Queueing Networks

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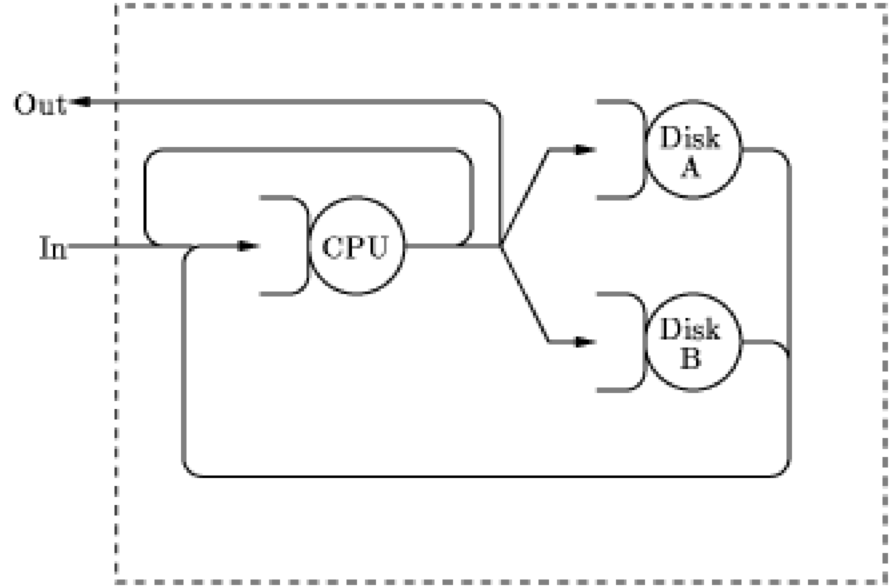
http://www.cse.wustl.edu/~jain/cse567-15/

1. Open and Closed Queueing Networks
2. Product Form Networks
3. Queueing Network Models of Computer Systems

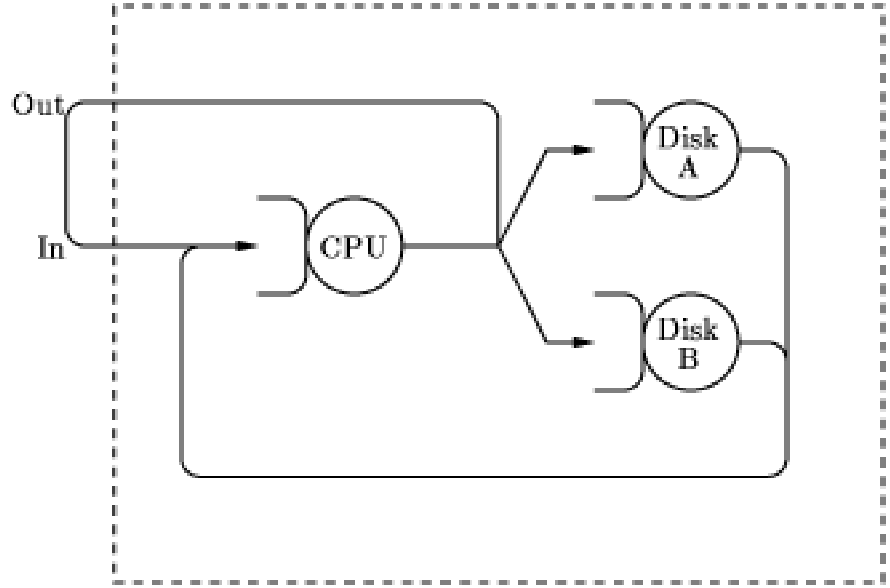
Open Queueing Networks

- **Queueing Network**: model in which jobs departing from one queue arrive at another queue (or possibly the same queue)
- **Open queueing network**: external arrivals and departures
  - Number of jobs in the system varies with time.
  - Throughput = arrival rate
  - Goal: To characterize the distribution of number of jobs in the system.



