Other Experimental Designs

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These slides are available on-line at:

http://www.cse.wustl.edu/~jain/cse567-17/

- Analysis of Covariance
- Plackett-Burman Designs
- Box-Behenken Designs
- Response Surface Analysis

Covariate

- Covariate: A factor that cannot be controlled but can be measured
  - Generally a continuous variable such as temperature
  - Can be added as a predictor in a regression
- Example: Two categorical factors A, B, and a covariate x

$$y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_{ABij} + c_x x_{ijk} + e_{ijk}$$

- Assumption: The effect of covariate is independent of other variables and is additive

$$\bar{y}_{...} = \mu + c_x x_{...}$$

$$y_{ijk} - y_{...} = \alpha_i + \beta_j + \gamma_{ABij} + c_x(x_{ijk} - x_{...}) + e_{ijk}$$

$$\begin{array}{ccccccccccc}
\sum(y_{ijk} - y_{...})^2 & = & \sum\alpha_i^2 & + & \sum\beta_j^2 & + & \sum\gamma_{ij}^2 & + & c_x^2\sum(x_{ijk} - x_{...})^2 & + & \sum e_{ijk}^2 \\
SST & = & SSA & + & SSB & + & SSAB & + & SSX & + & SSE \\
abr - 1 & = & (a-1) & + & (b-1) & + & (a-1)(b-1) & + & 1 & + & ab(r-1) - 1
\end{array}$$

General Full Factorial Designs With k Factors

- Model: k factors $\Rightarrow$ 2^k-1 effects

  k main effects

  $\binom{k}{2}$ two factor interactions,

  $\binom{k}{3}$ three factor interactions,

  and so on.

Example: 3 factors A, B, C:

$$
\begin{aligned}
y_{ijkl} &= \mu + \alpha_i + \beta_j + \xi_k + \gamma_{ABij} + \gamma_{ACik} + \gamma_{BCjk} + \\
&+ \gamma_{ABCijk} + e_{ijkl}
\end{aligned}
$$

$$i = 1, \ldots, a; \quad j = 1, \ldots, b; \quad k = 1, \ldots, c; \quad l = 1, \ldots, r;$$

Model Parameters

y_{ijkl}	$=$	Response in the lth replication with factors A, B, and C at levels i, j, and k, respectively.
μ	$=$	Mean response
α_i	$=$	Effect of factor A at level i
β_j	$=$	Effect of factor B at level j
ξ_k	$=$	Effect of factor C at level k
γ_{ABij}	$=$	Interaction between A and B at levels i and j.
γ_{ABCijk}	$=$	Interaction between A, B, C at levels i, j, and k.

and so on

- Analysis: Similar to that with two factors

$$\mu = \bar{y}_{....}$$

$$\alpha_i = \bar{y}_{i...} - \bar{y}_{....}$$

- The sums of squares, degrees of freedom, and F-test also extend as expected. }

Case Study 23.1: Paging Process

Factors and Levels for Page Swap Study

Symbol	Factor	Levels		
		1	2	3
A	Page Replacement Algorithm	LRUV	FIFO	RAND
D	Deck Arrangement	GROUP	FREQY	ALPHA
P	Problem Program	Small	Medium	Large
M	Memory Pages	24P	20P	16P

- Total 81 experiments.

Case Study 23.1 (Cont)

- Total Number of Page Swaps

Algor-ithm	Prog-ram	GROUP			FREQY			ALPHA		
		24P	20P	16P	24P	20P	16P	24P	20P	16P
LRUV	Small	32	48	538	52	244	998	59	536	1348
	Medium	53	81	1901	112	776	3621	121	1879	4639
	Large	142	197	5689	262	2625	10012	980	5698	12880
FIFO	Small	49	67	789	79	390	1373	85	814	1693
	Medium	100	134	3152	164	1255	4912	206	3394	5838
	Large	233	350	9100	458	3688	13531	1633	10022	17117
RAND	Small	62	100	1103	111	480	1782	111	839	2190
	Medium	96	245	2807	237	1502	6007	286	3092	7654
	Large	265	2012	12429	517	4870	18602	1728	8834	23134

- $y_{max}/y_{min} = 23134/32 = 723 \Rightarrow$ log transformation

Case Study 23.1 (Cont)

