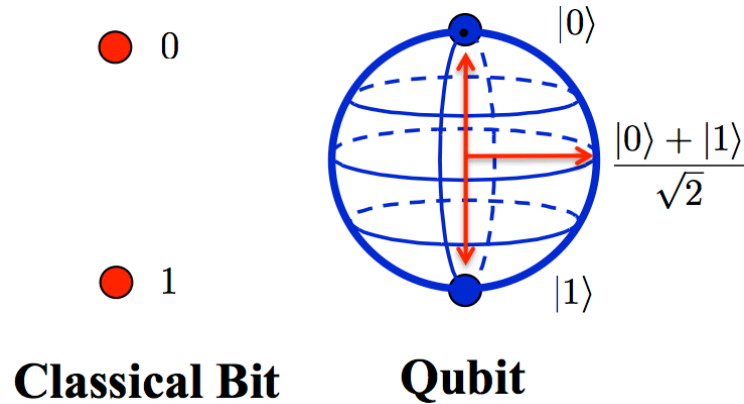


# Introduction to Quantum Computing and its Applications to Cyber Security



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These slides and audio/video recordings of this class lecture are at:  
<http://www.cse.wustl.edu/~jain/cse570-21/>

**Student Questions**

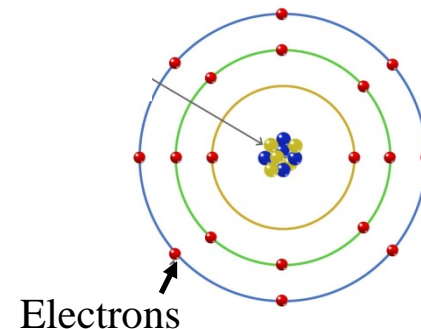
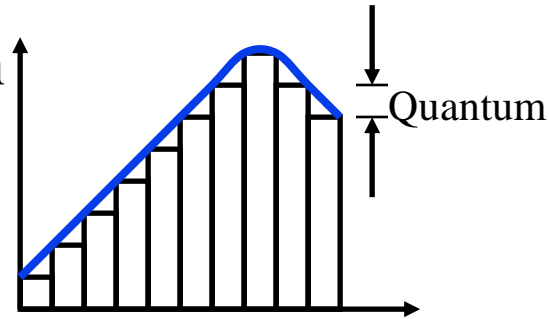


1. What is a Quantum and Quantum Bit?
2. Matrix Algebra Review
3. Quantum Gates: Not, And, or, Nand
4. Applications of Quantum Computing
5. Quantum Hardware and Programming

## Student Questions

# What is a Quantum?

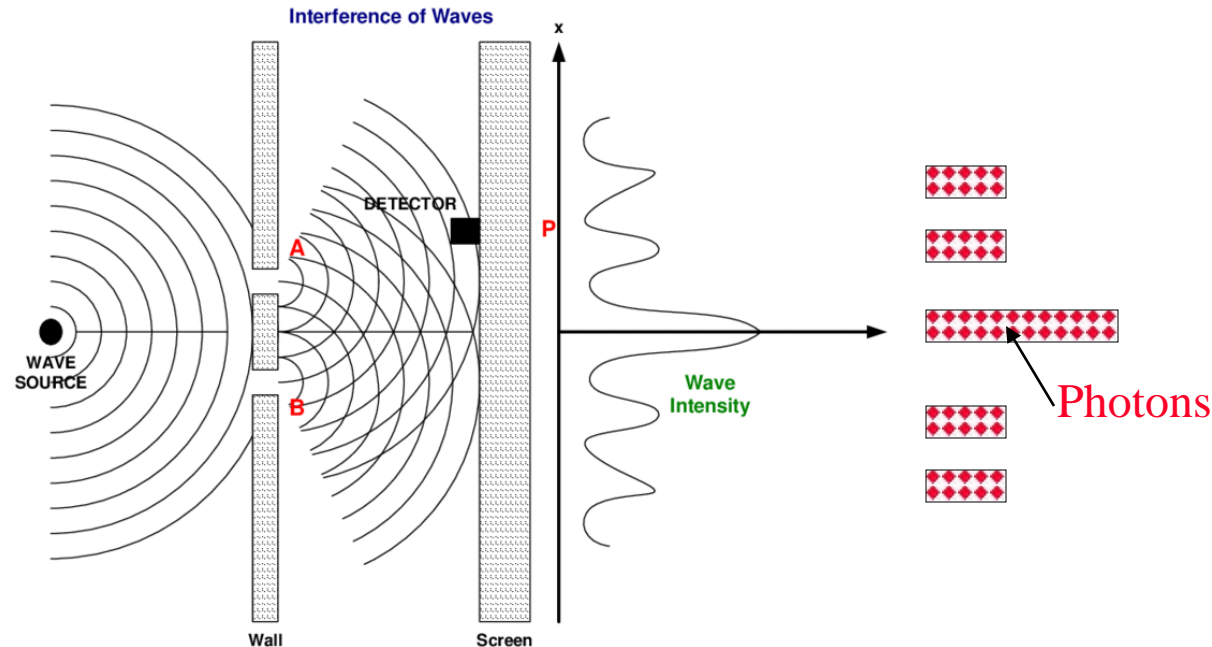
- ❑ Quantization: Analog to digital conversion
- ❑ Quantum = Smallest discrete unit
- ❑ **Wave Theory**: Light is a wave. It has a frequency, phase, amplitude
- ❑ **Quantum Mechanics**: Light behaves like discrete packets of energy that can be absorbed and released
- ❑ **Photon** = One quantum of light energy
- ❑ Photons can move an electron from one energy level to next higher level
- ❑ Photons are released when an electron moves from one level to lower energy level



