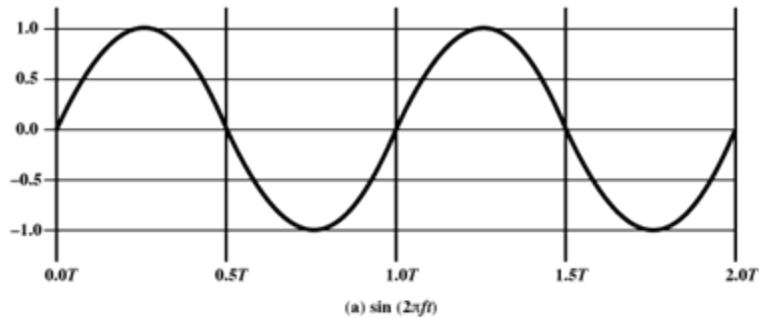
- ❑ Distance occupied by one cycle
- ❑ Distance between two points of corresponding phase in two consecutive cycles
- ❑ Wavelength =  $\lambda$
- ❑ Assuming signal velocity  $v$ 
  - $\lambda = vT$
  - $\lambda f = v$
  - $c = 3 \times 10^8 \text{ m/s}$  (speed of light in free space) = *300 m/μs*

# Example

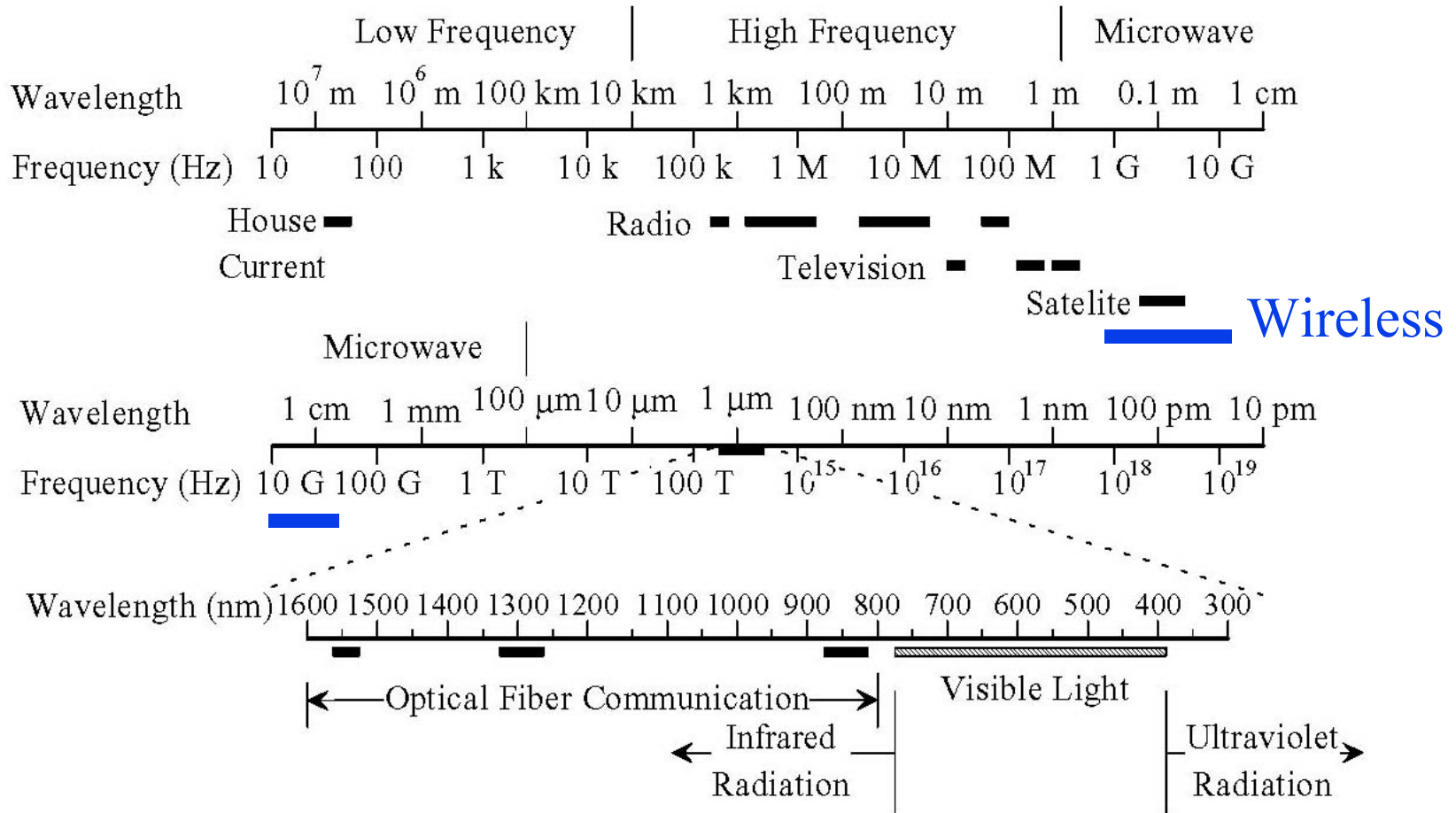
- Frequency = 2.5 GHz

$$\begin{aligned}\text{Wavelength} = \lambda &= \frac{c}{f} \\ &= \frac{300 \text{ m}/\mu\text{s}}{2.5 \times 10^9} \\ &= 120 \times 10^{-3} = 120 \text{ mm} = 12 \text{ cm}\end{aligned}$$

