$2^k r$ Factorial Designs

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These slides are available on-line at:

http://www.cse.wustl.edu/~jain/cse567-17/

- Computation of Effects
- Estimation of Experimental Errors
- Allocation of Variation
- Confidence Intervals for Effects
- Confidence Intervals for Predicted Responses
- Visual Tests for Verifying the assumptions
- Multiplicative Models

$2^k r$ Factorial Designs

- *r* replications of 2^k Experiments
  ⇒ $2^k r$ observations.
  ⇒ Allows estimation of experimental errors.

- Model:

$$y = q_0 + q_A x_A + q_B x_B + q_{AB} x_A x_B + e$$

- e = Experimental error

Computation of Effects

Simply use means of r measurements

I	A	B	A B	y	Mean $\bar{y}$
1	-1	-1	1	(15, 18, 12)	15
1	1	-1	-1	(45, 48, 51)	48
1	-1	1	-1	(25, 28, 19)	24
1	1	1	1	(75, 75, 81)	77
164	86	38	20		total
41	21.5	9.5	5		total/4

- Effects: $q_0 = 41$, $q_A = 21.5$, $q_B = 9.5$, $q_{AB} = 5$.

Estimation of Experimental Errors

- Estimated Response:

$$\hat{y}_i = q_0 + q_A x_{Ai} + q_B x_{Bi} + q_{AB} x_{Ai} x_{Bi}$$

Experimental Error = Measured - Estimated

$$e_{ij} = y_{ij} - \hat{y}_i$$
$$= y_{ij} - q_0 - q_A x_{Ai} - q_B x_{Bi} - q_{AB} x_{Ai} x_{Bi}$$

$$\sum_{i,j} e_{ij} = 0$$

Sum of Squared Errors: $\text{SSE} = \sum_{i=1}^{2^2} \sum_{j=1}^{r} e_{ij}^2$

Experimental Errors: Example

- Estimated Response:

$$\hat{y}_1 = q_0 - q_A - q_B + q_{AB} = 41 - 21.5 - 9.5 + 5 = 15$$

- Experimental errors:

$$e_{11} = y_{11} - \hat{y}_1 = 15 - 15 = 0$$

i	Effect I	Effect A	Effect B	Effect A B	Estimated Response	Measured Responses			Errors		
	41	21.5	9.5	5	$\hat{y}_i$	y_{i1}	y_{i2}	y_{i3}	e_{i1}	e_{i2}	e_{i3}
1	1	-1	-1	1	15	15	18	12	0	3	-3
2	1	1	-1	-1	48	45	48	51	-3	0	3
3	1	-1	1	-1	24	25	28	19	1	4	-5
4	1	1	1	1	77	75	75	81	-2	-2	4

Allocation of Variation

- Total variation or total sum of squares:

$$\text{SST} = \sum_{i,j} (y_{ij} - \bar{y}_{..})^2$$

$$y_{ij} = q_0 + q_A x_{Ai} + q_B x_{Bi} + q_{AB} x_{Ai} x_{Bi} + e_{ij}$$

$$\begin{array}{ccccccccc} \sum_{i,j}(y_{ij} - \bar{y}_{..})^2 & = & 2^2 r q_A^2 & + & 2^2 r q_B^2 & + & 2^2 r q_{AB}^2 & + & \sum_{i,j} e_{ij}^2 \\ \text{SST} & = & \text{SSA} & + & \text{SSB} & + & \text{SSAB} & + & \text{SSE} \end{array}$$

Example 18.3: Memory-Cache Study

$$\begin{aligned}
SSY &= 15^2 + 18^2 + 12^2 + 45^2 + \cdots + 75^2 + 75^2 + 81^2 \\
&= 27204 \\
SS0 &= 2^2 r q_0^2 = 12 \times 41^2 = 20172 \\
SSA &= 2^2 r q_A^2 = 12 \times (21.5)^2 = 5547 \\
SSB &= 2^2 r q_B^2 = 12 \times (9.5)^2 = 1083 \\
SSAB &= 2^2 r q_{AB}^2 = 12 \times 5^2 = 300 \\
SSE &= 27204 - 2^2 \times 3(41^2 + 21.5^2 + 9.5^2 + 5^2) \\
&= 102 \\
SST &= SSY - SS0 \\
&= 27204 - 20172 = 7032
\end{aligned}$$

Example 18.3 (Cont)

$$\text{SSA} + \text{SSB} + \text{SSAB} + \text{SSE}$$
$$= 5547 + 1083 + 300 + 102$$
$$= 7032 = \text{SST}$$

Factor A explains 5547/7032 or 78.88%

Factor B explains 15.40%

Interaction AB explains 4.27%

1.45% is unexplained and is attributed to errors.