## Student Questions

# Probabilistic Behavior

## □ Young's Double-Slit Experiment 1801



- The two waves exiting the slits interfere.
- Interference is constructive at some spots and destructive at others  $\Rightarrow$  Probabilistic

## Student Questions

# Quantum Bits

1. Computing bit is a binary scalar: 0 or 1
2. Quantum bit (**Qubit**) is a  $2 \times 1$  **vector**, e.g.,  $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$  or  $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$
3. Vector elements of Qubits are **complex numbers**  $x+iy$
4. **Modulus** of a complex Number  $|x+iy| = \sqrt{(x+iy)(x-iy)} = \sqrt{x^2 + y^2}$   
← Conjugate

Example:  $|1+2i| = \sqrt{(1+2i)(1-2i)} = \sqrt{1+4} = \sqrt{5}$

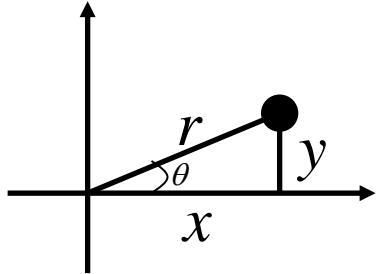
5. Probability of each element in a qubit vector is proportional to its modulus squared  $\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \Rightarrow \begin{matrix} P = |a_0|^2 / (|a_0|^2 + |a_1|^2) \\ P = |a_1|^2 / (|a_0|^2 + |a_1|^2) \end{matrix}$

$$\begin{bmatrix} 1+2i \\ 1-i \end{bmatrix} \Rightarrow \begin{matrix} |1+2i| = \sqrt{(1+2i)(1-2i)} = \sqrt{5} \\ |1-i| = \sqrt{(1-i)(1+i)} = \sqrt{2} \end{matrix} \Rightarrow P = \begin{cases} 5 / (5+2) = 5/7 \\ 2 / (5+2) = 2/7 \end{cases}$$

## Student Questions

# Polar Representation

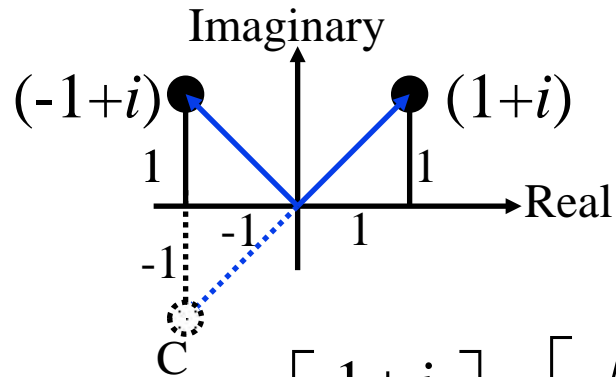
- Complex numbers in polar coordinates:



$$(x + iy) = re^{i\theta} = r(\cos(\theta) + i\sin(\theta))$$

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}(y/x)$$



$$2\pi = 360^\circ$$

$$\pi/4 = 45^\circ$$

$$\cos(\pi/4) = \frac{1}{\sqrt{2}}$$

$$\sin(\pi/4) = \frac{1}{\sqrt{2}}$$

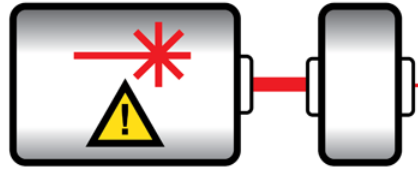
$$\begin{bmatrix} 1+i \\ -1+i \end{bmatrix} = \begin{bmatrix} \sqrt{2}e^{i\pi/4} \\ \sqrt{2}e^{3\pi/4} \end{bmatrix} = \begin{bmatrix} \sqrt{2}(\cos(\pi/4) + i\sin(\pi/4)) \\ \sqrt{2}(\cos(3\pi/4) + i\sin(3\pi/4)) \end{bmatrix}$$

- Exercise:** Find the complex and polar representation of C

## Student Questions

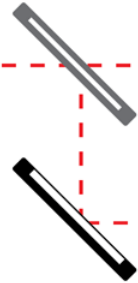
# Qubit Interpretation

single-photon source



photon

half-silvered mirror



mirror

0 A

1 B

photon detectors

[Source: Johnston, et al. 2019]

- ❑ If a single photon is emitted from the source, the photon reaches position A or B with some probability  
⇒ Photon has a *superposition* (rather than position)
- ❑ Each position has a different path length and, therefore, different amplitude and phase

Ref: E. R. Johnston, N. Harrigan, and M. Gimeno-Segovia, "Programming Quantum Computers," O'reilly, 2019, ISBN:9781492039686, 320 pp.

Washington University in St. Louis

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## Student Questions

# Bra-Ket Notation

- ❑ The vector  $\psi$  is denoted in bra-kets  $|\psi\rangle$
- ❑ Brackets:  $\{ \}$ ,  $[ ]$ ,  $\langle \rangle$
- ❑ Bra  $\langle a|$
- ❑ Ket  $|a\rangle$
- ❑ Example: Ket-zero and ket-one

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle \quad \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle$$

- ❑ Bra is the transpose of the complex-conjugate of a Ket.  
Example: Bra-zero and Bra-one

$$[1 \ 0] = \langle 0| \quad [0 \ 1] = \langle 1|$$

## Student Questions



# Matrix Multiplication

## □ Matrix multiplication $\times$ :

$$\begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \end{bmatrix} \times \begin{bmatrix} b_{00} & b_{01} \\ b_{10} & b_{11} \\ b_{20} & b_{21} \end{bmatrix}$$
$$= \begin{bmatrix} a_{00}b_{00} + a_{01}b_{10} + a_{02}b_{20} & a_{00}b_{01} + a_{01}b_{11} + a_{02}b_{21} \\ a_{10}b_{00} + a_{11}b_{10} + a_{12}b_{20} & a_{10}b_{01} + a_{11}b_{11} + a_{12}b_{21} \end{bmatrix}$$

