MEMS 5705
Project 3: Blade Dynamics
due Wednesday, April 12, 2017
Given all answers for angles in both radians and degrees (for clarity).

A wind turbine has the following structural properties:

- radius: \( R = 19.0 \text{ m} \)
- chord: \( c = 1.0 \text{ m} \)
- rotation speed: \( \Omega = 5.236 \text{ rad/sec (50 RPM)} \)
- number of blades: \( B = 3 \)
- flap inertia: \( I_b = 63,900 \text{ kg-m}^2 \)
- blade mass: \( M_b = 898 \text{ kg} \)
- cg location: \( r_b = 8.0 \text{ m} \)
- flap stiffness: \( K_\beta = 3.5 \times 10^5 \text{ N-m/grad} \)
- hinge offset ratio: \( e = 0.162 \)
- precone angle: \( \beta_{pc} = 0.0349 \text{ radians (2.0}^\circ) \)

The forcing parameters are:

- air density: \( \rho = 1.225 \text{ kg/m}^3 \)
- wind speed\(^*\): \( U = 8[1 - 0.15(r/R)\cos(\psi)] \text{ m/sec} \)
- induced flow: \( u = 2.46 \text{ m/sec (from BEM theory)} \)
- lift-curve slope: \( s = 5.73/\text{rad (0.1/deg)} \)
- pitch angle: \( \theta_{3/4} = 0.0174 \text{ rad (1.0}^\circ) \)
- acc. of gravity: \( g = 9.8 \text{ m/sec}^2 \)

\(^*\)Includes a 15\% boundary-layer gradient, and \( U = \text{average} = 8 \text{ m/sec} \).

1.) Find the following dimensionless system parameters:

- Lock number: \( \gamma \)
- Flap frequency: \( \bar{p} \) (book uses \( K^{1/2} \))
- Rotor solidity: \( \sigma \)
- inflow angle at \( r=3/4 \): \( \phi_{3/4} \)
- Mass flow parameter: \( \bar{V} \)
- Gravity ratio: \( \bar{G} \) (book \( G/\Omega R \))
- Equivalent Lock Number: \( \gamma^* \)

2.) Assume that: \( \beta = \beta_0 + \beta_s \sin(\psi) + \beta_c \cos(\psi) \)

   a.) Use harmonic balance of flapping equation with \( \gamma \) to solve for \( \beta_0, \beta_s, \beta_c \).
   
   b.) Repeat the calculations for \( \beta_s, \beta_c \) but with \( \gamma^* \).
   
   c.) Set \( u=0 \) in \( \phi_{3/4} \) and use that value with your \( \gamma^* \) to find \( \beta_0 \).
ROTOR DYNAMICS WITH PRE-CONE AND GRAVITY

\[ M_B = \kappa_0 \left( \beta - \beta_{pc} \right) \quad \bar{G} = \frac{m_B v_g}{I_B \Omega^2} \]

Uniform blade \( \bar{G} = \frac{3}{2} \frac{g}{R^2} \)

\[ \chi = \frac{d(1)}{d\Omega} = \frac{1}{\Omega} \frac{d(1)}{dt} \]

\[ \dot{\beta} + \frac{1}{8} \beta + \left[ 1 + \frac{3}{2} \left( \frac{e}{1-e} + \frac{\kappa_3}{R^2 I_b} \right) \right] \beta = \]

\[ \beta_{pc} \frac{\kappa_3}{R^2 I_b} - \beta_0 \bar{G} \cos \gamma + \frac{V}{8} \left( \Theta_{3/4} - \Theta_{3/4} \right) \]

\[ V = \rho_s c R^2 \]

\[ V^* = \frac{V}{1 + \frac{c}{8V}} \]

\[ \delta = \frac{BC}{\pi R} \]

\[ S = \delta C / \delta \Omega, \quad \bar{V} = \frac{\bar{U} - 2U}{\partial R} \]

\[ \Theta_{3/4} = \frac{U - 2U}{\partial R} \quad |r = 0.75 R \]

\[ p^2 = 1 + \frac{3}{2} \frac{e}{1-e} + \frac{\kappa_3}{R^2 I_b} \]