# Time and Frequency Domains



# Electromagnetic Spectrum



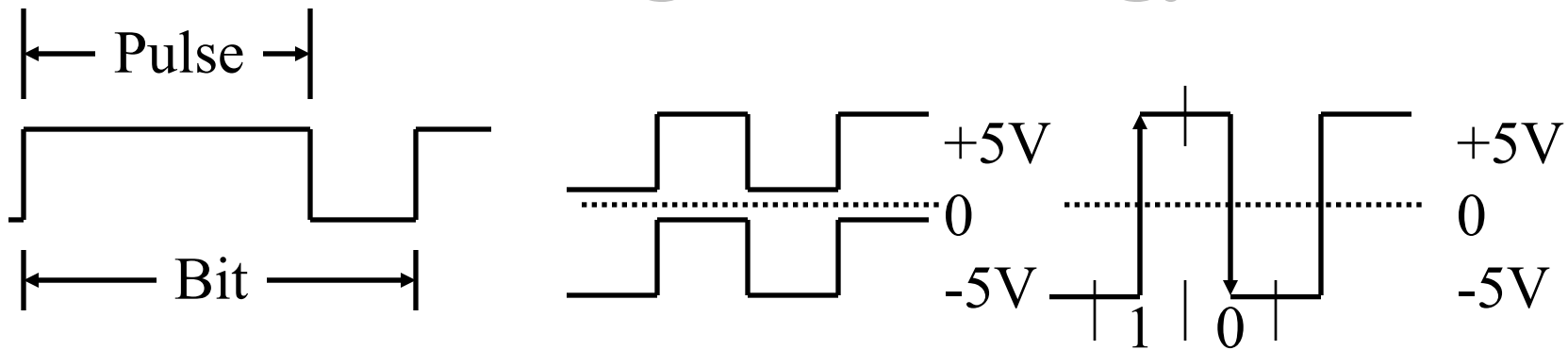
❑ Wireless communication uses 100 kHz to 60 GHz



# Decibels

- ❑ Attenuation =  $\text{Log}_{10} \frac{P_{in}}{P_{out}}$  Bel
- ❑ Attenuation =  $10 \text{ Log}_{10} \frac{P_{in}}{P_{out}}$  decibel
- ❑ Attenuation =  $20 \text{ Log}_{10} \frac{V_{in}}{V_{out}}$  decibel
- ❑ **Example 1:**  $P_{in} = 10 \text{ mW}$ ,  $P_{out} = 5 \text{ mW}$   
Attenuation =  $10 \log_{10} (10/5) = 10 \log_{10} 2 = 3 \text{ dB}$
- ❑ **Example 2:**  $P_{in} = 100 \text{ mW}$ ,  $P_{out} = 1 \text{ mW}$   
Attenuation =  $10 \log_{10} (100/1) = 10 \log_{10} 100 = 20 \text{ dB}$

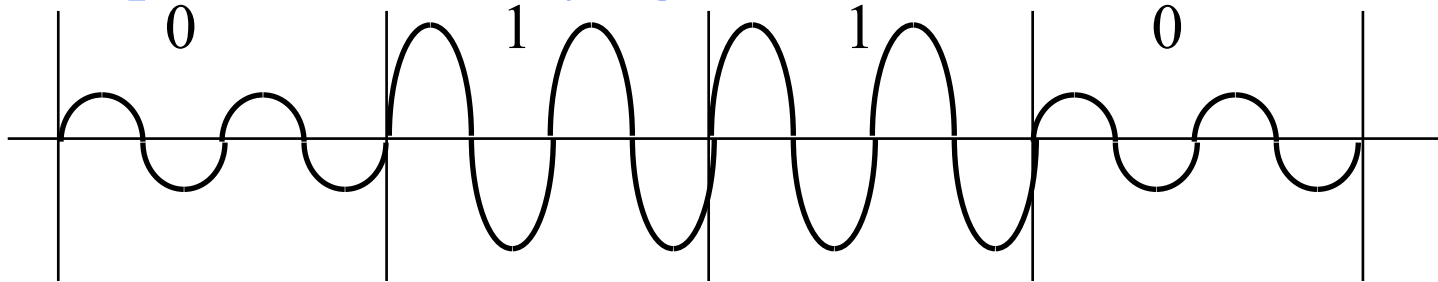
# Coding Terminology



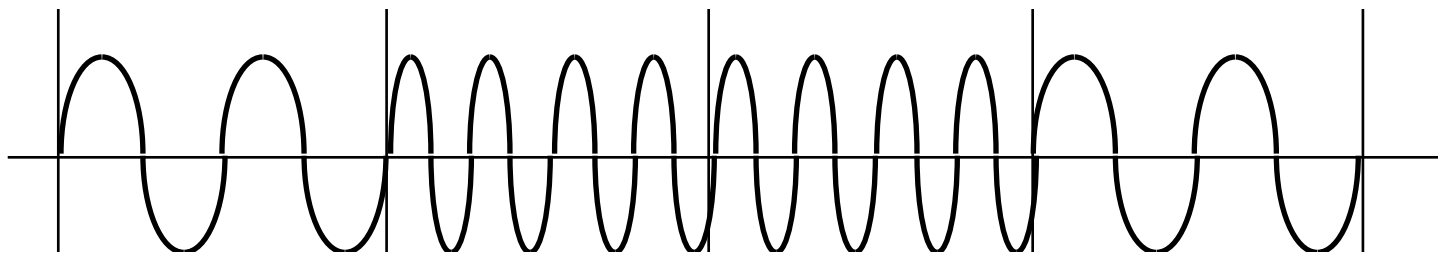
- **Signal element:** Pulse (of constant amplitude, frequency, phase) = **Symbol**
- **Modulation Rate:** 1/Duration of the smallest element = Baud rate
- **Data Rate:** Bits per second