Confidence Intervals For Effects

- Effects are random variables.
- Errors $\sim N(0,\sigma_e) \Rightarrow y \sim N(\bar{y}_{\cdot\cdot}, \sigma_e)$

$$q_0 = \frac{1}{2^2 r} \sum_{i,j} y_{ij}$$

- q_0 = Linear combination of normal variates
  $\Rightarrow q_0$ is normal with variance $\sigma_e^2/(2^2 r)$

Variance of errors:

$$s_e^2 = \frac{1}{2^2(r-1)} \sum_{ij} e_{ij}^2 = \frac{SSE}{2^2(r-1)} \triangleq MSE$$

- Denominator = $2^2(r-1)$ = # of independent terms in SSE
  $\Rightarrow$ SSE has $2^2(r-1)$ degrees of freedom.
  Estimated variance of q_0: $s_{q_0}^2 = s_e^2/(2^2 r)$

Conf. Intervals For Effects (Cont)

- Similarly,

$$s_{q_A} = s_{q_B} = s_{q_{AB}} = \frac{s_e}{\sqrt{2^2 r}}$$

- Confidence intervals (CI) for the effects:

$$q_i \mp t_{[1-\alpha/2;2^2(r-1)]} s_{q_i}$$

- CI does not include a zero ⇒ significant

Example 18.4

- For Memory-cache study: Standard deviation of errors:

$$s_e = \sqrt{\frac{SSE}{2^2(r-1)}} = \sqrt{\frac{102}{8}} = \sqrt{12.75} = 3.57$$

- Standard deviation of effects:

$$s_{q_i} = s_e/\sqrt{(2^2r)} = 3.57/\sqrt{12} = 1.03$$

- For 90% Confidence: $t_{[0.95,8]} = 1.86$

- Confidence intervals: $q_i \mp (1.86)(1.03) = q_i \mp 1.92$

q_0= (39.08, 42.91)

q_A=(19.58, 23.41)

q_B=(7.58, 11.41)

q_{AB}= (3.08, 6.91)

- No zero crossing ⇒ All effects are significant.

Confidence Intervals for Contrasts

- Contrast: Linear combination with $\sum$ coefficients = 0

$$\sum h_i q_i \text{ with } \sum h_i = 0$$

  For example, $q_A - q_B$ or $q_A + q_B - 2q_{AB}$

- Mean of $\sum h_i q_i = \sum h_i E[q_i]$

- Variance of $\sum h_i q_i$ $\quad s^2_{\sum h_i q_i} = \frac{s_e^2 \sum h_i^2}{2^2 r}$

- For 100(1-α)% confidence interval, use $t_{[1-\alpha/2;\, 2^2(r-1)]}$.

Example 18.5

Memory-cache study

$u = q_A + q_B - 2q_{AB}$

Coefficients= 0, 1, 1, and -2 ⇒ Contrast

$$\text{Mean } \bar{u} = 21.5 + 9.5 - 2 \times 5 = 21$$

$$\text{Variance } s_u^2 = \frac{s_e^2 \times 6}{2^2 \times 3} = 6.375$$

$$\text{Standard deviation } s_u = \sqrt{6.375} = 2.52$$

$t_{[0.95;8]} = 1.86$

90% Confidence interval for u:

$$\bar{u} \mp t s_u = 21 \mp 1.86 \times 2.52 = (16.31, 25.69)$$

Conf. Interval For Predictions

- Mean response $\hat{y}$:

$$\hat{y} = q_0 + q_A x_A + q_B x_B + q_{AB} x_A x_B$$

- The standard deviation of the mean of m responses:

$$s_{\hat{y}_m} = s_e \left(\frac{1}{n_{\text{eff}}} + \frac{1}{m} \right)^{1/2}$$

$$\begin{aligned} n_{\text{eff}} &= \text{Effective deg of freedom} \\ &= \frac{\text{Total number of runs}}{1 + \text{Sum of DFs of params used in } \hat{y}} \\ &= \frac{2^2 r}{5} \end{aligned}$$

Conf. Interval for Predictions (Cont)

100(1-α)% confidence interval:

$$\hat{y} \mp t_{[1-\alpha/2;2^2(r-1)]} s_{\hat{y}_m}$$

- A single run (m=1): $s_{\hat{y}_1} = s_e \left(\frac{5}{2^2 r} + 1 \right)^{1/2}$
- Population mean (m=∞): $s_{\hat{y}} = s_e \left(\frac{5}{2^2 r} \right)^{1/2}$

Example 18.6: Memory-cache Study

- For $x_A = -1$ and $x_B = -1$:
- A single confirmation experiment:

$$\begin{aligned} \hat{y}_1 &= q_0 - q_A - q_B + q_{AB} \\ &= 41 - 21.5 - 9.5 + 5 = 15 \end{aligned}$$

- Standard deviation of the prediction:

$$s_{\hat{y}_1} = s_e \left(\frac{5}{2^2 r} + 1 \right)^{1/2} = 3.57 \sqrt{\frac{5}{12} + 1} = 4.25$$

- Using $t_{[0.95;8]} = 1.86$, the 90% confidence interval is:

$$15 \mp 1.86 \times 4.25 = (7.09,\ 22.91)$$

Example 18.6 (Cont)

- Mean response for 5 experiments in future:

$$
\begin{aligned}
s_{\hat{y}_1} &= s_e \left(\frac{5}{2^2 r} + \frac{1}{m} \right)^{1/2} \\
&= 3.57 \sqrt{\frac{5}{12} + \frac{1}{5}} = 2.80
\end{aligned}
$$