□ Example:

$$\begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$3 \times 2$                    $2 \times 3$                    $3 \times 3$

## Student Questions

# Tensor Product

□ **Tensor Product**  $\otimes$ :  $m \times n \otimes k \times l$  results in  $mk \times nl$  matrix

$$\begin{aligned}
 A &= \begin{bmatrix} a_{00} & a_{01} \\ a_{10} & a_{11} \end{bmatrix} \quad B = \begin{bmatrix} b_{00} & b_{01} & b_{02} \\ b_{10} & b_{11} & b_{12} \\ b_{20} & b_{21} & b_{22} \end{bmatrix} \\
 A \otimes B &= \begin{bmatrix} a_{00} \begin{bmatrix} b_{00} & b_{01} & b_{02} \\ b_{10} & b_{11} & b_{12} \\ b_{20} & b_{21} & b_{22} \end{bmatrix} & a_{01} \begin{bmatrix} b_{00} & b_{01} & b_{02} \\ b_{10} & b_{11} & b_{12} \\ b_{20} & b_{21} & b_{22} \end{bmatrix} \\
 a_{10} \begin{bmatrix} b_{00} & b_{01} & b_{02} \\ b_{10} & b_{11} & b_{12} \\ b_{20} & b_{21} & b_{22} \end{bmatrix} & a_{11} \begin{bmatrix} b_{00} & b_{01} & b_{02} \\ b_{10} & b_{11} & b_{12} \\ b_{20} & b_{21} & b_{22} \end{bmatrix} \end{bmatrix} \\
 &= \begin{bmatrix} a_{00}b_{00} & a_{00}b_{01} & a_{00}b_{02} & a_{01}b_{00} & a_{01}b_{01} & a_{01}b_{02} \\
 a_{00}b_{10} & a_{00}b_{11} & a_{00}b_{12} & a_{01}b_{10} & a_{01}b_{11} & a_{01}b_{12} \\
 a_{00}b_{20} & a_{00}b_{21} & a_{00}b_{22} & a_{01}b_{20} & a_{01}b_{21} & a_{01}b_{22} \\
 a_{10}b_{00} & a_{10}b_{01} & a_{10}b_{02} & a_{11}b_{00} & a_{11}b_{01} & a_{11}b_{02} \\
 a_{10}b_{10} & a_{10}b_{11} & a_{10}b_{12} & a_{11}b_{10} & a_{11}b_{11} & a_{11}b_{12} \\
 a_{10}b_{20} & a_{10}b_{21} & a_{10}b_{22} & a_{11}b_{20} & a_{11}b_{21} & a_{11}b_{22} \end{bmatrix}
 \end{aligned}$$

## Student Questions

# Tensor Product (Cont)

□ Example 1:

$$\begin{array}{ccc} \begin{bmatrix} a_{00} \\ a_{10} \\ a_{20} \end{bmatrix} & \otimes & \begin{bmatrix} b_{00} \\ b_{10} \end{bmatrix} \\ \mathbf{3 \times 1} & & \mathbf{2 \times 1} \end{array} = \begin{array}{c} \begin{bmatrix} a_{00} \begin{bmatrix} b_{00} \\ b_{10} \end{bmatrix} \\ a_{10} \begin{bmatrix} b_{00} \\ b_{10} \end{bmatrix} \\ a_{20} \begin{bmatrix} b_{00} \\ b_{10} \end{bmatrix} \end{bmatrix} \\ \mathbf{6 \times 1} \end{array} = \begin{bmatrix} a_{00}b_{00} \\ a_{00}b_{10} \\ a_{10}b_{00} \\ a_{10}b_{10} \\ a_{20}b_{00} \\ a_{20}b_{10} \end{bmatrix}$$

□ Example 2:

$$\begin{array}{ccc} \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} & \otimes & \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \\ \mathbf{2 \times 2} & & \mathbf{1 \times 3} \end{array} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} \\ & & & \mathbf{2 \times 6} \end{array}$$

**Student Questions**

# Multiple Qubits and QuBytes

One Qbit:  $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$   $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Two Qbits:  $|00\rangle = |0\rangle \otimes |0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$   $|01\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$   $|10\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$   $|11\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$

Tensor Product

Three Qbits:

$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$
$ 000\rangle$	$ 001\rangle$	$ 010\rangle$	$ 011\rangle$	$ 100\rangle$	$ 101\rangle$	$ 110\rangle$	$ 111\rangle$

- ❑ In a k-qubit register, each of the  $2^k$  positions can be any complex number
- ❑ QuByte=8-Qubits = 256-element vector

## Student Questions

# Homework 19A

- Given two matrices:

$$A = \begin{bmatrix} 1+i & 1 \\ 1-i & i \end{bmatrix} B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

- Compute:  $A \times B, A \otimes B$
- Compute the probabilities of each element of  $A \times B$

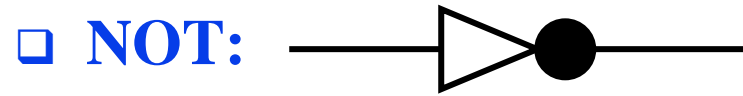
## Student Questions

# Quantum Gates

1. Quantum NOT Gate
2. Quantum AND Gate
3. Quantum OR Gate
4. Quantum NAND Gate
5. Quantum  $\sqrt{\text{NOT}}$  Gate

## Student Questions

# Quantum NOT Gate



$$NOT = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$NOT |1\rangle = |0\rangle$$