# Modulation

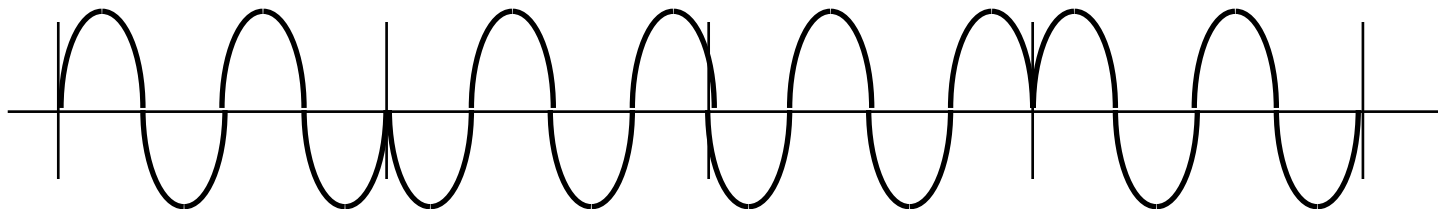
- ❑ Digital version of modulation is called **keying**
- ❑ **Amplitude Shift Keying (ASK):**



- ❑ **Frequency Shift Keying (FSK):**

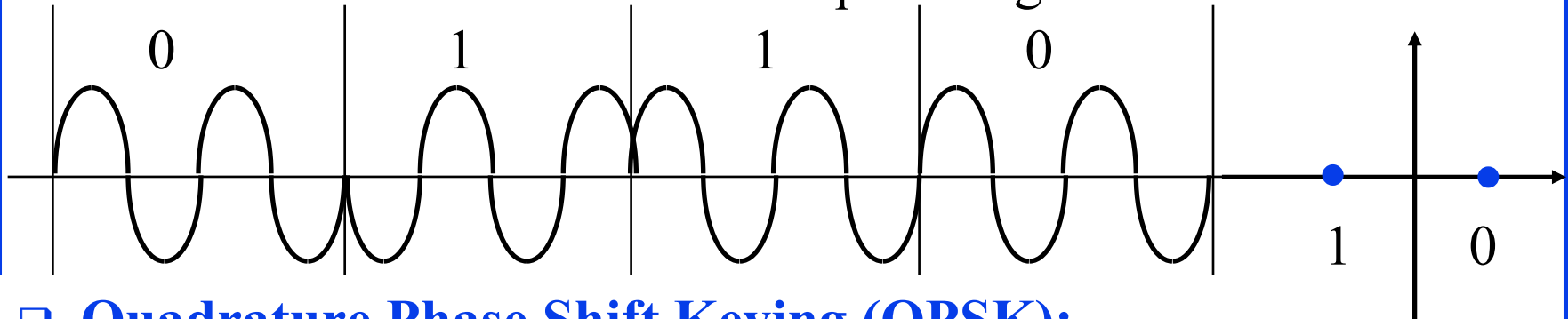


- ❑ **Phase Shift Keying (PSK):** Binary PSK (BPSK)