- The 90% confidence interval is:

$$15 \mp 1.86 \times 2.80 = (9.79,\ 20.21)$$

Example 18.6 (Cont)

- Mean response for a large number of experiments in future:

$$s_{\hat{y_1}} = s_e \left(\frac{5}{2^2 r}\right)^{1/2} = 3.57\sqrt{\frac{5}{12}} = 2.30$$

- The 90% confidence interval is:
  $15 \mp 1.86 \times 2.30 = (10.72,\ 19.28)$
- Current mean response: Not for future. Use contrasts formula.

$$s_{\hat{y_1}} = \sqrt{\frac{s_e^2 \sum h_i^2}{2^2 r}} = \sqrt{\frac{12.75 \times 4}{12}} = 2.06$$

- 90% confidence interval:

  $15 \mp 1.86 \times 2.06 = (11.17,\ 18.83)$

Homework 18A

Updated Exercise 18.1: The following table lists measured CPU times for two processors on two workloads. Each experiment was repeated three times. Determine the effects.

Table 18.12 $2^2 3$ Experimental Design Exercise

Workload	Processor	
	A	B
I	(41.16, 39.02, 42.56)	(65.17, 69.25, 64.23)
J	(53.50, 55.50, 50.50)	(50.08, 48.98, 47.10)

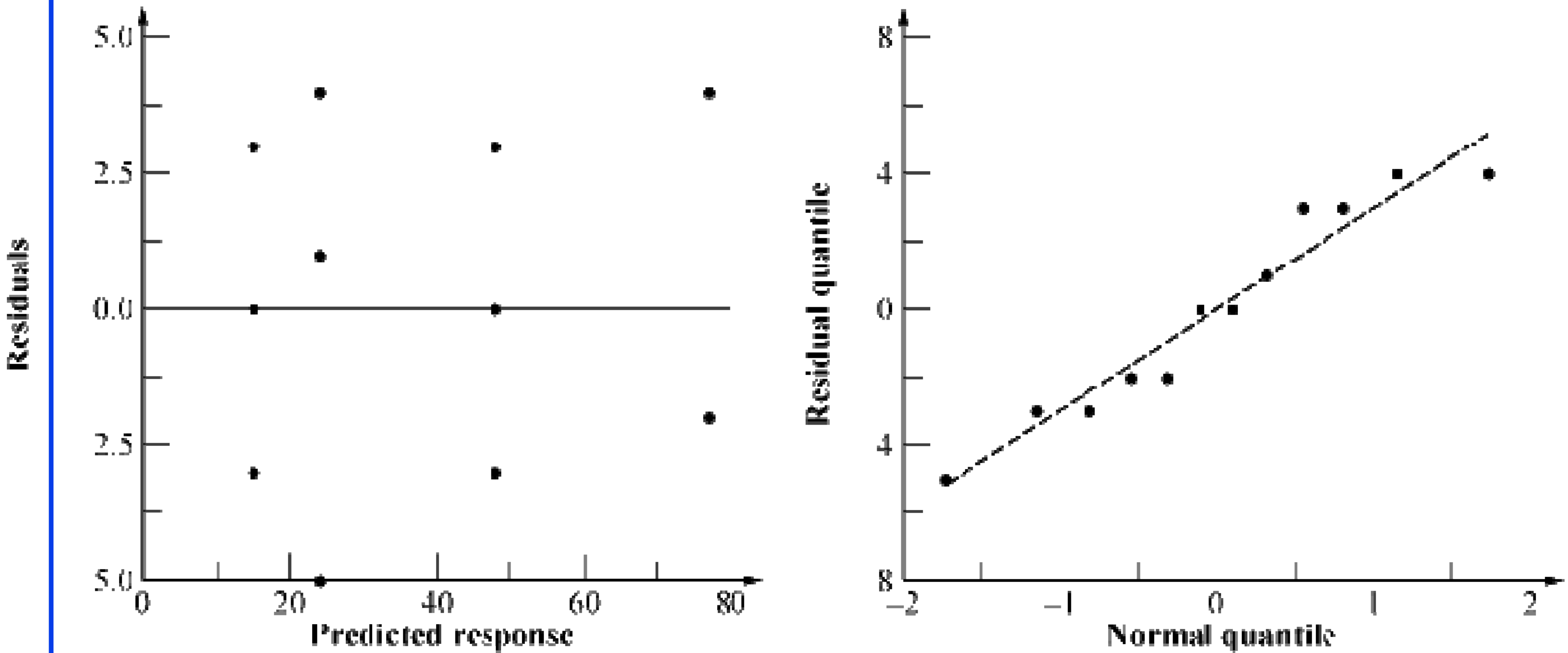
Assumptions

1. Errors are statistically independent.
2. Errors are additive.
3. Errors are normally distributed.
4. Errors have a constant standard deviation σ_e.
5. Effects of factors are additive
   $\Rightarrow$ observations are independent and normally distributed with constant variance.

