# **Introduction to Experimental Design**

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http://www.cse.wustl.edu/~jain/cse567-06/



### **Experimental Design and Analysis**

How to:

- Design a proper set of experiments for measurement or simulation.
- Develop a model that best describes the data obtained.
- Estimate the contribution of each alternative to the performance.
- □ Isolate the measurement errors.
- □ Estimate confidence intervals for model parameters.
- □ Check if the alternatives are significantly different.
- □ Check if the model is adequate.

### Example

Personal workstation design

- 1. Processor: 68000, Z80, or 8086.
- 2. Memory size: 512K, 2M, or 8M bytes
- 3. Number of Disks: One, two, three, or four
- 4. Workload: Secretarial, managerial, or scientific.
- 5. User education: High school, college, or postgraduate level.

Five **Factors** at 3x3x4x3x3 **levels** 



### Terminology

- **Response Variable**: Outcome.
  - E.g., throughput, response time
- **Factors**: Variables that affect the response variable.

E.g., CPU type, memory size, number of disk drives, workload used, and user's educational level.

Also called predictor variables or predictors.

□ Levels: The values that a factor can assume, E.g., the CPU type has three levels: 68000, 8080, or Z80.

# of disk drives has four levels.

Also called **treatment**.

□ **Primary Factors**: The factors whose effects need to be quantified.

E.g., CPU type, memory size only, and number of disk drives.

## **Terminology (Cont)**

- Secondary Factors: Factors whose impact need not be quantified.
  - E.g., the workloads.
- **Replication**: Repetition of all or some experiments.
- Design: The number of experiments, the factor level and number of replications for each experiment.

E.g., Full Factorial Design with 5 replications:  $3 \times 3 \times 4 \times 3 \times 3$  or 324 experiments, each repeated five times.

- Experimental Unit: Any entity that is used for experiments.
   E.g., users. Generally, no interest in comparing the units.
- Goal minimize the impact of variation among the units.

### **Terminology (Cont)**

# □ Interaction $\Rightarrow$ Effect of one factor depends upon the level of the other.

 Table 1: Noninteracting Factors

	$A_1$	$A_2$
$B_1$	3	5
$B_2$	6	8

 Table 2: Interacting Factors

		$A_1$	$A_2$
	$B_1$	3	5
	$B_2$	6	9
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#### **Common Mistakes in Experimentation**

- □ The variation due to experimental error is ignored.
- □ Important parameters are not controlled.
- □ Effects of different factors are not isolated
- □ Simple one-factor-at-a-time designs are used
- □ Interactions are ignored
- □ Too many experiments are conducted.

Better: two phases.

### **Types of Experimental Designs**

□ **Simple Designs**: Vary one factor at a time

# of Experiments = 
$$1 + \sum_{i=1}^{k} (n_i - 1)$$

- > Not statistically efficient.
- > Wrong conclusions if the factors have interaction.

> Not recommended.

□ **Full Factorial Design**: All combinations.

# of Experiments = 
$$\prod_{i=1}^{n} n_i$$

- > Can find the effect of all factors.
- > Too much time and money.
- ▹ May try 2<sup>k</sup> design first.

### **Types of Experimental Designs (Cont)**

□ Fractional Factorial Designs: Less than Full Factorial

- > Save time and expense.
- Less information.
- > May not get all interactions.
- > Not a problem if negligible interactions

## **A Sample Fractional Factorial Design**

- Workstation Design:
  - (3 CPUs)(3 Memory levels)(3 workloads)(3 ed levels)

= 81 experiments

Experiment	CPU	Memory	Workload	Educational
Number		Level	Type	Level
1	68000	512K	Managerial	High School
2	68000	2M	Scientific	Post-graduate
3	68000	$8\mathrm{M}$	Secretarial	College
4	Z80	512K	Scientific	College
5	Z80	2M	Secretarial	High School
6	Z80	$8\mathrm{M}$	Managerial	Post-graduate
7	8086	512K	Secretarial	Post-graduate
8	8086	2M	Managerial	College
9	8086	$8\mathrm{M}$	Scientific	High School
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- Goal of proper experimental design is to get the maximum information with minimum number of experiments
- □ Factors, levels, full-factorial designs

### Exercise 16.1

The performance of a system being designed depends upon the following three factors:

- > CPU type: 68000, 8086, 80286
- Operating System type: CPM, MS-DOS, UNIX
- Disk drive type: A, B, C
  - a. There is significant interaction among factors.
  - b. There is no interaction among factors.
  - c. The interactions are small compared to main effects.