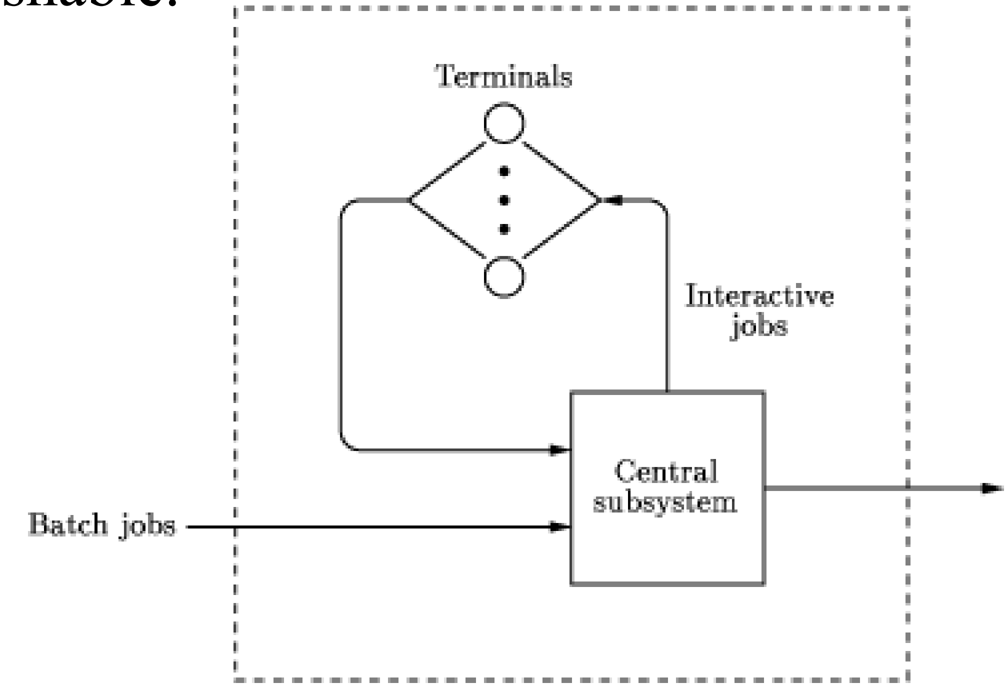
Closed Queueing Networks

- Closed queueing network: No external arrivals or departures
  - Total number of jobs in the system is constant
  - `OUT' is connected back to `IN.'
  - Throughput = flow of jobs in the OUT-to-IN link
  - Number of jobs is given, determine the throughput

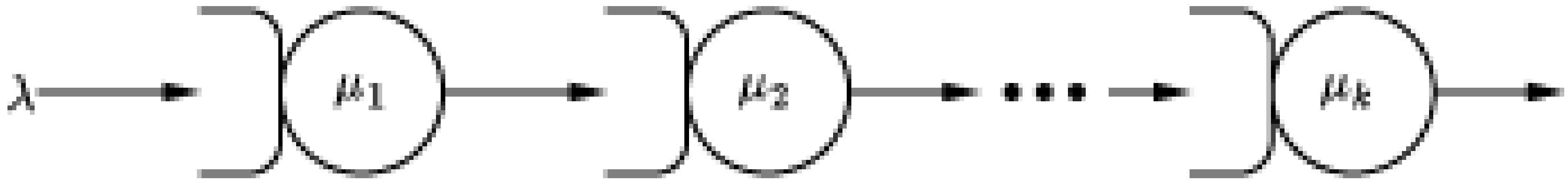


Mixed Queueing Networks

- **Mixed queueing networks**: Open for some workloads and closed for others ⇒ Two classes of jobs. **Class** = types of jobs. All jobs of a single class have the same service demands and transition probabilities. Within each class, the jobs are indistinguishable.