Visual Tests

1. **Independent Errors**:
   - Scatter plot of residuals versus the predicted response $\hat{y}_i$
   - Magnitude of residuals < Magnitude of responses/10
     $\Rightarrow$ Ignore trends
   - Plot the residuals as a function of the experiment number
   - Trend up or down $\Rightarrow$ other factors or side effects
2. **Normally distributed errors**:
   Normal quantile-quantile plot of errors
3. **Constant Standard Deviation of Errors**:
   Scatter plot of y for various levels of the factor
   Spread at one level significantly different than that at other
   $\Rightarrow$ Need transformation

Example 18.7: Memory-cache



Multiplicative Models

- Additive model:

  $$y_{ij} = q_0 + q_A x_A + q_B x_B + q_{AB} x_A x_B + e_{ij}$$

- Not valid if effects do not add.
  E.g., execution time of workloads.

  *i*th processor speed= v_i instructions/second.

  *j*th workload Size= w_j instructions

- The two effects multiply. Logarithm ⇒ additive model:

  $$\text{Execution Time } y_{ij} = w_j / v_i$$

  $$\log(y_{ij}) = \log(w_j) - \log(v_i)$$

- Correct Model:

  $$y'_{ij} = q_0 + q_A x_A + q_B x_B + q_{AB} x_A x_B + e_{ij}$$

  Where, $y'_{ij} = \log(y_{ij})$

Multiplicative Model (Cont)

- Taking an antilog of effects:

  $u_A = 10^{q_A}$, $u_B = 10^{q_B}$, and $u_{AB} = 10^{q_{AB}}$
- u_A = ratio of MIPS rating of the two processors
- u_B = ratio of the size of the two workloads.
- Antilog of additive mean $q_0 \Rightarrow$ geometric mean

$$\dot{y} = 10^{q_0} = (y_1 y_2 \cdots y_n)^{1/n} \quad n = 2^2 r$$

Example 18.8: Execution Times

Analysis Using an Additive Model

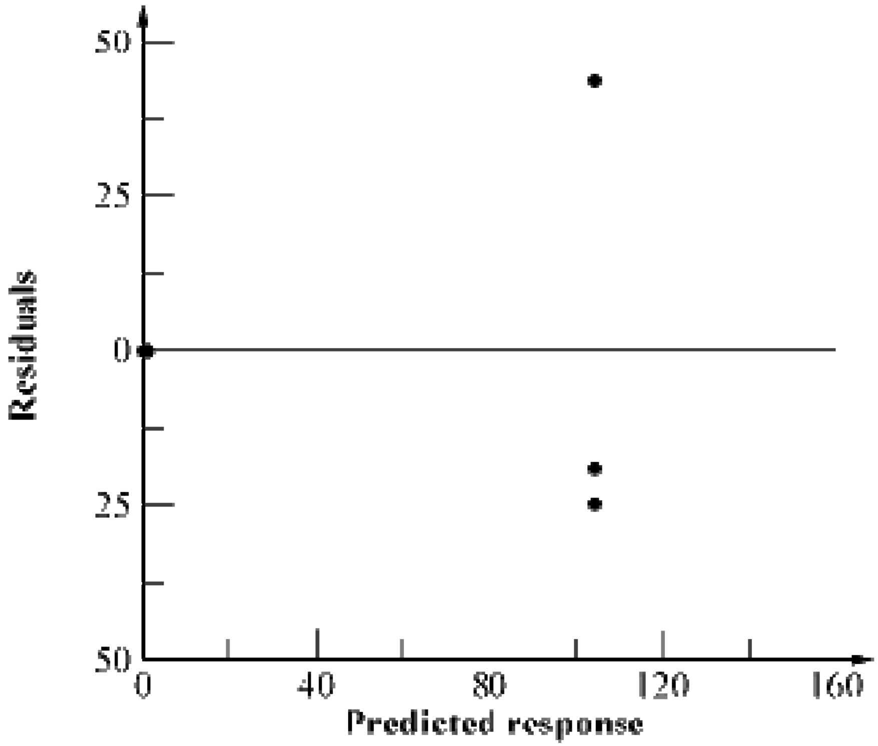
I	A	B	AB	y	Mean $\bar{y}$
1	-1	-1	1	(85.10, 79.50, 147.90)	104.170
1	1	-1	-1	(0.891, 1.047, 1.072)	1.003
1	-1	1	-1	(0.955, 0.933, 1.122)	1.003
1	1	1	1	(0.0148, 0.0126, 0.0118)	0.013
106.19	-104.15	-104.15	102.17	total	
26.55	-26.04	-26.04	25.54	total/4	

Additive model is not valid because:

- Physical consideration ⇒ effects of workload and processors do not add. They multiply.
- Large range for y. y_{max}/y_{min}= 147.90/0.0118 or 12,534 ⇒ log transformation
- Taking an arithmetic mean of 104.17 and 0.013 is inappropriate.