- Transformed Data For the Paging Study

Algorithm	Program	GROUP			FREQY			ALPHA		
		24P	20P	16P	24P	20P	16P	24P	20P	16P
LRUV	Small	1.51	1.68	2.73	1.72	2.39	3.00	1.77	2.73	3.13
	Medium	1.72	1.91	3.28	2.05	2.89	3.56	2.08	3.27	3.67
	Large	2.15	2.29	3.76	2.42	3.42	4.00	2.99	3.76	4.11
FIFO	Small	1.69	1.83	2.90	1.90	2.59	3.14	1.93	2.91	3.23
	Medium	2.00	2.13	3.50	2.21	3.10	3.69	2.31	3.53	3.77
	Large	2.37	2.54	3.96	2.66	3.57	4.13	3.21	4.00	4.23
RAND	Small	1.79	2.00	3.04	2.05	2.68	3.25	2.05	2.92	3.34
	Medium	1.98	2.39	3.58	2.37	3.18	3.78	2.46	3.49	3.88
	Large	2.42	2.30	4.09	2.71	3.69	4.27	3.24	3.95	4.36

Case Study 23.1 (Cont)

- Effects: $\alpha_1 = y_{1\ldots} - y_{\ldots.} = 2.74 - 2.90 = -0.16$

Main Effects

	Level		
Factor	1	2	3
A	-0.16	0.02	0.14
D	-0.36	0.07	0.29
P	-0.47	-0.02	0.49
M	-0.69	-0.01	0.70

- Also
  - Six two-factor interactions,
  - Four three-factor interactions, and
  - One four-factor interaction.

Case Study 23.1: ANOVA Table

Component	Sum of Squares	%Variation	DF	Mean Square
y	730.01		81	
$\bar{y}_{...}$	681.21		1	
y-$\bar{y}_{....}$	48.80	100%	80	
Main Effects	45.80	93.85%	8	5.7
A	1.30		2	
D	6.10		2	
P	12.30		2	
M	26.20		2	
First-order Interactions	2.40	4.91%	24	0.1
AD	0.07		4	
AP	0.02		4	
AM	0.03		4	
DP	0.15		4	
DM	1.96		4	
PM	0.14		4	
Second-order Interactions	0.48	0.98%	32	0.015
ADP	0.05		8	
ADM	0.13		8	
APM	0.04		8	
DPM	0.26		8	
Third-order Interaction (ADPM)	0.07	0.14%	16	0.004

Case Study 23.1: Simplified model

- Most interactions except DM are small.

$$y_{ijkl} = \mu + \alpha_i + \beta_j + \gamma_k + \delta_l + \xi_{jl}$$

Where,

$$\begin{aligned}
\mu &= \text{grand mean} \\
\alpha_i &= \text{Effect of A} \\
\beta_j &= \text{Effect of D} \\
\gamma_k &= \text{Effect of P} \\
\delta_l &= \text{Effect of M} \\
\xi_{jl} &= \text{Interaction between D and M.}
\end{aligned}$$

Case Study 23.1: Simplified Model (Cont)

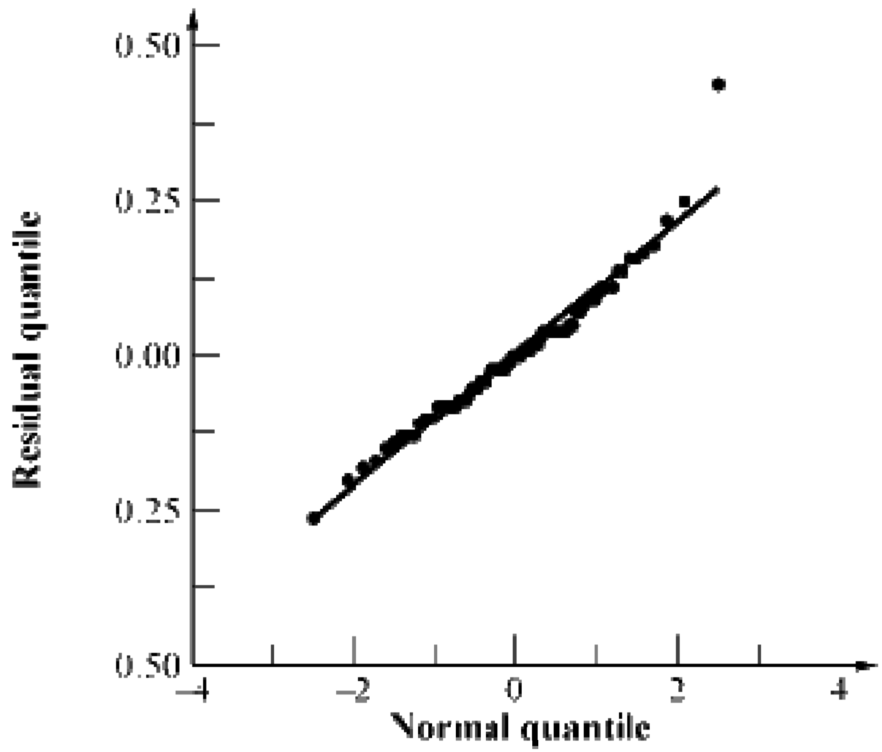
- Interactions Between Deck Arrangement and Memory Pages