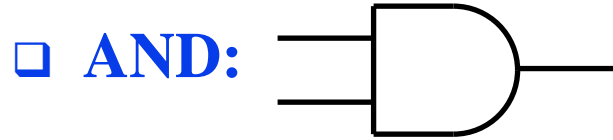
$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

$$NOT |0\rangle = |?\rangle$$

□ **Exercise:** Fill in the ?'s

**Student Questions**

# Quantum AND Gate



$$\text{AND} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} ? \\ ? \end{bmatrix}$$

$$\text{AND} \quad |00\rangle \quad |01\rangle \quad |10\rangle \quad |11\rangle = |0\rangle \quad |0\rangle \quad |0\rangle \quad |?\rangle$$

2×4

4×1

2×1

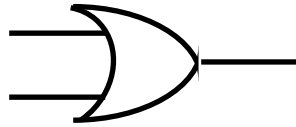
□ **Exercise:** Fill in the ?'s

**Student Questions**



# Quantum OR Gate

□ OR:



$$OR = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

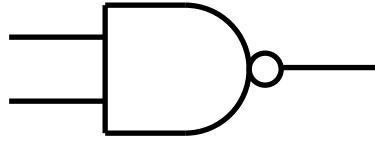
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

OR             $|00\rangle$     $|01\rangle$     $|10\rangle$     $|11\rangle$  =    $|0\rangle$     $|1\rangle$     $|1\rangle$     $|1\rangle$

**Student Questions**

# Quantum NAND Gate

□ **NAND:**



$$\text{NAND} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

NAND       $|00\rangle$     $|01\rangle$     $|10\rangle$     $|11\rangle$  =  $|1\rangle$     $|1\rangle$     $|1\rangle$     $|0\rangle$

**Student Questions**

# Quantum $\sqrt{\text{NOT}}$ Gate

□  $\sqrt{\text{NOT}}$ :  $\sqrt{\text{NOT}} \times \sqrt{\text{NOT}} = \text{NOT}$

$$\sqrt{\text{NOT}} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$|1\rangle$

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \times \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$|0\rangle$

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$|0\rangle$

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \times \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

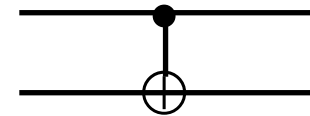
$|1\rangle$

## Student Questions

# Controlled NOT Gate

- **CNOT:** If the control bit is 0, no change to the 2<sup>nd</sup> bit

If control bit is 1, the 2<sup>nd</sup> bit is complemented



$$\text{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

CNOT      |00>   |01>   |10>   |11> = |00>   |01>   |11>   |10>

- Controlled NOT gate can be used to produce two bits that are **entangled** ⇒ Two bits behave similarly even if far apart  
⇒ Can be used for teleportation of information

## Student Questions

# Quantum Gates: Summary

$$\text{NOT} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\text{AND} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{OR} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

$$\text{NAND} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

$$\sqrt{\text{NOT}} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$\text{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Classical

Non-Classical

- ❑ The first 4 gates above are similar to the classical gates. The last two are non-classical gate.
- ❑ There are many other classical/non-classical quantum gates, e.g., Rotate, Copy, Read, Write, ...
- ❑ Using such gates one can design **quantum circuits**

## Student Questions

# Quantum Applications

- ❑ It has been shown that quantum computation makes several problems easy that are hard currently. Including:
  - **Fourier Transforms**
  - **Factoring large numbers**
  - Error correction
  - Searching a large unordered list
- ❑ There are some new methods:
  - Quantum Key Exchange
  - Quantum Teleportation (transfer states from one location to another)

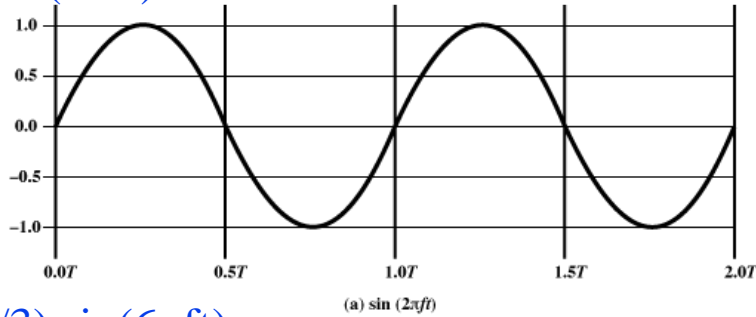
Quantum-Safe Cryptography is being standardized

