3.1 Introduction

The steady-state behavior of circuits energized by sinusoidal sources is an important area of study for several reasons. First, the generation, transmission, distribution, and consumption of electric energy occur under essentially sinusoidal steady-state conditions. Second, an understanding of sinusoidal behavior makes possible the prediction of circuit behavior when nonsinusoidal sources are used through the use of techniques such as Fourier analysis and superposition. Finally, by specifying the performance of a circuit in terms of its steady-state sinusoidal behavior, the design of the circuit can often be simplified. Needless to say, the importance of sinusoidal steady-state behavior cannot be overemphasized, and many of the topics in future experiments are based on a thorough understanding of the techniques used to analyze circuits driven by sinusoidal sources.

In this experiment, the behavior of several types of circuits will be examined to determine their behavior when excited by sinusoidal sources. First, the behavior of both RC and RLC circuits will be examined when driven by a sinusoidal source at a
given frequency. Subsequently, the frequency response of both a low-pass filter and a high-pass filter will be considered.

### 3.2 Objectives

At the end of this experiment, the student will be able to:

1. Determine the steady-state behavior of linear circuits driven by sinusoidal sources,
2. Use the oscilloscope to measure the phase difference between two sinusoidal signals,
3. Determine analytically the frequency response of a network,
4. Construct Bode plots relating the magnitude and phase response of the voltage ratio of a linear network as a function of frequency, and
5. Design primitive low- and high-pass filters using one resistor and one capacitor.

### 3.3 Theory

#### 3.3.1 Sinusoidal Steady-State Analysis

As stated previously, the steady-state behavior of circuits that are energized by sinusoidal sources is an important area of study. A sinusoidal voltage source produces a voltage that varies sinusoidally with time. Using the cosine function, we can write a sinusoidally varying voltage as follows

\[ v(t) = V_m \cos (\omega t + \phi) \]  

and the corresponding sinusoidally varying current as

\[ i(t) = I_m \cos (\omega t + \phi). \]
The plot of sinusoidal voltage versus time is shown in Figure 3.1 below.

![Figure 3.1: A Sinusoidal Voltage](image)

Our first observation is that a sinusoidal function is periodic. A periodic function is defined such that $y(t) = y(t + T)$ where $T$ is called the period of the function and is the length of time it takes the sinusoidal function to pass through all of its possible values. The reciprocal of $T$ gives the number of cycles per second, or frequency, of the sine wave and is denoted by $f$; thus

$$f = 1/T. \tag{3.2}$$

A cycle per second is referred to as a Hertz and is abbreviated Hz. Omega, $(\omega)$, is used to represent the angular or radian frequency of the sinusoidal function; thus

$$\omega = 2\pi f = 2\pi / T \text{ radians/second.} \tag{3.3}$$
Equation (3.3) is derived from the fact that the cosine function passes through a complete set of values each time its argument passes through $2\pi$ radians (360°).

From Equation (3.3), we see that whenever $t$ is an integral multiple of $T$, the argument $\omega t$ increases by an integral multiple of $2\pi$ radians.

The maximum amplitude of the sinusoidal voltage shown in Figure 3.1 is given by the coefficient $V_m$ and is called the peak voltage. It follows that the peak-to-peak voltage is equal to $2V_m$. The phase angle of the sinusoidal voltage is the angle $\phi$. It determines the value of the sinusoidal function at $t = 0$. Changing the value of the phase angle $\phi$ shifts the sinusoidal function along the time axis, but has no effect on either the amplitude ($V_m$) or the angular frequency ($\omega$). Note that $\phi$ is normally given in degrees. It follows that $\omega t$ must be converted from radians to degrees before the two quantities can be added.

In summary, a major point to recognize is that a sinusoidal function can be completely specified by giving its maximum amplitude (either peak or peak-to-peak value), frequency, and phase angle. In general, there are three important criteria to remember regarding the steady-state response of a linear network (i.e., circuits consisting of resistors, inductors, and capacitors) excited by a sinusoidal source.

1. The steady-state response is a sinusoidal function.
2. The frequency of the response signal is identical to the frequency of the source signal.
3. The amplitude and phase angle of the response will most likely differ from the amplitude and phase angle of the source and are dependent on the values of the resistors, inductors, and capacitors in the circuit as well as the frequency of the source signal.
It follows then the problem of finding the steady-state response is reduced to finding the maximum amplitude and phase angle of the response signal since the waveform and frequency of the response are the same as the source signal. In analyzing the steady-state response of a linear network, we use the phasor, which is a complex number that carries the amplitude and phase angle information of a sinusoidal function. Given the sinusoidal voltage defined previously in Equation (3.1a), we define the phasor representation as

\[ V = V_m e^{j\phi} \]  