Example 18.8 (Cont)

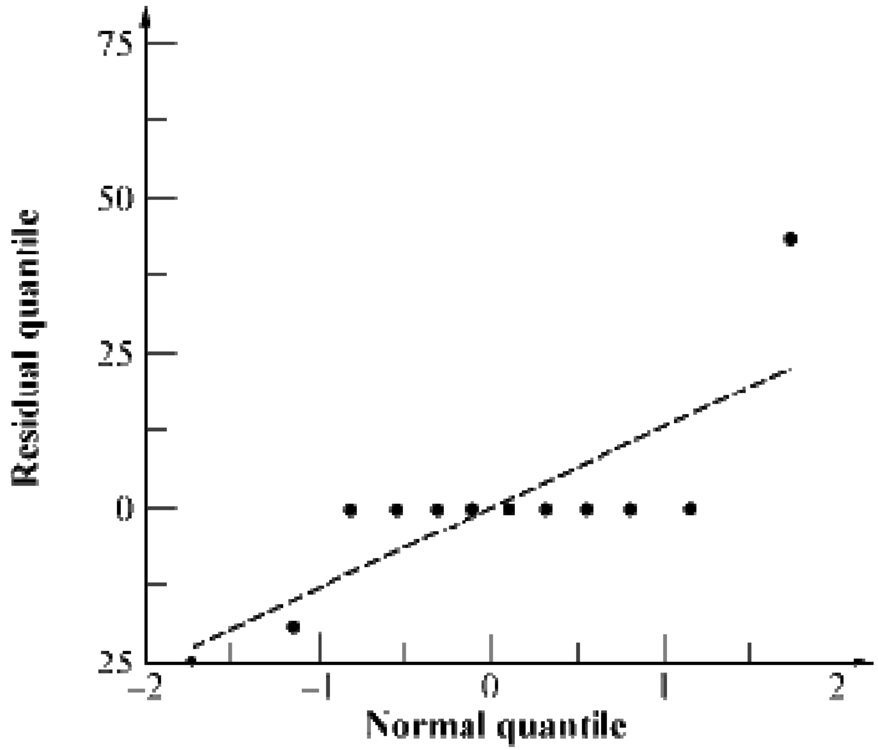
- The residuals are not small as compared to the response.



- The spread of residuals is large at larger value of the response.
  $\Rightarrow$ log transformation

Example 18.8 (Cont)

- Residual distribution has a longer tail than normal



Analysis Using Multiplicative Model

Data After Log Transformation

I	A	B	AB	y	Mean $\bar{y}$
1	-1	-1	1	(1.93, 1.90, 2.17)	2.00
1	1	-1	-1	(-0.05, 0.02, 0.03)	0.00
1	-1	1	-1	(-0.02, -0.03, 0.05)	0.00
1	1	1	1	(-1.83, -1.90, -1.93)	-1.89
0.11	-3.89	-3.89	0.11	total	
0.03	-0.97	-0.97	0.03	total/4	