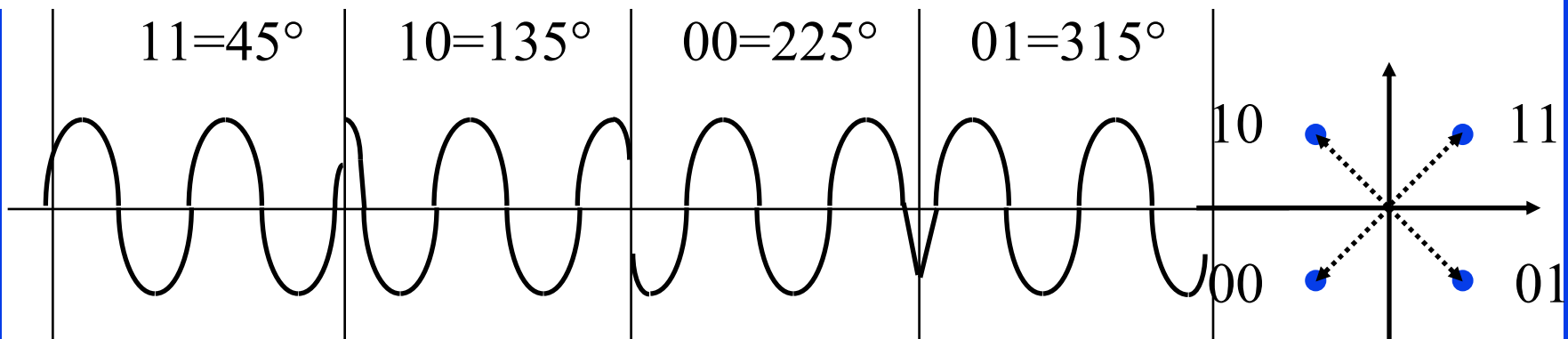


# Modulation (Cont)

- **Differential BPSK:** Does not require original carrier



- **Quadrature Phase Shift Keying (QPSK):**



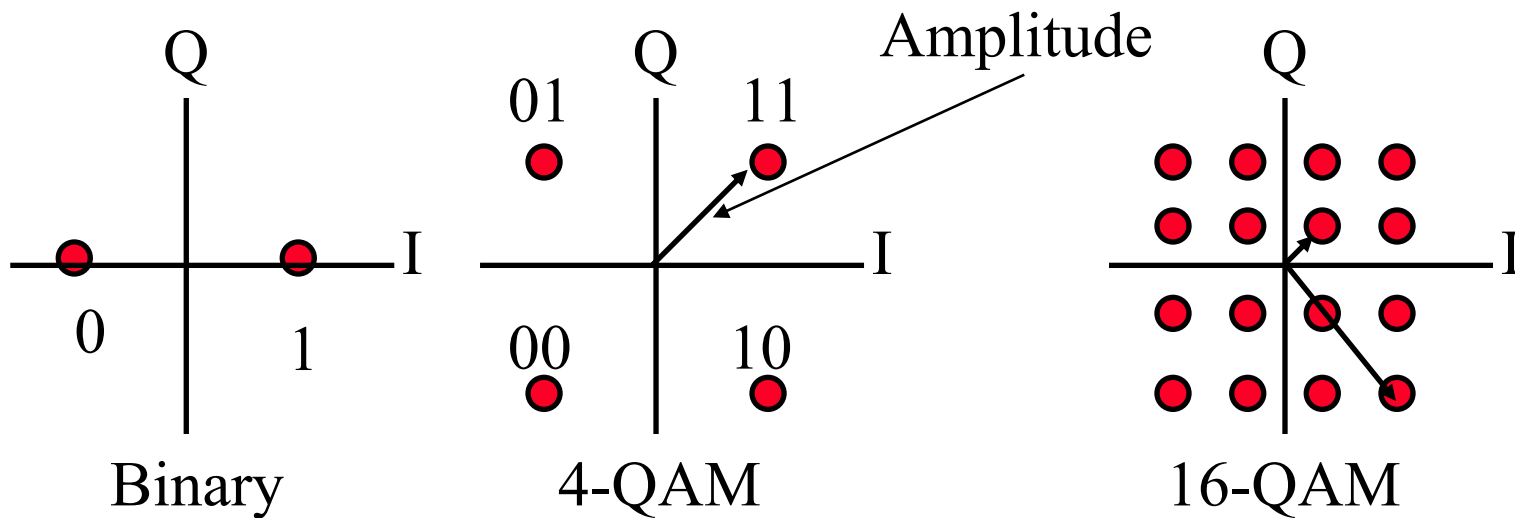
- In-phase (I) and Quadrature (Q) or  $90^\circ$  components are added

Ref: Electronic Design, "Understanding Modern Digital Modulation Techniques,"

<http://electronicdesign.com/communications/understanding-modern-digital-modulation-techniques>

# QAM

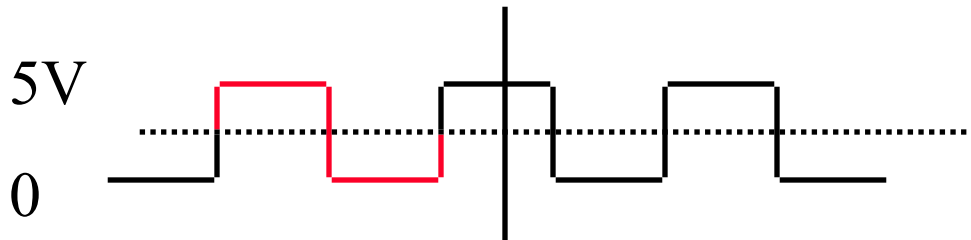
- ❑ Quadrature Amplitude and Phase Modulation
- ❑ 4-QAM, 16-QAM, 64-QAM, 256-QAM
- ❑ Used in DSL and wireless networks



- ❑ 4-QAM  $\Rightarrow$  2 bits/symbol, 16-QAM  $\Rightarrow$  4 bits/symbol, ...

# Channel Capacity

- Capacity = Maximum data rate for a channel
- **Nyquist Theorem:** Bandwidth = B  
Data rate  $\leq 2 B$
- Bi-level Encoding: Data rate =  $2 \times$  Bandwidth



- Multilevel: Data rate =  $2 \times$  Bandwidth  $\times \log_2 M$



**Example:**  $M=4$ , Capacity =  $4 \times$  Bandwidth

# Shannon's Theorem

- Bandwidth = B Hz  
Signal-to-noise ratio = S/N
- Maximum number of bits/sec =  $B \log_2 (1+S/N)$
- Example: Phone wire bandwidth = 3100 Hz

$$S/N = 30 \text{ dB}$$

$$10 \text{ Log}_{10} S/N = 30$$

$$\text{Log}_{10} S/N = 3$$

$$S/N = 10^3 = 1000$$

$$\begin{aligned} \text{Capacity} &= 3100 \log_2 (1+1000) \\ &= 30,894 \text{ bps} \end{aligned}$$

# Hamming Distance

- Hamming Distance between two sequences  
= Number of bits in which they disagree

- Example:           011011  
                          110001

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Difference           101010  $\Rightarrow$  Distance = 3