		M		
		1	2	3
D	1	0.11	-0.30	0.19
	2	-0.05	0.09	-0.04
	3	-0.06	0.21	-0.15

Case Study 23.1: Error Computation

Algor-ithm	Prog-ram	GROUP			FREQY			ALPHA		
		24P	20P	16P	24P	20P	16P	24P	20P	16P
LRUV	Small	0.18	0.08	-0.07	0.11	-0.04	-0.02	-0.05	-0.04	0.01
	Medium	-0.05	-0.13	0.04	0.01	0.02	0.10	-0.18	0.07	0.11
	Large	-0.13	-0.26	0.01	-0.14	0.04	0.03	0.22	0.04	0.04
FIFO	Small	0.17	0.04	0.09	0.11	-0.02	-0.07	-0.08	-0.04	-0.08
	Medium	0.05	-0.10	0.07	-0.02	0.04	0.05	-0.13	0.14	0.02
	Large	-0.10	-0.20	0.02	-0.00	0.00	-0.03	0.25	0.09	-0.02
RAND	Small	0.16	0.09	-0.06	0.14	-0.05	-0.07	-0.08	-0.08	-0.08
	Medium	-0.10	0.04	0.04	-0.02	0.00	0.01	-0.11	-0.02	-0.02
	Large	-0.17	0.44	0.04	-0.15	0.00	-0.01	0.16	-0.08	-0.01

Case Study 23.1: Visual Test



- Almost a straight line.
- Outlier was verified.

Case Study 23.1: Final Model

$$\text{LPS} = 2.90 + A\left\{\begin{array}{r}-0.16\\+0.02\\+0.14\end{array}\right. + D\left\{\begin{array}{r}-0.36\\+0.07\\+0.29\end{array}\right. + P\left\{\begin{array}{r}-0.47\\-0.02\\+0.49\end{array}\right. + M\left\{\begin{array}{r}-0.69\\-0.01\\+0.70\end{array}\right.$$

Log (Pages Swapped)

Replacement Algorithm
1. LRUV
2. FIFO
3. RAND

1. GROUP
2. FREQY
3. ALPHA

1. Small
2. Medium
3. Large

1. 24P
2. 20P
3. 16P

$$+D\left\{\overbrace{\begin{array}{rrr}-0.27 & 1.40 & 1.30\\-0.60 & 0.80 & -0.20\\3.30 & -2.20 & -1.10\end{array}}^{M}\right. \pm 0.12$$

Standard Error
= Stdv of sample mean
= Stdv of Error

Observation Method

- To find the best combination.
- Example: Scheduler Design
- Three Classes of Jobs:
  - Word processing
  - Interactive data processing
  - Background data processing
- Five Factors 2^{5-1} design

Example 23.1: Measured Throughputs

No.	A	B	C	D	E	T_W	T_I	T_B
1	-1	-1	-1	-1	1	15.0	25.0	15.2
2	1	-1	-1	-1	-1	11.0	41.0	3.0
3	-1	1	-1	-1	-1	25.0	36.0	21.0
4	1	1	-1	-1	1	10.0	15.7	8.6
5	-1	-1	1	-1	-1	14.0	63.9	7.5
6	1	-1	1	-1	1	10.0	13.2	7.5
7	-1	1	1	-1	1	28.0	36.3	20.2
8	1	1	1	-1	-1	11.0	23.0	3.0
9	-1	-1	-1	1	-1	14.0	66.1	6.4
10	1	-1	-1	1	1	10.0	9.1	8.4
11	-1	1	-1	1	1	27.0	34.6	15.7
12	1	1	-1	1	-1	11.0	23.0	3.0
13	-1	-1	1	1	1	14.0	26.0	12.0
14	1	-1	1	1	-1	11.0	38.0	2.0
15	-1	1	1	1	-1	25.0	35.0	17.2
16	1	1	1	1	1	11.0	22.0	2.0

Example 23.1: Conclusions

To get high throughput for word processing jobs,:

1. There should not be any preemption (A=-1)
2. The time slice should be large (B=1)
3. The fairness should be on (E=1)
4. The settings for queue assignment and re-queueing do not matter.