(3.4a)

where \( V \) is the maximum amplitude i.e., the peak voltage and \( \phi \) is the phase angle of the sinusoidal function. Equation (3.4) is the polar form of a phasor. A phasor can also be expressed in rectangular form. Thus, Equation (3.4a) can be written as

\[ V = V_m \cos \phi + jV_m \sin \phi \]  

(3.4b)

In an analogous manner, the phasor representation of the sinusoidal current defined in Equation (3.1b) is defined as

\[ I = I_m e^{j\theta} = I_m \cos \theta + jI_m \sin \theta. \]  

(3.5)

We will find both the polar form and rectangular form useful in circuit applications of the phasor concept. Also, the frequent occurrence of the exponential function \( e^{j\theta} \) has led to a shorthand notation called the angle notation such that

\[ V_m / \phi \equiv V_m e^{j\phi} \]  

(3.6a)

and

\[ I_m / \theta \equiv I_m e^{j\theta} \]  

(3.6b)
Thus far, we have shown how to move from a sinusoidal function (i.e., the time domain) to its phasor transform (i.e., the complex domain). It should be apparent that we can reverse the process. Thus, given the phasor, we can write the expression for the sinusoidal function. As an example, if we are given the phasor \( V = 5/36^\circ \), the expression for \( v(t) \) in the time domain is \( v(t) = 5 \cos(\omega t + 36^\circ) \), since we are using the cosine function for our sinusoidal reference function.

Now, the systematic application of the phasor transform in circuit analysis requires that we introduce the concept of impedance. In general, we find that the voltage-current relationship of a linear circuit element is given by

\[
V = Z I
\]  

(3.7)

where \( Z \) is the impedance of the circuit element.

The impedance of each of the three linear circuit elements is given as follows:

(a) The impedance \( (Z_R) \) of a resistor is \( R \) in rectangular form and \( R/\theta^\circ \) in angle form. From Equation (3.7), we see that the phasor voltage at the terminals of a resistor equals \( R \) times the phasor current. The phasor-domain equivalent circuit for the resistor circuit is shown in Figure 3.2 (a). If it is assumed that the current through the resistor is given by the phasor \( I = I_m/\theta \), then the voltage across the resistor is

\[
V = R I = (R/\theta^\circ)(I_m/\theta) = R I_m/\theta
\]  

(3.8)

from which we observe that there is no phase shift between the current and voltage.
Figure 3.2. Phasor Domain Equivalent Circuits

(b) The impedance ($Z_L$) of an inductor is $j\omega L$ in rectangular form and $\omega L/90^\circ$ in angle form. Again, from Equation (3.7), we see that the phasor voltage at the terminals of an inductor equals $j\omega L$ times the phasor current. The phasor-domain equivalent circuit for the inductor is shown in Figure 3.2(b).

If we assume $I = I_m / \theta$, then the voltage is given by

$$V = (j\omega L) I = (\omega L / 90^\circ)(I_m / \theta) = \omega LI_m / \theta + 90^\circ \quad (3.9)$$

from which we observe that the voltage and current will be out of phase by exactly $90^\circ$. In particular, the voltage will lead the current by $90^\circ$ or, what is equivalent, the current will lag behind the voltage by $90^\circ$.

(c) The impedance ($Z_C$) of a capacitor is $1/j\omega C$ (or $-j/\omega C$) in rectangular form and $1/\omega C/90^\circ$ in angle form. Equation (3.7) indicates that the phasor voltage at the terminals of a capacitor equals $1/j\omega C$ times the phasor current. The phasor-domain equivalent circuit for the capacitor is shown in Figure 3.2(c).
If we assume $I = I_m/\theta$, then the voltage is given by

$$V = (-j/\omega C)I = (1/\omega C / -90^\circ)(I_m / \theta) = I_m/\omega C / \theta - 90^\circ \quad (3.10)$$

where in this case, the voltage across the terminals of the capacitor will lag behind the capacitor current by $90^\circ$. The alternative way to express the phase relationship is to say that the current leads the voltage by $90^\circ$.

It can be shown that Ohm's law and Kirchhoff's laws apply to the phasor domain. Thus, all of the techniques used for analyzing dc resistive circuits can be used to find phasor currents and voltages. Thus, new analysis techniques are not needed to analyze circuits in the phasor domain. The basic tools of series-parallel simplifications, source transformations, Thevenin-Norton equivalent circuits, superposition, node-voltage analysis, and mesh-current analysis can all be used in the analysis of circuits in the phasor domain in order to determine the steady-state response of a network to sinusoidal sources. The problem of learning phasor circuit analysis involves two parts. First, you must be able to construct the phasor-domain model of the circuit and, second, you must be able to algebraically manipulate complex numbers and/or quantities to arrive at a solution.

We will demonstrate this by considering the example circuit shown in Figure 3.3, which consists of a resistor, inductor, and capacitor connected in series across the terminals of a sinusoidal voltage source. Assume the steady-state voltage source is a sine wave that we can represent as $v_s(t) = 5 \cos \omega t$ at a frequency of $f = 1000$Hz.