# Error Correction Example

- 2-bit words transmitted as 5-bit/word

<u>Data</u>	<u>Codeword</u>
00	00000
01	00111
10	11001
11	11110

Received = 00100  $\Rightarrow$  Not one of the code words  $\Rightarrow$  Error

Distance (00100,00000) = 1    Distance (00100,00111) = 2

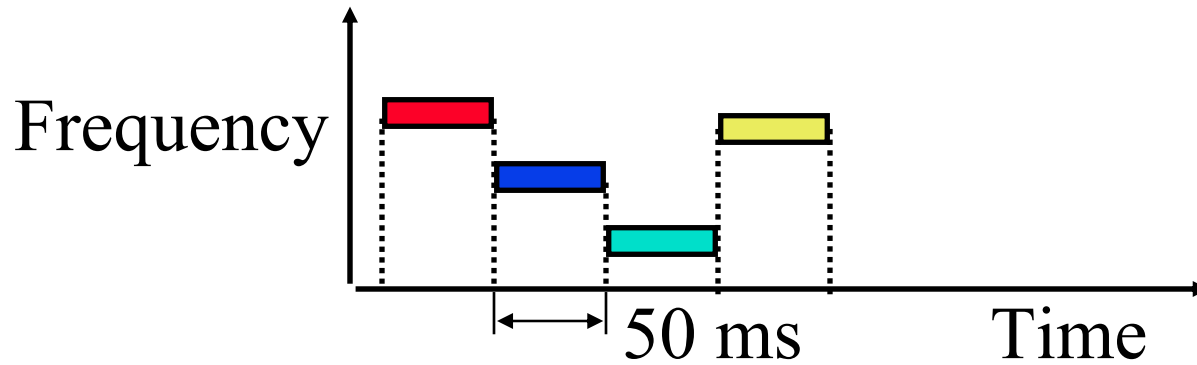
Distance (00100,11001) = 4    Distance (00100,11110) = 3

$\Rightarrow$  Most likely 00000 was sent. Corrected data = 00

b. Received = 01010 Distance(...,00000) = 2 = Distance(...,11110)  
Error detected but cannot be corrected

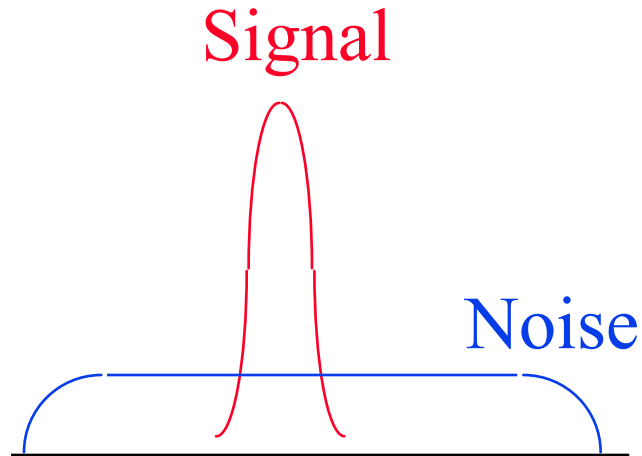
c. Three bit errors will not be detected. Sent 00000, Received 00111.

# Frequency Hopping Spread Spectrum



- ❑ Pseudo-random frequency hopping
- ❑ Spreads the power over a wide spectrum  
⇒ ❑ Spread Spectrum
- ❑ Developed initially for military
- ❑ Patented by actress Hedy Lamarr
- ❑ Narrowband interference can't jam