## Student Questions

# Fourier Transforms

## Time Domain

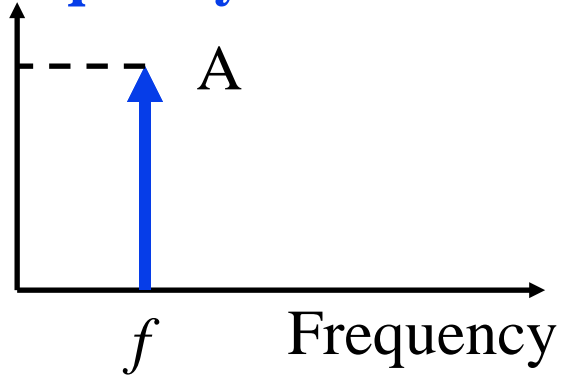
$$A \sin(2\pi ft)$$



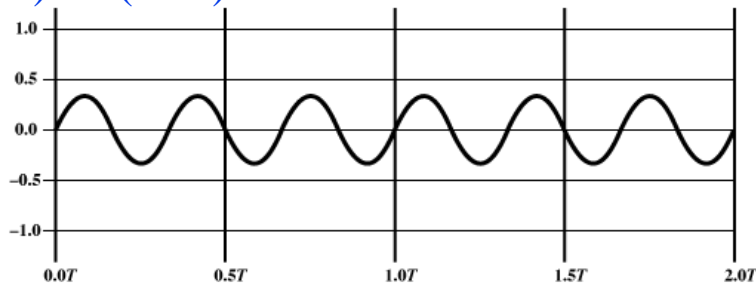
Amplitude

Fast  
Fourier  
Transform

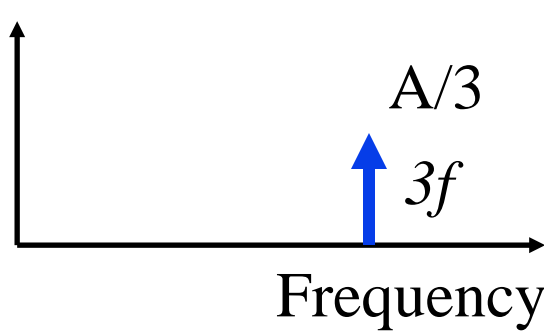
## Frequency Domain



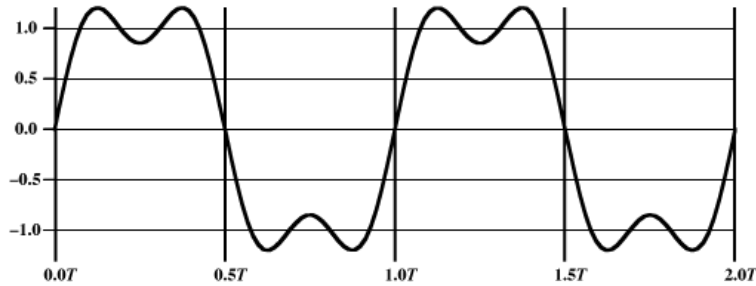
$$(A/3) \sin(6\pi ft)$$



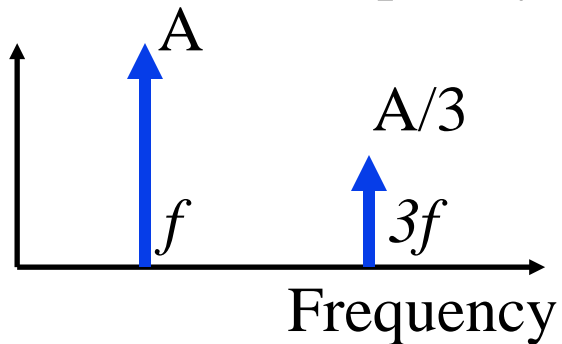
Amplitude



$$A(\sin(2\pi ft) + (1/3) \sin(6\pi ft))$$



Amplitude



## Student Questions

# Quantum Fourier Transform (QFT)

- ❑ Fourier transform is used to find periodic components of signals
- ❑ QFT is faster than classical FT for large *inputs*

## Student Questions



# GCD

- Greatest Common Divisor of any two numbers
  - Divide the larger number with the smaller number and get the remainder less than the divisor
  - Divide the previous divisor with the remainder
  - Continue this until the remainder is zero.  
The last divisor is the GCD

$$\begin{array}{r} 15 \overline{) 35} \quad (2 \\ \underline{30} \\ 05 \end{array} \quad \begin{array}{r} 15 \quad (3 \\ \underline{15} \\ 0 \end{array}$$

gcd  $\nearrow$

## Student Questions

# Shor's Factoring Algorithm

- ❑ Peter Shor used QFT and showed that Quantum Computers can find prime factors of large numbers exponentially faster than conventional computers
- ❑ **Step 1:** Find the period of  $a^i \bmod N$  sequence.  
Here  $a$  is co-prime to  $N \Rightarrow a$  is a prime such that  $\gcd(a, N) = 1 \Rightarrow a$  and  $N$  have no common factors.
  - Example:  $N=15, a=2;$   
 $2^i \bmod 15$  for  $i=0, 1, 2, \dots$   
 $= 1, 2, 4, 8, 1, \dots \Rightarrow p=4$
  - This is the classical method for finding period.  
QFT makes it fast.
- ❑ **Step 2:** Prime factors of  $N$  might be  $\gcd(N, a^{p/2}+1)$  and  $\gcd(N, a^{p/2}-1)$ 
  - Example:  $\gcd(15, 2^2-1) = 3; \gcd(15, 2^2+1) = 5;$

## Student Questions

# Homework 19B

- ❑ Find factors of 35 using Shor's algorithm. Show all steps.
- ❑ Optional: Try factoring 407 (Answer:  $11 \times 37$ )

## Student Questions

# Quantum Machine Learning (QML)

- ❑ Quantum for solving systems of linear equation
- ❑ Quantum Principal Component Analysis
- ❑ Quantum Support Vector Machines (QSVM)
  - Classical SVM has runtime of  $O(\text{poly}(m,n))$ ,  $m$  data points,  $n$  features
  - QSVM has runtime of  $O(\log(mn))$ 
    - ❑ Currently limited to data that can be represented with small number of qubits
- ❑ QML can process data directly from Quantum sensors with full range of quantum information

## Student Questions

Ref: E. R. Johnston, N. Harrigan, and M. Gimeno-Segovia, "Programming Quantum Computers," O'reilly, 2019, ISBN:9781492039686, 320 pp.