Series Networks



- *k M/M/1* queues in series
- Each individual queue can be analyzed independently of other queues
- Arrival rate= λ. If μ_i is the service rate for i^{th} server:

$$\text{Utilization of } i^{th} \text{ server} \rho_i = \lambda/\mu_i$$

$$\text{Probability of } n_i \text{ jobs in the } i^{\text{th}} \text{ queue } = (1-\rho_i)\rho_i^{n_i}$$

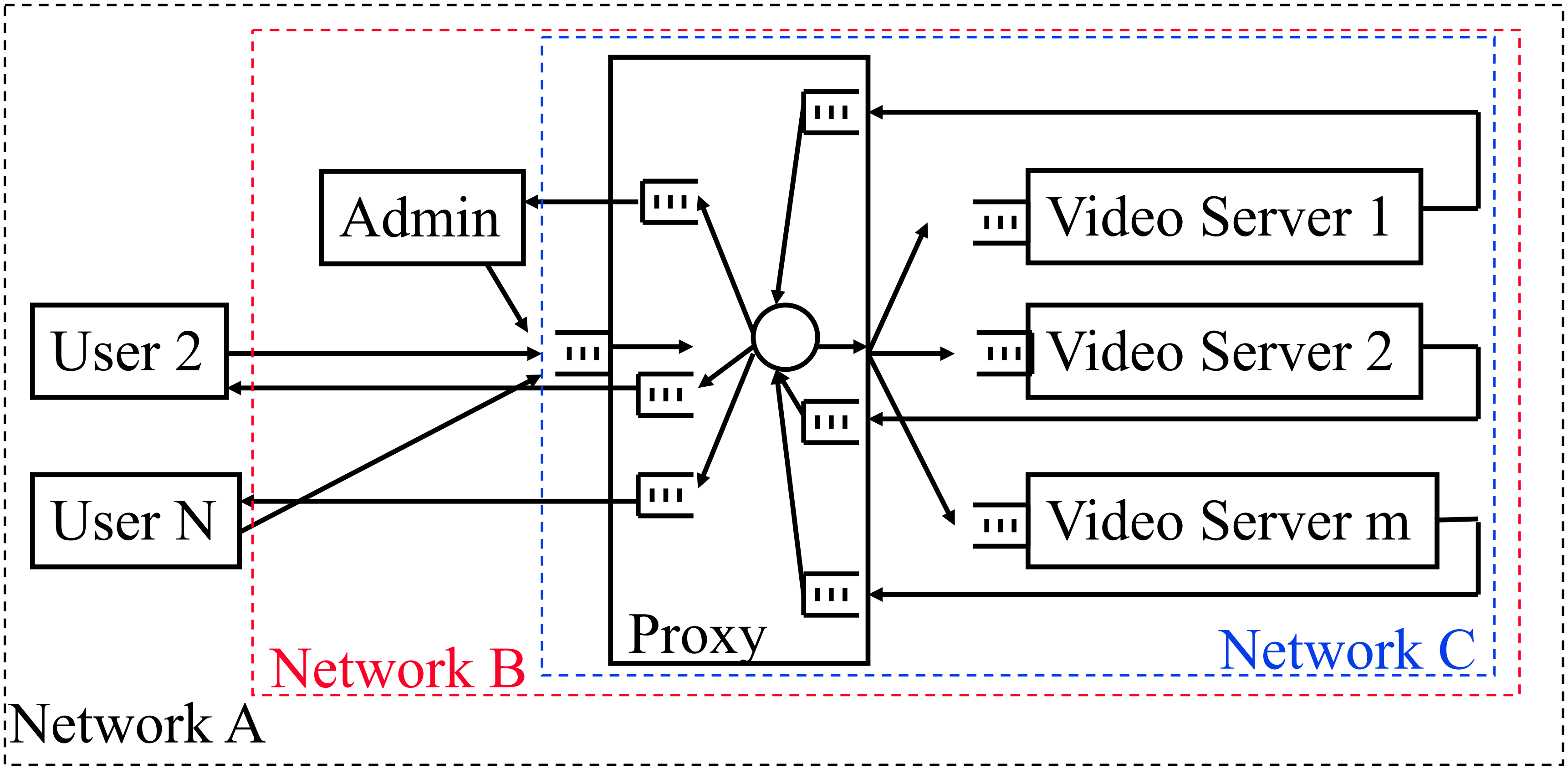
Series Networks (Cont)