# Spectrum

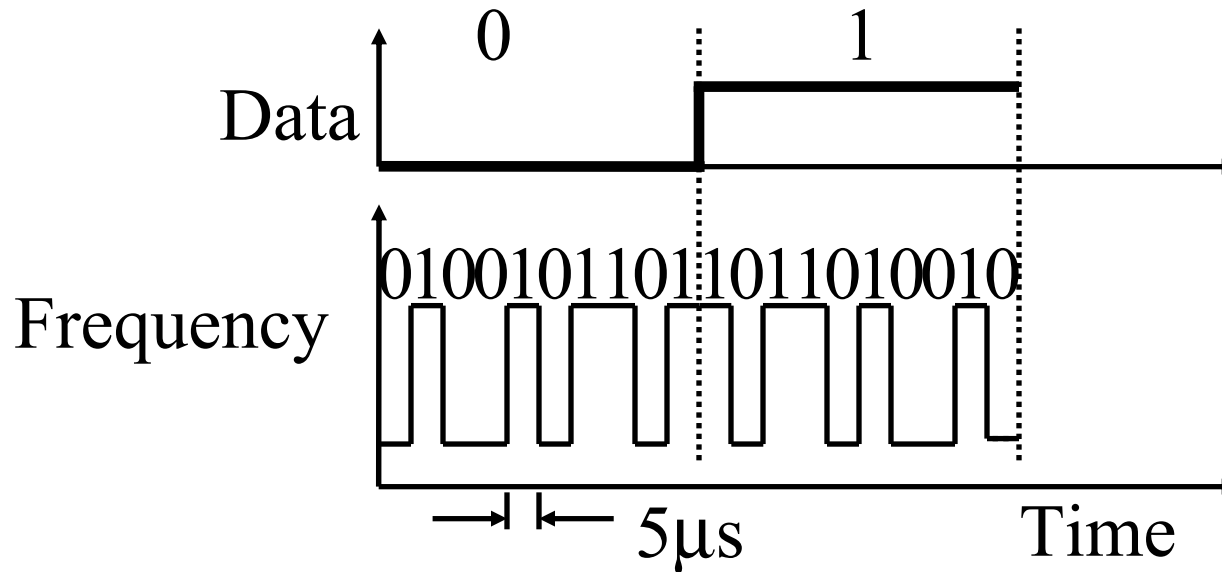


(a) Normal



(b) Frequency Hopping

# Direct-Sequence Spread Spectrum

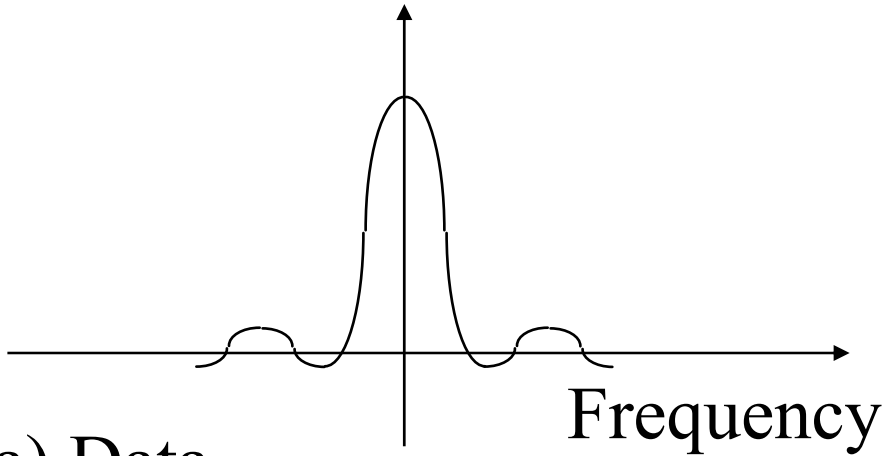
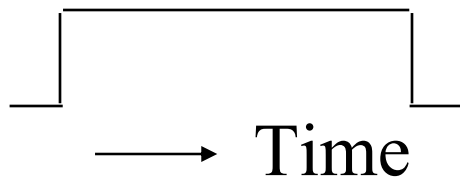


- ❑ Spreading factor = Code bits/data bit, 10-100 commercial (Min 10 by FCC), 10,000 for military
- ❑ Signal bandwidth  $> 10 \times$  data bandwidth
- ❑ Code sequence synchronization
- ❑ Correlation between codes  $\Rightarrow$  Interference   Orthogonal

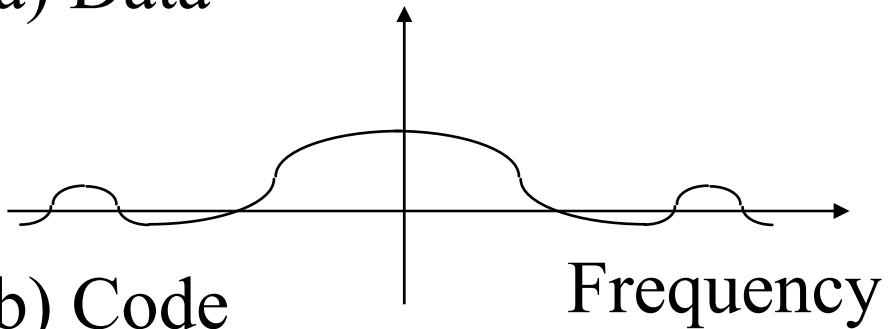
# DS Spectrum

Time Domain

Frequency Domain

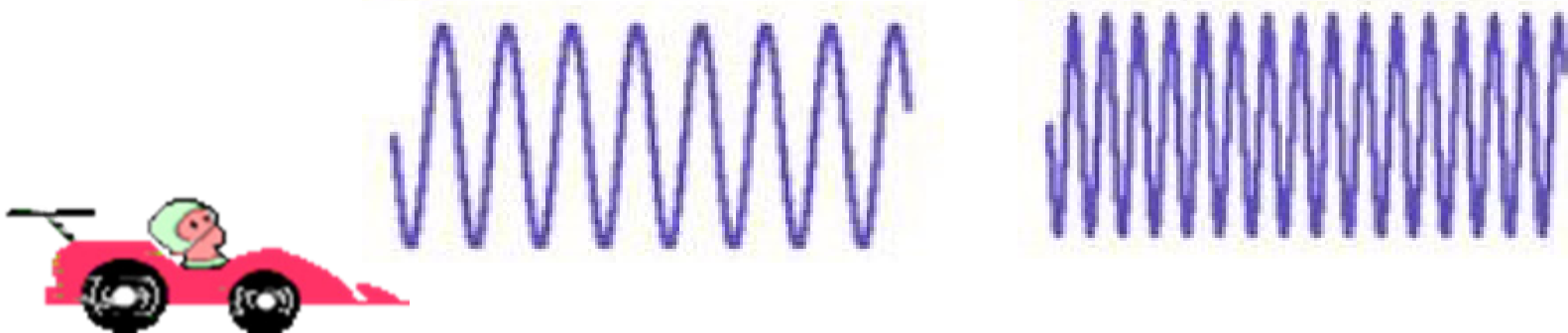


(a) Data



(b) Code

# Doppler Shift



- ❑ If the transmitter or receiver or both are mobile the frequency of received signal changes
- ❑ Moving towards each other => Frequency increases
- ❑ Moving away from each other => Frequency decreases