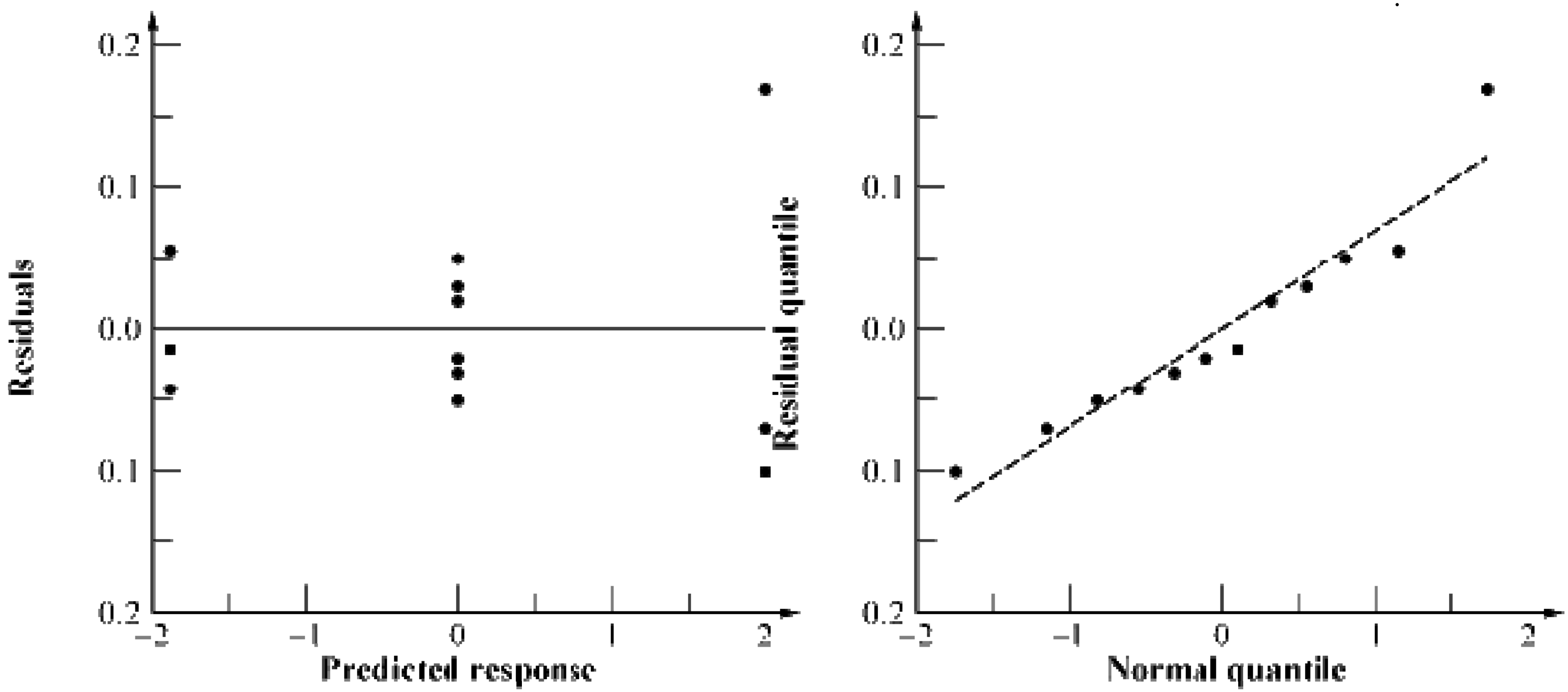
Variation Explained by the Two Models

	Additive Model			Multiplicative Model		
Factor	Effect	% Var.	Conf. Interval	Effect	% Var.	Conf. Interval
I	26.55		(16.35, 36.74)	0.03		(-0.02, 0.07)†
A	-26.04	30.1%	(-36.23, -15.84)	-0.97	49.9%	(-1.02, -0.93)
B	-26.04	30.1%	(-36.23, -15.84)	-0.97	49.9%	(-1.02, -0.93)
AB	25.54	29.0%	(15.35, 35.74)	0.03	0.0%	(-0.02, 0.07)†
e		10.8%			0.2%	

† ⇒ Not Significant

- With multiplicative model:
  - Interaction is almost zero.
  - Unexplained variation is only 0.2%

Visual Tests



- **Conclusion**: Multiplicative model is better than the additive model.

Interpretation of Results

$$\log(y) = q_0 + q_A x_A + q_B x_B + q_{AB} x_A x_B + e$$

$$\begin{aligned}\Rightarrow y &= 10^{q_0} 10^{q_A x_A} 10^{q_B x_B} 10^{q_{AB} x_A x_B} 10^e \\ &= 10^{0.03} 10^{-0.97 x_A} 10^{-0.97 x_B} 10^{0.03 x_A x_B} 10^e \\ &= 1.07 \times 0.107^{x_A} \times 0.107^{x_B} \times 1.07^{x_A x_B} 10^e\end{aligned}$$

- The time for an average processor on an average benchmark is 1.07.
- The time on processor A_1 is 9.35 times (0.107^{-1}) that on an average processor. The time on A_2 is 1/9.35 (0.107^1) of that on an average processor.
- MIPS rate for A_2 is 87.4 times that of A_1.
- Benchmark B_1 executes 87.4 times more instructions than B_2.
- The interaction is negligible.

  ⇒ Results apply to all benchmarks and processors.

Transformation Considerations

- y_{max}/y_{min} small $\Rightarrow$ Multiplicative model results similar to additive model.
- Many other transformations possible.
- Box-Cox family of transformations:

$$w = \begin{cases} \frac{y^a - 1}{a g^{a-1}}, & a \neq 0 \\ (\ln y) g, & a = 0 \end{cases}$$

- Where *g* is the geometric mean of the responses:

$$g = (y_1 y_2 \cdots y_n)^{1/n}$$

- w has the same units as y.
- *a* can have any real value, positive, negative, or zero.
- Plot SSE as a function of *a* $\Rightarrow$ optimal *a*
- Knowledge about the system behavior should always take precedence over statistical considerations.

General $2^k r$ Factorial Design

- Model:

$$y_{ij} = q_0 + q_A x_{Ai} + q_B x_{Bi} + q_{AB} x_{Ai} x_{Bi} + \cdots + e_{ij}$$

- Parameter estimation:

$$q_j = \frac{1}{2^k} \sum_{i=1}^{2^k} S_{ij} \bar{y}_i$$

S_{ij} = *(i,j)*th entry in the sign table.

- Sum of squares:

$$\text{SSY} = \sum_{i=1}^{2^k} \sum_{j=1}^{r} y_{ij}^2$$
$$\text{SS0} = 2^k r q_0^2$$
$$\text{SST} = \text{SSY} - \text{SS0}$$
$$\text{SS}j = 2^k r q_j^2 \qquad \text{j} = 1, 2, \ldots, 2^k - 1$$
$$\text{SSE} = \text{SST} - \sum_{j=1}^{2^k-1} \text{SS}j$$

$$\begin{array}{ccccccccc} \text{SST} & = & \text{SSY} & - & \text{SS0} & = & \sum_{j=1}^{2^k} \text{SSj} & + & \text{SSE} \\ 2^k r - 1 & = & 2^k r & - & 1 & = & \sum_{j=1}^{2^k} 1 & + & 2^k(r-1) \end{array}$$

General $2^k r$ Factorial Design (Cont)

- Percentage of y's variation explained by *j*th effect =

$$(\text{SS}j/\text{SST}) \times 100\%$$

- Standard deviation of errors:

$$s_e = \sqrt{\frac{\text{SSE}}{2^k(r-1)}}$$

- Standard deviation of effects:

$$s_{q_0} = s_{q_A} = s_{q_B} = s_{q_{AB}} = s_e/\sqrt{2^k r}$$

- Variance of contrast $\sum h_i q_i$, where $\sum h_i = 0$ is:

$$s^2_{\sum h_i q_i} = (s_e^2 \sum h_i^2)/2^k r$$

General $2^k r$ Factorial Design (Cont)

- Standard deviation of the mean of m future responses:

$$s_{\hat{y}_p} = s_e \left(\frac{1+2^k}{2^k r} + \frac{1}{m} \right)^{1/2}$$

- Confidence intervals are calculated using $t_{[1-\alpha/2;2^k(r-1)]}$.
- Modeling assumptions:
  - Errors are IID normal variates with zero mean.
  - Errors have the same variance for all values of the predictors.
  - Effects and errors are additive.