- Joint probability of queue lengths:

$$P(n_1, n_2, n_3, \ldots, n_M)$$
$$= (1-\rho_1)\rho_1^{n_1}(1-\rho_2)\rho_2^{n_2}(1-\rho_3)\rho_3^{n_3}\cdots(1-\rho_M)\rho_M^{n_M}$$
$$= p_1(n_1)p_2(n_2)p_3(n_3)\cdots p_M(n_M)$$

$\Rightarrow$ product form network

Quiz 32A



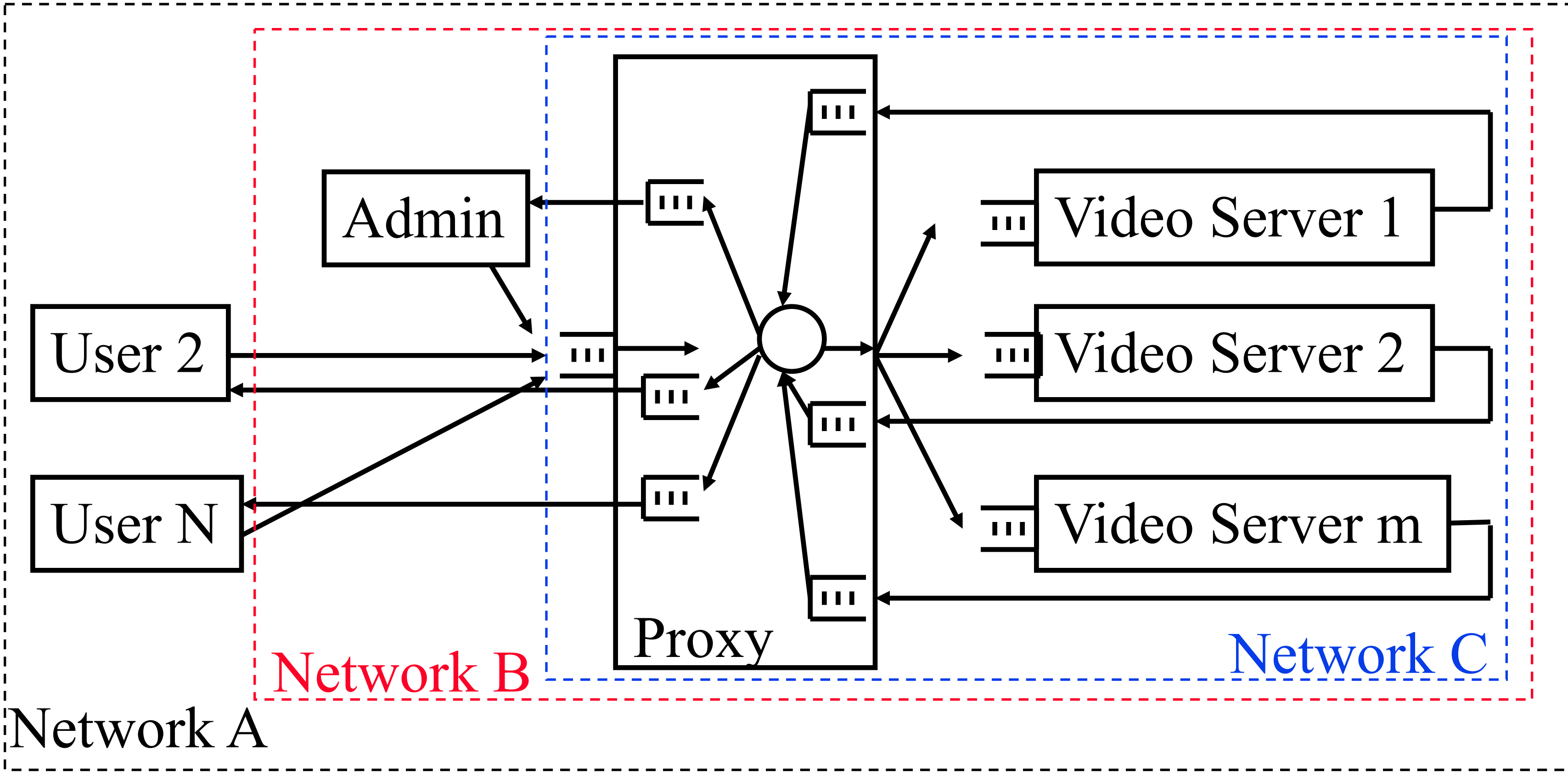
Identify open/closed/mixed networks:

A. Network A is ______________

B. Network B is ______________

C. Network C is ______________

Solution to Quiz 32A



Identify open/closed/mixed networks:

A. Network A is Closed.
B. Network B is Mixed.
C. Network C is Open.

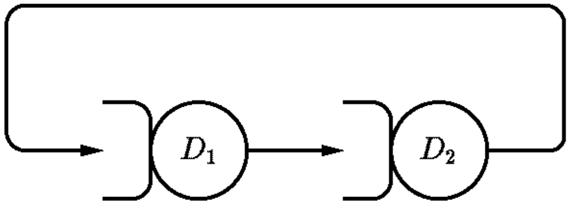
Product-Form Network

- Any queueing network in which:

$$P(n_1, n_2, \ldots, n_M) = \frac{1}{G(N)} \prod_{i=1}^{M} f_i(n_i)$$

- When $f_i(n_i)$ is some function of the number of jobs at the ith facility, $G(N)$ is a normalizing constant and is a function of the total number of jobs in the system.

Example 32.1

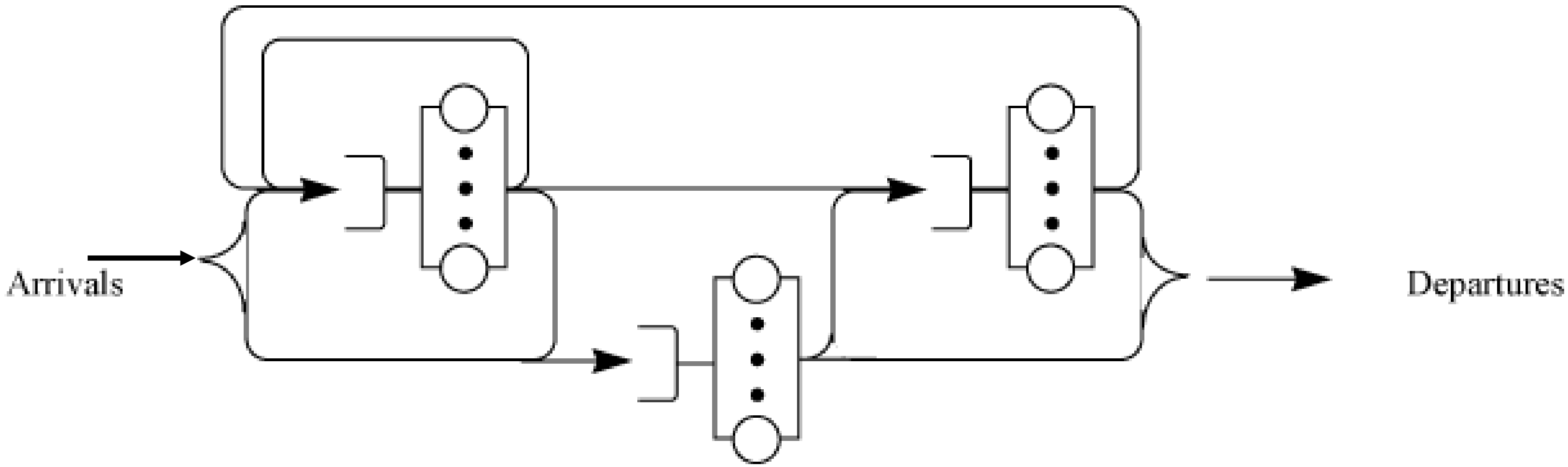


- Consider a closed system with two queues and N jobs circulating among the queues:
- Both servers have an exponentially distributed service time. The mean service times are 2 and 3, respectively. The probability of having n_1 jobs in the first queue and $n_2=N-n_1$ jobs in the second queue can be shown to be:

$$P(n_1, n_2) = \frac{1}{3^{N+1} - 2^{N+1}} \left(2^{n_1} \times 3^{n_2}\right)$$

- In this case, the normalizing constant $G(N)$ is $3^{N+1}-2^{N+1}$.
- The state probabilities are products of functions of the number of jobs in the queues. Thus, this is a ***product form network***.