*Frequency difference = velocity/Wavelength*

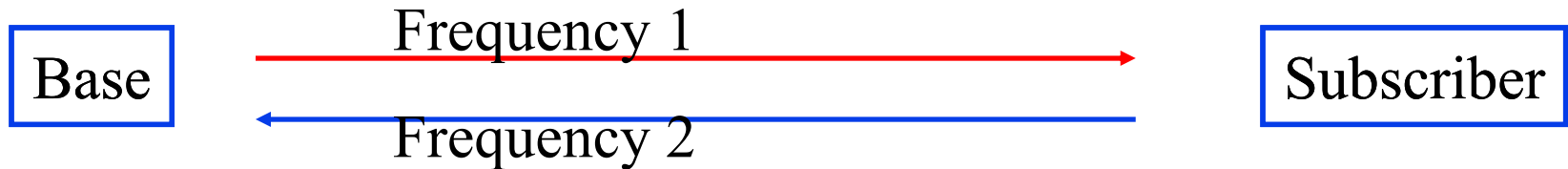
**Example:**  $2.4 \text{ GHz} \Rightarrow \lambda = 3 \times 10^8 / 2.4 \times 10^9 = .125 \text{ m}$

$120 \text{ km/hr} = 120 \times 1000 / 3600 = 33.3 \text{ m/s}$

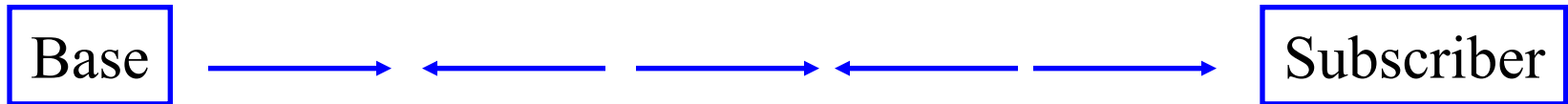
$\text{Freq diff} = 33.3 / .125 = 267 \text{ Hz}$

# Duplexing

- ❑ Duplex = Bi-Directional Communication
- ❑ Frequency division duplexing (FDD) (Full-Duplex)

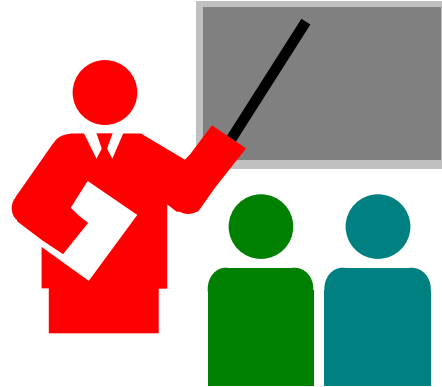


- ❑ Time division duplex (TDD): Half-duplex



- ❑ Most WiMAX deployments will use TDD.
  - Allows more flexible sharing of DL/UL data rate
  - Does not require paired spectrum
  - Easy channel estimation  $\Rightarrow$  Simpler transceiver design
  - Con: All neighboring BS should time synchronize

# Summary



1. Electric, Radio, Light, X-Rays, are all electromagnetic waves
2. Wireless radio waves travel at the speed of light  $300 \text{ m}/\mu\text{s}$   
Wavelength  $\lambda = c/f$
3. 16-QAM uses 16 combinations of amplitude and phase using 4 bits per symbol.
4. Hertz and Bit rate are related by Nyquist and Shannon's Theorems
5. Frequency hopping and Direct Sequence are two methods of code division multiple access (CDMA).



## Homework 3

- A. What is wavelength of a signal at 60 GHz?
- B. How many Watts of power is 30dBm?
- C. A telephone line is known to have a loss of 20 dB. The input signal power is measured at 1 Watt, and the output signal noise level is measured at 1 mW. Using this information, calculate the output signal to noise ratio in dB.
- D. What is the maximum data rate that can be supported on a 10 MHz noise-less channel if the channel uses eight-level digital signals?
- E. What signal to noise ratio (in dB) is required to achieve 10 Mbps through a 5 MHz channel?
- F. Compute the average Doppler frequency shift at 36 km/hr using 3 GHz band? Doppler spread is twice the Doppler shift. What is the channel coherence time?

# Reading List

- ❑ Electronic Design, “Understanding Modern Digital Modulation Techniques,”  
<http://electronicdesign.com/communications/understanding-modern-digital-modulation-techniques>

# Wikipedia Links

- ❑ <http://en.wikipedia.org/wiki/Frequency>
- ❑ <http://en.wikipedia.org/wiki/Wavelength>
- ❑ [http://en.wikipedia.org/wiki/Phase\\_\(waves\)](http://en.wikipedia.org/wiki/Phase_(waves))
- ❑ [http://en.wikipedia.org/wiki/Quadrature\\_phase](http://en.wikipedia.org/wiki/Quadrature_phase)
- ❑ [http://en.wikipedia.org/wiki/Frequency\\_domain](http://en.wikipedia.org/wiki/Frequency_domain)
- ❑ [http://en.wikipedia.org/wiki/Time\\_domain](http://en.wikipedia.org/wiki/Time_domain)
- ❑ [http://en.wikipedia.org/wiki/Fourier\\_transform](http://en.wikipedia.org/wiki/Fourier_transform)
- ❑ [http://en.wikipedia.org/wiki/Electromagnetic\\_spectrum](http://en.wikipedia.org/wiki/Electromagnetic_spectrum)
- ❑ <http://en.wikipedia.org/wiki/Decibel>
- ❑ <http://en.wikipedia.org/wiki/DBm>
- ❑ <http://en.wikipedia.org/wiki/Modulation>
- ❑ [http://en.wikipedia.org/wiki/Amplitude-shift\\_keying](http://en.wikipedia.org/wiki/Amplitude-shift_keying)
- ❑ [http://en.wikipedia.org/wiki/Phase-shift\\_keying](http://en.wikipedia.org/wiki/Phase-shift_keying)
- ❑ [http://en.wikipedia.org/wiki/Frequency-shift\\_keying](http://en.wikipedia.org/wiki/Frequency-shift_keying)
- ❑ [http://en.wikipedia.org/wiki/Quadrature\\_phase-shift\\_keying](http://en.wikipedia.org/wiki/Quadrature_phase-shift_keying)