# Building Quantum Computers

1. **Neural Atom:** Group of cesium or rubidium atoms are cooled down to a few degree Kelvin and controlled using lasers
2. **Nuclear Magnetic Resonance (NMR)**
3. **Nitrogen-Vacancy Center-in-Diamond:** Some carbon atoms in diamond lattice are replaced by nitrogen atoms
4. **Photonics:** Mirrors, beam splitters, and phase shifters are used to control photons
5. **Spin Qubits:** Using semiconductor materials
6. **Topological Quantum Computing:** Uses Anyon which are quasi-particles different from photons or electrons
7. **Superconducting Qubits:** Requires cooling down to 10mK

## Student Questions

Ref: J. D. Hidary, "Quantum Computing: An Applied Approach," Springer, 2019, 380 pp.

Washington University in St. Louis

<http://www.cse.wustl.edu/~jain/cse570-21/>

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# Quantum Hardware

- ❑ IBM Q Experience: 5-Qubit quantum processor  
Open to public for experiments using their cloud,  
<https://www.ibm.com/quantum-computing/technology/experience/>



## Student Questions

Ref: <https://www.ibm.com/blogs/research/2018/04/ibm-startups-accelerate-quantum/>

Washington University in St. Louis

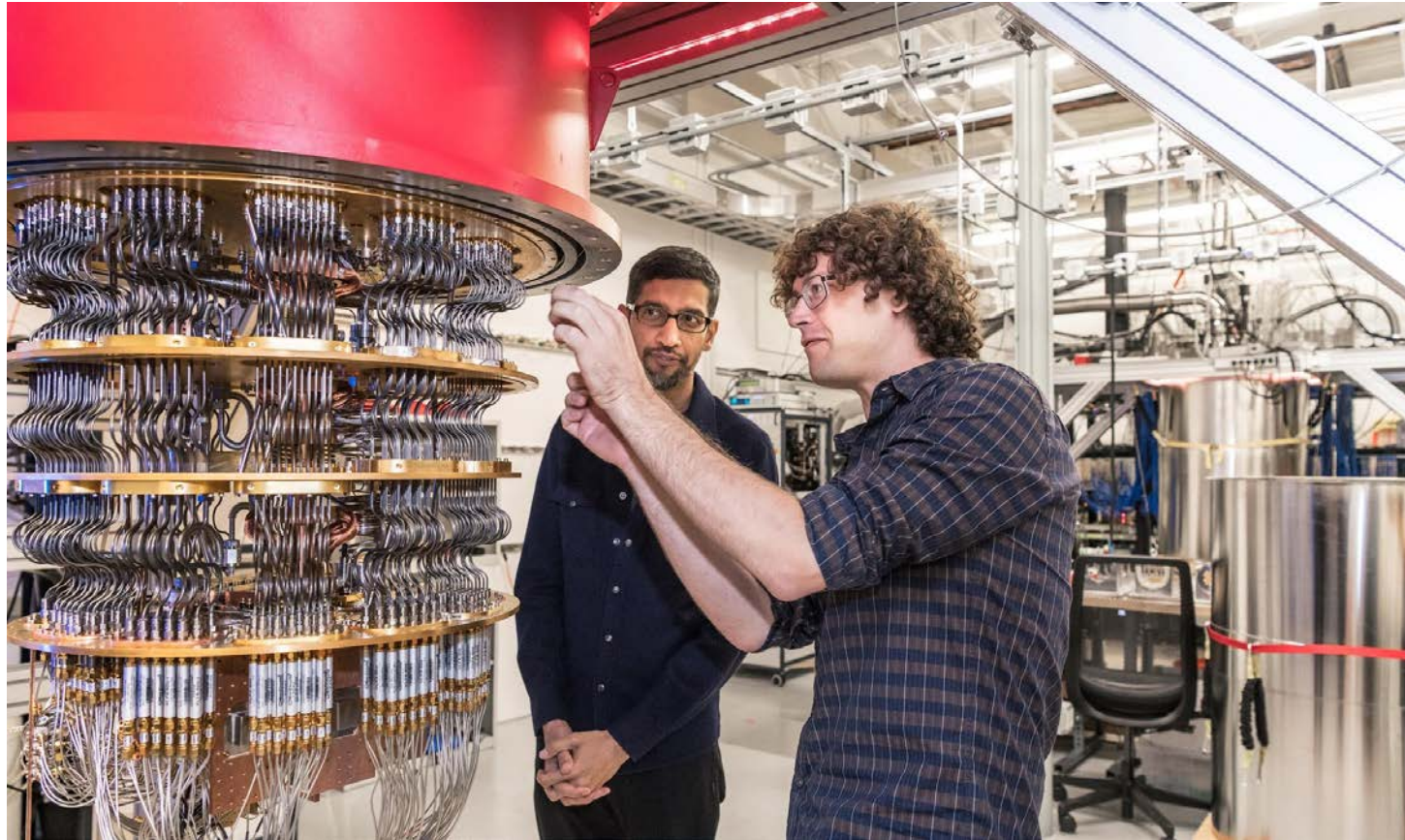
<http://www.cse.wustl.edu/~jain/cse570-21/>

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# Quantum Hardware (Cont)

- ❑ Google's Quantum computer in Santa Barbara Lab



## Student Questions

Ref: <https://www.nbcnews.com/mach/science/google-claims-quantum-computing-breakthrough-ibm-pushes-back-ncna1070461>