Visual Tests for $2^k r$ Designs

- The scatter plot of errors versus predicted responses should not have any trend.
- The normal quantile-quantile plot of errors should be linear.
- Spread of y values in all experiments should be comparable.

Example 18.9: A $2^3 3$ Design

I	A	B	C	A B	A C	B C	A B C	y	Mean $\bar{\mathbf{y}}$
1	-1	-1	-1	1	1	1	-1	(14, 16, 12)	14
1	1	-1	-1	-1	-1	1	1	(22, 18, 20)	20
1	-1	1	-1	-1	1	-1	1	(11, 15, 19)	15
1	1	1	-1	1	-1	-1	-1	(34, 30, 35)	33
1	-1	-1	1	1	-1	-1	1	(46, 42, 44)	44
1	1	-1	1	-1	1	-1	-1	(58, 62, 60)	60
1	-1	1	1	-1	-1	1	-1	(50, 55, 54)	53
1	1	1	1	1	1	1	1	(86, 80, 74)	80
319	67	43	155	23	19	15	-1	total	
39.87	8.375	5.375	19.37	2.875	2.375	1.875	-0.125	total/8	

Example 18.9 (Cont)

- Sum of Squares:

Component	Sum of Squares	Percent Variation
y	4.9E4	
$\bar{y}$	3.8E4	
y-$\bar{y}$	1.1E4	100.00%
A	1683.0	14.06%
B	693.3	5.79%
C	9009.0	75.27%
AB	198.3	1.66%
AC	135.4	1.13%
BC	84.4	0.70%
ABC	0.4	0.00%
Errors	164.0	1.37%

Example 18.9 (Cont)

- The errors have $2^3(3-1)$ or 16 degrees of freedom. Standard deviation of errors:

$$s_e = \sqrt{\frac{\text{SSE}}{2^k(r-1)}} = \sqrt{\frac{164}{16}} = 3.20$$

- Standard deviation of effects:

$$s_{q_i} = s_e/\sqrt{(2^3 3)} = 3.20/\sqrt{24} = 0.654$$

Example 18.9 (Cont)

- % Variation:

Component	Sum of Squares	Percent Variation
y	4.9E4	
$\bar{y}$	3.8E4	
y-$\bar{y}$	1.1E4	100.00%
A	1683.0	14.06%
B	693.3	5.79%
C	9009.0	75.27%
AB	198.3	1.66%
AC	135.4	1.13%
BC	84.4	0.70%
ABC	0.4	0.00%
Errors	164.0	1.37%

Example 18.9 (Cont)

- $t_{[0.90, 16]}$=1.337
- 80% confidence intervals for parameters: $q_i \mp (1.337)(0.654)$ $= q_i \mp 0.874$

$$
\begin{aligned}
q_0 &= (39.00, 40.74)\\
q_A &= (7.50, 9.25)\\
q_B &= (4.50, 6.25)\\
q_C &= (18.50, 20.24)\\
q_{AB} &= (2.00, 3.75)\\
q_{AC} &= (1.50, 3.25)\\
q_{BC} &= (1.00, 2.75)\\
q_{ABC} &= (-1.00, 0.75)
\end{aligned}
$$

- All effects except q_{ABC} are significant.

Example 18.9 (Cont)

- For a single confirmation experiment (m = 1)

  With A = B = C = -1:

$$\hat{y} = 14$$

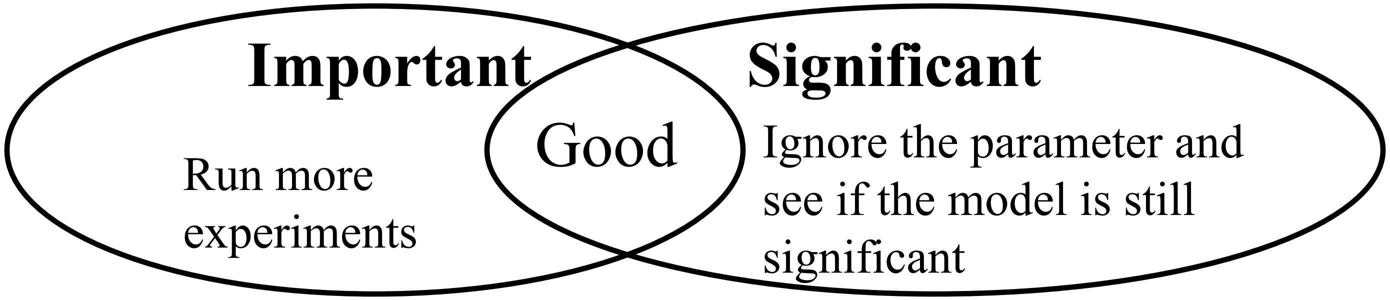
$$\begin{aligned} s_{\hat{y}} &= s_e \left(\frac{9}{2^k r} + \frac{1}{m} \right)^{1/2} \\ &= 3.2 \left(\frac{9}{24} + 1 \right)^{1/2} \\ &= 3.75 \end{aligned}$$