Ranking Method

- Sort the experiments.

No.	A	B	C	D	E	T_W	T_I	T_B
7	-1	1	1	-1	1	28.0	36.3	20.2
11	-1	1	-1	1	1	27.0	34.6	15.7
15	-1	1	1	1	-1	25.0	35.0	17.2
3	-1	1	-1	-1	-1	25.0	36.0	21.0
1	-1	-1	-1	-1	1	15.0	25.0	15.2
5	-1	-1	1	-1	-1	14.0	63.9	7.5
9	-1	-1	-1	1	-1	14.0	66.1	6.4
13	-1	-1	1	1	1	14.0	26.0	12.0
2	1	-1	-1	-1	-1	11.0	41.0	3.0
8	1	1	1	-1	-1	11.0	23.0	3.0
12	1	1	-1	1	-1	11.0	23.0	3.0
14	1	-1	1	1	-1	11.0	38.0	2.0
16	1	1	1	1	1	11.0	22.0	2.0
6	1	-1	1	-1	1	10.0	13.2	7.5
4	1	1	-1	-1	1	10.0	15.7	8.6
10	1	-1	-1	1	1	10.0	9.1	8.4

Example 23.2: Conclusions

1. A=-1 (no preemption) is good for word processing jobs and also that A=1 is bad.
2. B=1 (large time slice) is good for such jobs. No strong negative comment can be made about B=-1.
3. Given a choice C should be chosen at 1, that is, there should be two queues.
4. The effect of E is not clear.
5. If top rows chosen, then E=1 is a good choice.

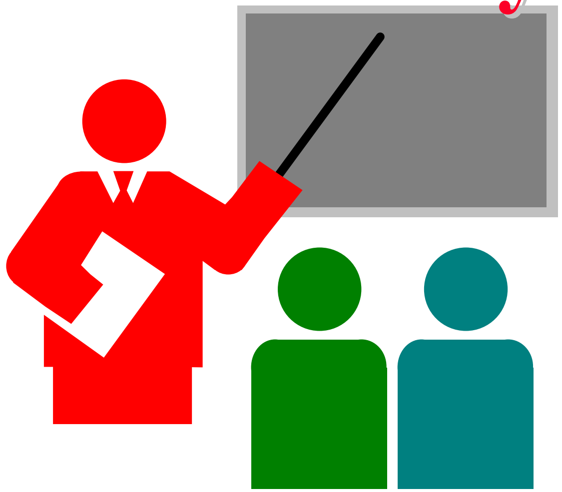
Range Method

- Range = Maximum-Minimum
- Factors with large range are important.

Factor	Level 1	Level 2	Level 3	Range of of Averages
Replacement Algorithm	2056	2986	3781	1725
Deck Arrangement	1584	2913	4326	2742
Problem Program	592	2047	6185	5593
Memory Size	305	2006	6512	6207

- Memory size is the most influential factor.
- Problem program, deck arrangement, and replacement algorithm are next in order.

Summary



- A general k factor design can have k main effects, two factor interactions, three factor interactions, and so on.
- Information Methods:
  - Observation: Find the highest or lowest response
  - Ranking: Sort all responses
  - Range: Largest - smallest average response

Homework 23

- Analyze the following results using observation and ranking methods.

No.	A	B	C	D	E	T
1	-1	-1	-1	-1	1	13.2
2	1	-1	-1	-1	-1	4.0
3	-1	1	-1	-1	-1	22.0
4	1	1	-1	-1	1	9.6
5	-1	-1	1	-1	-1	6.5
6	1	-1	1	-1	1	8.5
7	-1	1	1	-1	1	21.2
8	1	1	1	-1	-1	2.0
9	-1	-1	-1	1	-1	7.4
10	1	-1	-1	1	1	7.4
11	-1	1	-1	1	1	14.7
12	1	1	-1	1	-1	4.0
13	-1	-1	1	1	1	13.0
14	1	-1	1	1	-1	3.0
15	-1	1	1	1	-1	18.2
16	1	1	1	1	1	3.0

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