Washington University in St. Louis

<http://www.cse.wustl.edu/~jain/cse570-21/>

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# Quantum Simulators

- ❑ QCEngine: <https://oreilly-qc.github.io/>
- ❑ Qiskit, <https://qiskit.org/>
  - Qiskit OpenQASM (Quantum Assembly Language),  
<https://github.com/QISKit/openqasm/blob/master/examples/generic/adder.qasm>
- ❑ Q# (Qsharp), <https://docs.microsoft.com/en-gb/quantum/?view=qsharp-preview>
- ❑ Cirq, <https://arxiv.org/abs/1812.09167>
- ❑ Forest, <https://www.rigetti.com/forest>
- ❑ List of QC Simulators, <https://quantiki.org/wiki/list-qc-simulators>
- ❑ See the complete list at:  
[https://en.wikipedia.org/wiki/Quantum\\_programming](https://en.wikipedia.org/wiki/Quantum_programming)

Ref: E. R. Johnston, N. Harrigan, and M. Gimeno-Segovia, "Programming Quantum Computers," O'reilly, 2019, ISBN:9781492039686, 320 pp.

## Student Questions

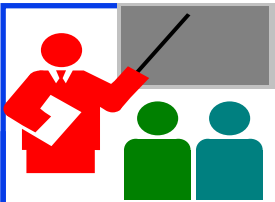


# Quantum Supremacy

- ❑ Quantum Supremacy: Solve a problem on quantum computer that can not be solved on a classical computer
- ❑ Google announced it has achieved Quantum Supremacy on October 23, 2019
  - Google built a 54-qubit quantum computer using programmable superconducting processor
- ❑ Vendors: IBM, Microsoft, Google, Alibaba Cloud, D-Wave Systems, 1QBit, QC Ware, QinetiQ, Rigetti Computing, Zapata Computing
- ❑ Global Competition: China, Japan, USA, EU are also competing

## Student Questions

Ref: F. Arute, K. Arya, R. Babbush, *et al.*, “Quantum supremacy using a programmable superconducting processor,” *Nature* **574**, 505–510 (Oct. 23, 2019), <https://www.nature.com/articles/s41586-019-1666-5>



# Summary

1. Qubits are two element vectors. Each element is a complex number that indicate the probability of that level
2. Multi-qubits are represented by tensor products of single-qubits
3. Qbit operations are mostly matrix operations. The number of possible operations is much larger than the classic computing.
4. Shor's factorization algorithm is an example of algorithms that can be done in significantly less time than in classic computing
5. Quantum computing is here. IBM, Microsoft, Google all offer platforms that can be used to write simple quantum computing programs and familiarize yourself.
6. Quantum-Safe Crypto is in standardization

## Student Questions

# Reading List

- ❑ J. D. Hidary, “Quantum Computing: An Applied Approach,” Springer, 2019, 380 pp.
- ❑ Mercedes Gimeno-Segovia, Nic Harrigan, Eric R. Johnston, "Programming Quantum Computers," O'Reilly Media, Inc., July 2019, ISBN:9781492039686 (Safari Book). **Recommended.**
- ❑ N. S. Yanofsky and M. A. Mannucci, “Quantum Computing for Computer Scientists,” Cambridge, 2008, 380 pp.
- ❑ N. D. Mermin, “Quantum Computer Science: An Introduction,” Cambridge, 2007, 220 pp.

## Student Questions

# References

- ❑ Gerd Leuchs, Dagmar Bruss, "Quantum Information," 2 Volume Set, 2nd Edition, Wiley-VCH, June 2019, ISBN:9783527413539 (Safari Book).
- ❑ Vladimir Silva, "Practical Quantum Computing for Developers: Programming Quantum Rigs in the Cloud using Python, Quantum Assembly Language and IBM QExperience," Apress, December 2018, ISBN:9781484242186 (Safari Book).
- ❑ Mingsheng Ying, "Foundations of Quantum Programming," Morgan Kaufmann, March 2016, ISBN:9780128025468 (Safari Book).
- ❑ F.J. Duarte, "Quantum Optics for Engineers," CRC Press, November 2017, ISBN:9781351832618 (Safari Book).
- ❑ Quantum Algorithm Zoo, (Compiled list of Quantum algorithms), <http://quantumalgorithmzoo.org/>

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# Wikipedia Links

- ❑ <https://en.wikipedia.org/?title=Inner-product&redirect=no>
- ❑ [https://en.wikipedia.org/wiki/Bra%E2%80%93ket\\_notation](https://en.wikipedia.org/wiki/Bra%E2%80%93ket_notation)
- ❑ [https://en.wikipedia.org/wiki/Complex\\_number](https://en.wikipedia.org/wiki/Complex_number)
- ❑ [https://en.wikipedia.org/wiki/Controlled\\_NOT\\_gate](https://en.wikipedia.org/wiki/Controlled_NOT_gate)
- ❑ [https://en.wikipedia.org/wiki/Dot\\_product](https://en.wikipedia.org/wiki/Dot_product)
- ❑ [https://en.wikipedia.org/wiki/Fourier\\_transform](https://en.wikipedia.org/wiki/Fourier_transform)
- ❑ [https://en.wikipedia.org/wiki/Greatest\\_common\\_divisor](https://en.wikipedia.org/wiki/Greatest_common_divisor)
- ❑ [https://en.wikipedia.org/wiki/List\\_of\\_quantum\\_processors](https://en.wikipedia.org/wiki/List_of_quantum_processors)
- ❑ [https://en.wikipedia.org/wiki/Matrix\\_multiplication](https://en.wikipedia.org/wiki/Matrix_multiplication)
- ❑ [https://en.wikipedia.org/wiki/Polar\\_coordinate\\_system](https://en.wikipedia.org/wiki/Polar_coordinate_system)
- ❑ <https://en.wikipedia.org/wiki/Quantum>
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- ❑ [https://en.wikipedia.org/wiki/Quantum\\_error\\_correction](https://en.wikipedia.org/wiki/Quantum_error_correction)
- ❑ [https://en.wikipedia.org/wiki/Quantum\\_Fourier\\_transform](https://en.wikipedia.org/wiki/Quantum_Fourier_transform)