- 80% confidence interval:

$$14 \mp 1.337 \times 3.75 = 14 \mp 5.02 = (8.98,\ 19.02)$$

Importance vs. Significance

- Important = Parameters or models that explain a high percent of variation
- Significant = Parameters or models whose confidence interval does not include zero



Case Study 18.1: Garbage collection

Factors and Levels

Variable	Factor	-1	1
A	Workload	Single Task	Several parallel tasks
B	Compiler	Simple	Deallocating
C	Limbo List	Enabled	Disabled
D	Chunk Size	4K bytes	16K bytes

Case Study 18.1 (Cont)

I	A	B	C	D	y	Mean $\bar{y}$
1	-1	-1	-1	-1	(97, 97, 97)	97.00
1	1	-1	-1	-1	(31, 31, 32)	31.33
1	-1	1	-1	-1	(97, 97, 97)	97.00
1	1	1	-1	-1	(31, 32, 31)	31.33
1	-1	-1	1	-1	(97, 97, 97)	97.00
1	1	-1	1	-1	(32, 32, 31)	31.67
1	-1	1	1	-1	(97, 97, 97)	97.00
1	1	1	1	-1	(32, 32, 32)	32.00
1	-1	-1	-1	1	(407, 407, 407)	407.00
1	1	-1	-1	1	(135, 136, 135)	135.33
1	-1	1	-1	1	(409, 409, 409)	409.00
1	1	1	-1	1	(135, 135, 136)	135.33
1	-1	-1	1	1	(407, 407, 407)	407.00
1	1	-1	1	1	(139, 140, 139)	139.33
1	-1	1	1	1	(409, 409, 409)	409.00
1	1	1	1	1	(139, 139, 140)	139.33
2695.67	-1344.33	4.33	9.00	1667.00		total
168.48	-84.02	0.27	0.56	104.19		total/16

Case Study 18.1 (Cont)

Factor	Effect	% Variation	Conf. Interval
I	168.48		(168.386, 168.573)
A	-84.02	34.4%	(-84.114, -83.927)
B	0.27	0.0%	(0.177, 0.364)
C	0.56	0.0%	(0.469, 0.656)
D	104.19	52.8%	(104.094, 104.281)
AB	-0.23	0.0%	(-0.323, -0.136)
AC	0.56	0.0%	(0.469, 0.656)
AD	-51.31	12.8%	(-51.406, -51.219)
BC	0.02	0.0%	(-0.073, 0.114)†
BD	0.23	0.0%	(0.136, 0.323)
CD	0.44	0.0%	(0.344, 0.531)
ABC	0.02	0.0%	(-0.073, 0.114)†
ABD	-0.27	0.0%	(-0.364, -0.177)
ACD	0.44	0.0%	(0.344, 0.531)
BCD	-0.02	0.0%	(-0.114, 0.073)†
ABCD	-0.02	0.0%	(-0.114, 0.073)†

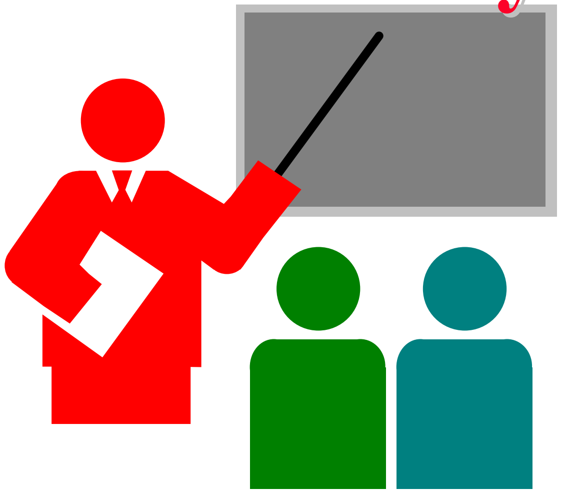
† $\Rightarrow$ Not Significant

Case Study 18.1: Conclusions

- Most of the variation is explained by factors A (Workload), D (Chunk size), and the interaction A D between the two.
- The variation due to experimental error is small

  ⇒ Several effects that explain less than 0.05% of variation (listed as 0.0%) are statistically significant.
- Only effects A, D, and AD are both practically significant and statistically significant.

Summary



- Replications allow estimation of measurement errors
  $\Rightarrow$ Confidence Intervals of parameters
  $\Rightarrow$ Confidence Intervals of predicted responses
- Allocation of variation is proportional to square of effects
- Multiplicative models are appropriate if the factors multiply
- Visual tests for independence normal errors

Homework 18B

Updated Exercise 18.1: For the data of Homework 18A, determine percentage of variation explained, find confidence intervals of the effects, and conduct visual tests.

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