General Open Network of Queues



- Product form networks are easier to analyze
- Jackson (1963) showed that any arbitrary open network of m-server queues with exponentially distributed service times has a product form

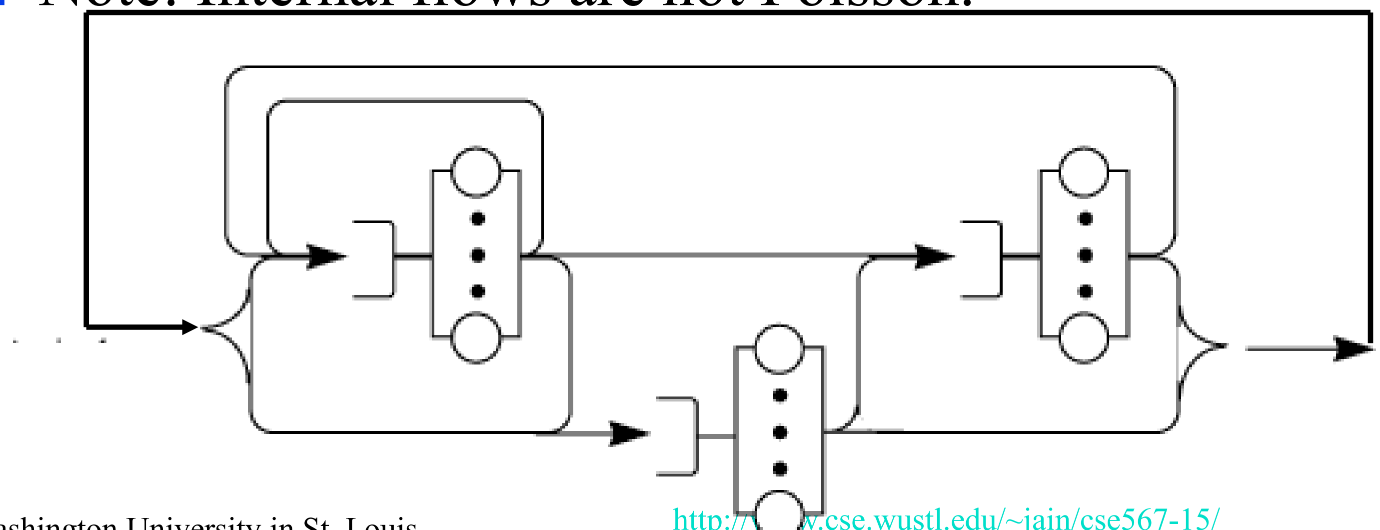
General Open Network of Queues (Cont)

- If all queues are single-server queues, the queue length distribution is:

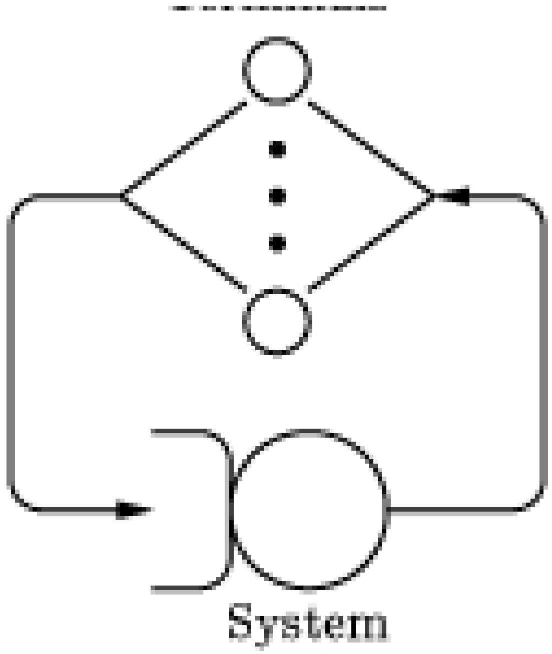
$$P(n_1, n_2, n_3, \ldots, n_M)$$
$$= (1-\rho_1)\rho_1^{n_1}(1-\rho_2)\rho_2^{n_2}(1-\rho_3)\rho_3^{n_3}\cdots(1-\rho_M)\rho_M^{n_M}$$
$$= p_1(n_1)p_2(n_2)p_3(n_3)\cdots p_M(n_M)$$