# Wikipedia Links (Cont)

- ❑ [http://en.wikipedia.org/wiki/Differential\\_coding](http://en.wikipedia.org/wiki/Differential_coding)
- ❑ [http://en.wikipedia.org/wiki/Quadrature\\_amplitude\\_modulation](http://en.wikipedia.org/wiki/Quadrature_amplitude_modulation)
- ❑ [http://en.wikipedia.org/wiki/Shannon%E2%80%93Hartley\\_theorem](http://en.wikipedia.org/wiki/Shannon%E2%80%93Hartley_theorem)
- ❑ [http://en.wikipedia.org/wiki/Channel\\_capacity](http://en.wikipedia.org/wiki/Channel_capacity)
- ❑ [http://en.wikipedia.org/wiki/Hamming\\_distance](http://en.wikipedia.org/wiki/Hamming_distance)
- ❑ [http://en.wikipedia.org/wiki/Channel\\_access\\_method](http://en.wikipedia.org/wiki/Channel_access_method)
- ❑ [http://en.wikipedia.org/wiki/Time\\_division\\_multiple\\_access](http://en.wikipedia.org/wiki/Time_division_multiple_access)
- ❑ [http://en.wikipedia.org/wiki/Frequency-division\\_multiple\\_access](http://en.wikipedia.org/wiki/Frequency-division_multiple_access)
- ❑ <http://en.wikipedia.org/wiki/CDMA>
- ❑ [http://en.wikipedia.org/wiki/Spread\\_spectrum](http://en.wikipedia.org/wiki/Spread_spectrum)
- ❑ [http://en.wikipedia.org/wiki/Direct-sequence\\_spread\\_spectrum](http://en.wikipedia.org/wiki/Direct-sequence_spread_spectrum)
- ❑ [http://en.wikipedia.org/wiki/Frequency-hopping\\_spread\\_spectrum](http://en.wikipedia.org/wiki/Frequency-hopping_spread_spectrum)
- ❑ [http://en.wikipedia.org/wiki/Doppler\\_effect](http://en.wikipedia.org/wiki/Doppler_effect)
- ❑ [http://en.wikipedia.org/wiki/Duplex\\_\(telecommunications\)](http://en.wikipedia.org/wiki/Duplex_(telecommunications))
- ❑ [http://en.wikipedia.org/wiki/Time-division\\_duplex](http://en.wikipedia.org/wiki/Time-division_duplex)
- ❑ [http://en.wikipedia.org/wiki/Frequency\\_division\\_duplex](http://en.wikipedia.org/wiki/Frequency_division_duplex)

# References

- Lars Lundheim, “On Shannon and Shannon's law,”  
<http://www.iet.ntnu.no/projects/beats/Documents/LarsTelektronikk02.pdf>

# Optional Listening Material

Those not familiar with modulation, coding, CRC, etc may want to listen to the following lectures from CSE473S:

- ❑ Transmission Media,  
[http://www.cse.wustl.edu/~jain/cse473-11/i\\_1cni.htm](http://www.cse.wustl.edu/~jain/cse473-11/i_1cni.htm)
- ❑ Signal Encoding Techniques,  
[http://www.cse.wustl.edu/~jain/cse473-05/i\\_5cod.htm](http://www.cse.wustl.edu/~jain/cse473-05/i_5cod.htm)
- ❑ Digital Communications Techniques,  
[http://www.cse.wustl.edu/~jain/cse473-05/i\\_6com.htm](http://www.cse.wustl.edu/~jain/cse473-05/i_6com.htm)

# Acronyms

- ❑ ASK            Amplitude Shift Keying
- ❑ BPSK          Binary Phase Shift Keying
- ❑ BS             Base Station
- ❑ CDMA         Code division multiple access
- ❑ CRC          Cyclic Redundancy Check
- ❑ dB            Decibel
- ❑ dBm          Decibel milliWatt
- ❑ DL            Downlink
- ❑ DS            Direct Sequence
- ❑ DSL          Digital Subscriber Line
- ❑ FCC          Federal Communications Commission
- ❑ FDD          Frequency Division Duplexing
- ❑ FSK          Frequency Shift Keying
- ❑ GHz          Giga Hertz
- ❑ LAN          Local Area Network
- ❑ MHz          Mega Hertz

# Acronyms (Cont)

- ❑ mW            milli Watt
- ❑ OFDM        Orthogonal Frequency Division Multiplexing
- ❑ PSK          Phase Shift Keying
- ❑ QAM         Quadrature Amplitude Modulation
- ❑ QPSK        Quadrature Phase Shift Keying
- ❑ SS            Subscriber Station
- ❑ TDD         Time Division Duplexing
- ❑ UL            Uplink