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# Wikipedia Links (Cont)

- ❑ [https://en.wikipedia.org/wiki/Quantum\\_logic\\_gate](https://en.wikipedia.org/wiki/Quantum_logic_gate)
- ❑ [https://en.wikipedia.org/wiki/Quantum\\_machine\\_learning](https://en.wikipedia.org/wiki/Quantum_machine_learning)
- ❑ [https://en.wikipedia.org/wiki/Quantum\\_mechanics](https://en.wikipedia.org/wiki/Quantum_mechanics)
- ❑ [https://en.wikipedia.org/wiki/Quantum\\_simulator](https://en.wikipedia.org/wiki/Quantum_simulator)
- ❑ [https://en.wikipedia.org/wiki/Quantum\\_supremacy](https://en.wikipedia.org/wiki/Quantum_supremacy)
- ❑ [https://en.wikipedia.org/wiki/Quantum\\_technology](https://en.wikipedia.org/wiki/Quantum_technology)
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- ❑ <https://en.wikipedia.org/wiki/Qubit>
- ❑ [https://en.wikipedia.org/wiki/Shor%27s\\_algorithm](https://en.wikipedia.org/wiki/Shor%27s_algorithm)
- ❑ [https://en.wikipedia.org/wiki/Superconducting\\_quantum\\_computing](https://en.wikipedia.org/wiki/Superconducting_quantum_computing)
- ❑ [https://en.wikipedia.org/wiki/Sycamore\\_processor](https://en.wikipedia.org/wiki/Sycamore_processor)
- ❑ [https://en.wikipedia.org/wiki/Tensor\\_product](https://en.wikipedia.org/wiki/Tensor_product)
- ❑ [https://en.wikipedia.org/wiki/Timeline\\_of\\_quantum\\_computing](https://en.wikipedia.org/wiki/Timeline_of_quantum_computing)
- ❑ [https://en.wikipedia.org/wiki/Category:Quantum\\_gates](https://en.wikipedia.org/wiki/Category:Quantum_gates)

## Student Questions

# Classic Papers on Quantum Computing

- ❑ R. P. Feynman, “Simulating Physics with Computers,” *International journal of theoretical physics* 21.6 (1982): 467-488, <http://www.springerlink.com/index/t2x8115127841630.pdf>
- ❑ D. E. Deutsch, “Quantum theory, the Church-Turing principle and the universal quantum computer,” *Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences* 400.1818 (1985): 97-117, , <https://royalsocietypublishing.org/doi/abs/10.1098/rspa.1985.0070>
- ❑ D. E. Deutsch, “Quantum Computational Networks,” *Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences* 425.1868 (1989), 73-90. , <https://royalsocietypublishing.org/doi/pdf/10.1098/rspa.1989.0099> (subscribers only)
- ❑ P. W. Shor, “Algorithms for Quantum Computation: Discrete Log and Factoring,” *Proceedings of the 35th Annual Symposium on the Foundations of Computer Science*, IEEE, 1994, p. 124
- ❑ A. Barenco et al., “Elementary gates for quantum computation,” *Physical Review A*, March 22, 1995, <https://arxiv.org/pdf/quant-ph/9503016>

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# Classic Papers (Cont)

- ❑ L. K. Grover, “A fast quantum mechanical algorithm for database search,” Proceedings, STOC 1996, Philadelphia PA, USA, pp. 212-219, <https://arxiv.org/pdf/quant-ph/9605043>
- ❑ G. Brassard et al., “Quantum Counting,” 1998, <https://arxiv.org/pdf/quant-ph/9805082>
- ❑ G. Brassard et al., “Quantum Amplitude Amplification and Estimation,” 2000, <https://arxiv.org/pdf/quant-ph/0005055>
- ❑ S. Lloyd, “Quantum Algorithm for Solving Linear Systems of Equations,” American Physical Society, APS March Meeting 2010, March 15-19,2010

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[http://www.cse.wustl.edu/~jain/cse570-21/m\\_19qnt.htm](http://www.cse.wustl.edu/~jain/cse570-21/m_19qnt.htm)

# Related Modules



CSE567M: Computer Systems Analysis (Spring 2013),

[https://www.youtube.com/playlist?list=PLjGG94etKypJEKjNAa1n\\_1X0bWWNyZcof](https://www.youtube.com/playlist?list=PLjGG94etKypJEKjNAa1n_1X0bWWNyZcof)

CSE473S: Introduction to Computer Networks (Fall 2011),

[https://www.youtube.com/playlist?list=PLjGG94etKypJWOSPMh8Azcg5e\\_10TiDw](https://www.youtube.com/playlist?list=PLjGG94etKypJWOSPMh8Azcg5e_10TiDw)



Wireless and Mobile Networking (Spring 2016),

[https://www.youtube.com/playlist?list=PLjGG94etKypKeb0nzyN9tSs\\_HCd5c4wXF](https://www.youtube.com/playlist?list=PLjGG94etKypKeb0nzyN9tSs_HCd5c4wXF)

CSE571S: Network Security (Fall 2011),

<https://www.youtube.com/playlist?list=PLjGG94etKypKvzfVtutHcPFJXumyyg93u>



Video Podcasts of Prof. Raj Jain's Lectures,

<https://www.youtube.com/channel/UCN4-5wzNP9-ruOzQMs-8NUw>

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