Closed Product-Form Networks

- Gordon and Newell (1967) showed that any arbitrary closed networks of m-server queues with exponentially distributed service times also have a product form solution.
- Baskett, Chandy, Muntz, and Palacios (1975) and then Denning and Buzen (1978) showed that product form solutions exist for an even broader class of networks.
- Note: Internal flows are not Poisson.



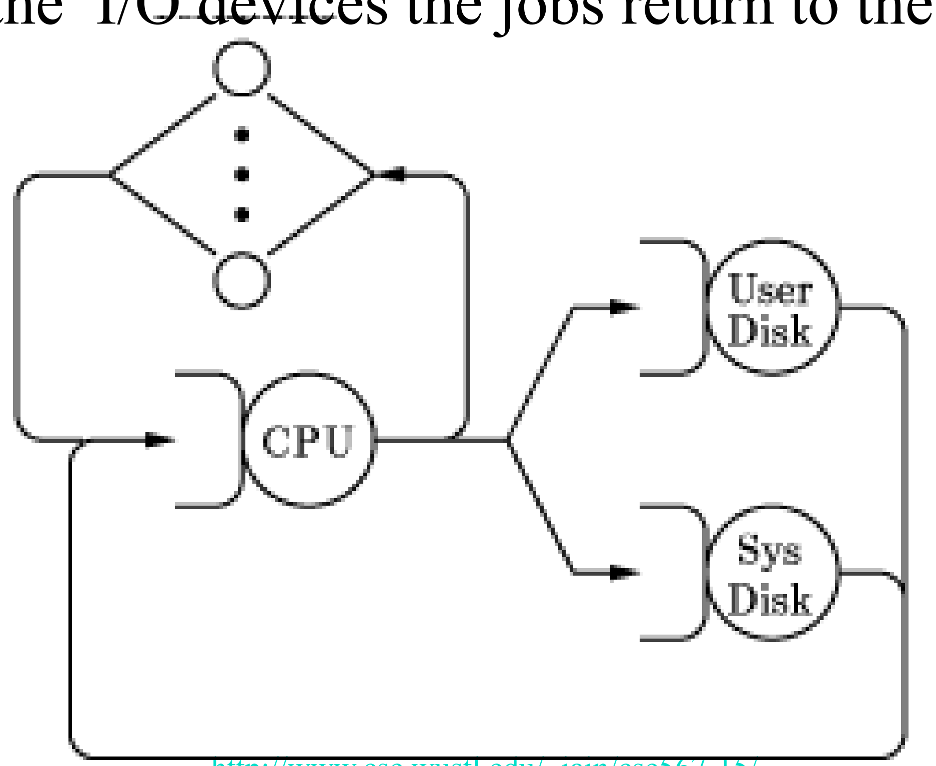
Machine Repairman Model



- Originally for machine repair shops
- A number of working machines with a repair facility with one or more servers (repairmen).
- Whenever a machine breaks down, it is put in the queue for repair and serviced as soon as a repairman is available
- Scherr (1967) used this model to represent a timesharing system with *n* terminals.
- Users sitting at the terminals generate requests (jobs) that are serviced by the system which serves as a repairman.
- After a job is done, it waits at the user-terminal for a random ``think-time'' interval before cycling again.

Central Server Model

- Introduced by Buzen (1973)
- The CPU is the ``central server'' that schedules visits to other devices
- After service at the I/O devices the jobs return to the CPU



Types of Service Centers

Three kinds of devices

1. **Fixed-capacity service centers**: Service time does not depend upon the number of jobs in the device

For example, the CPU in a system may be modeled as a fixed-capacity service center.

2. **Delay centers or infinite server**: No queueing. Jobs spend the same amount of time in the device regardless of the number of jobs in it. A group of dedicated terminals is usually modeled as a delay center.

3. **Load-dependent service centers**: Service rates may depend upon the load or the number of jobs in the device., e.g., *M/M/m* queue (with $m \geq 2$)

A group of parallel links between two nodes in a computer network is another example

Quiz 32B

- The probability function for jobs in a system with m queues is:

$$P(n_1, n_2, , n_m) = \frac{g(n_1)g(n_2)g(n_{m-1})}{g(n_m)}$$

Is this a product form network? ________

- Identify the type of server:
  A. Multi-core CPU: ____________
  B. Single-core CPU (No dynamic frequency scaling): ____________
  C. Single-core CPU (with dynamic frequency scaling): ____________
  D. Hard disk drives: ____________
  E. Solid state drives: ____________
  F. Multiple users each handling one window: ____________
  G. A user handling multiple windows: ____________

Solution to Quiz 32B

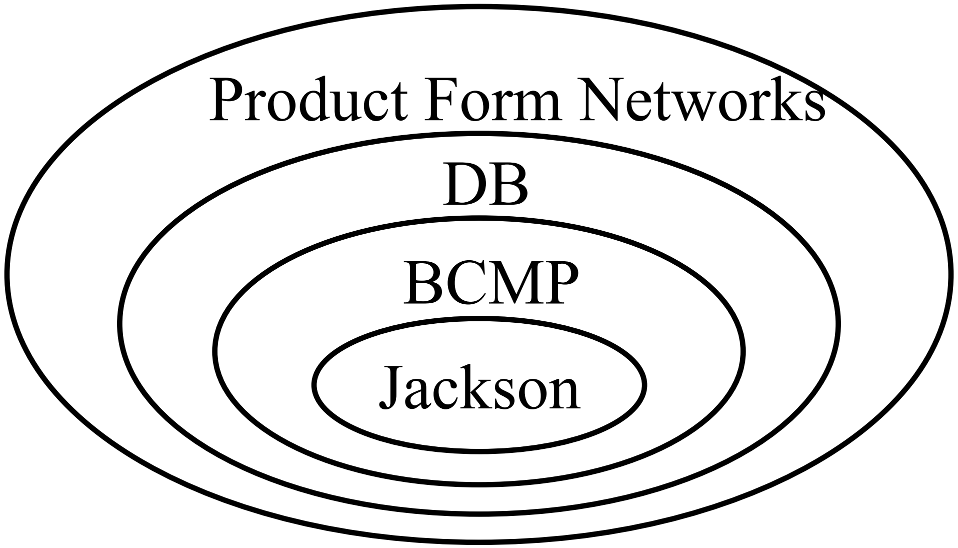
- The probability function for jobs in a system with m queues is:

$$P(n_1, n_2, , n_m) = \frac{g(n_1)g(n_2)g(n_{m-1})}{g(n_m)}$$

Is this a product form network? YES

- Identify the type of server:
  A. Multi-core CPU: Load dependent
  B. Single-core CPU (No dynamic frequency scaling): Fixed Capacity
  C. Single-core CPU (with dynamic frequency scaling): Load Dependent
  D. Hard disk drives: Load dependent
  E. Solid state drives: Fixed capacity
  F. Multiple users each handling one window: Delay Center
  G. A user handling multiple windows: Fixed capacity

Summary



- Open, Closed, and Mixed queueing networks
- Product form networks: Any network in which the system state probability is a product of device state probabilities
- Jackson: Network of M/M/m queues.
  BCMP: More general conditions
  Denning and Buzen: Even more general conditions
- Service centers: Fixed capacity, delay centers, load dependent

Homework 32

- In a series network of three routers, the packets arrive at the rate of 100 packets/second. The service rate of the three routers is 250 packets/s, 150 packets/s, and 200 packets/s.
- Write an expression for the state probability of the system.
- Calculate the probability of having 2 packets at each of the three routers.