The first step in the solution is to determine the phasor-domain equivalent circuit. Given a frequency of 1000 Hz, then $\omega = 2\pi f = 6283$ radians/second. Therefore, the
impedance of the inductor is

\[ Z_L = j\omega L = j(6283)(0.030) = j188.5\Omega \quad (3.11) \]

and the impedance of the capacitor is

\[ Z_C = -j\left(\frac{1}{\omega C}\right) = -j\frac{10^6}{(6283)(5)} = -j31.8\Omega \quad (3.12) \]

The phasor transform of \( v_s(t) \) is

\[ V_s = 5\angle 0^\circ \quad (3.13) \]

The phasor domain equivalent circuit for the circuit shown in Figure 3.3 is illustrated in Figure 3.4.

We can compute the phasor current by simply dividing the phasor voltage of the voltage source by the total impedance across its terminals. The total impedance is

\[ Z_T = R + Z_L + Z_C \quad (3.14) \]
since the resistor, inductor, and capacitor are connected in series. Substituting values for $R$, $Z_L$, and $Z_C$ gives

$$Z_T = 100 + j 188.5 - j 31.8 = 100 + j 156.7. \quad (3.15)$$

Converting this to angle form, we have

$$Z_T = \sqrt{(100)^2 + (156.7)^2} \tan^{-1}(156.7/100), \quad (3.16)$$

which simplifies to

$$Z_T = 185.9/ + 57.5^\circ. \quad (3.17)$$

Thus, the current is equal to

$$I = \frac{V_S}{Z_T} = (5 / 0^\circ)/(185.9 / 57.5^\circ) = 0.0269 / -57.5^\circ \text{ A.} \quad (3.18)$$

and the voltage across the capacitor is

$$V_C = Z_C I = (31.8 / -90^\circ)(0.0269 / -57.5^\circ) = 0.855 / -147.5^\circ \text{ V.} \quad (3.19)$$
We can now write the stead-state expressions for $I(t)$ and $v_C(t)$ directly:

$$i(t) = 26.9 \cos (6283t - 57.5^\circ) \text{ mA} \quad (3.20)$$

and

$$v_C(t) = 0.855 \cos (6283t - 147.5^\circ) \text{ V.} \quad (3.21)$$

### 3.3.2 Frequency Response of AC Circuits: Bode Diagrams

As indicated above, network connections of linear circuit elements used to transmit signals generally result in an output signal with an amplitude and time dependence that differs from that of the input signal. If the input signal is sinusoidal, so too will be the output for a linear network. However, usually both the amplitude ratio and the phase difference between the input and the output signals will depend on the frequency. A particularly convenient graphical representation is obtained if logarithmic scales are used for both the frequency and the amplitude ratio while a linear scale is used for the phase difference. Also, extremely useful approximate graphical representations can be obtained for many networks that result in a series of straight lines on these graphs. Both representations are called Bode Diagrams. In this part of the experiment, the measured response of two networks will be compared to that theoretically predicted and the straight line approximations. In addition, decibel notation will be introduced.

The frequency response of the elementary circuit shown in Figure 3.5, where $v_i(t)$ corresponds to the input signal and $v_o(t)$ to the output signal, will initially be determined. Sinusoidal signals will be assumed. In analyzing this circuit, the
The impedance across $v_i$ is a function of the operating frequency and can be expressed as

$$Z(\omega) = R + \frac{1}{j\omega C}.$$  \hspace{1cm} (3.22)

Using Ohm's law in the phasor domain, the current in the circuit is

$$I(\omega) = \frac{V_i}{Z(\omega)} = \frac{V_i}{R + \frac{1}{j\omega C}},$$ \hspace{1cm} (3.23)

and the output voltage, which is equal to the voltage across the capacitor, is

$$V_o = Z_c I = \left(\frac{1}{j\omega C}\right) \left(\frac{V_i}{R + \frac{1}{j\omega C}}\right) = \frac{V_i}{1 + j\omega RC}.$$ \hspace{1cm} (3.24)

Examination of Equation (3.24) above indicates that for low frequencies (that is, for $\omega \to 0$), the output voltage is approximately equal to the input voltage (i.e. $V_o \to V_i$). Also, for high frequencies (that is for $\omega \to \infty$), the output voltage becomes negligibly small (i.e. $V_o \to 0$). Since low frequency signals are virtually unaffected, a network of this type is called a **low-pass filter**. It should be noted that many complex circuits comprised of active elements, such as transistors, may be reduced to an equivalent circuit of this type. The capacitance, $C$, is often the
result of unavoidable circuit and element capacitances, while the resistance, $R$, is the Thevenin equivalent resistance of the complex circuit.

If we let $A$ be defined as the ratio of the output voltage to the input voltage, then

$$ A(\omega) = \frac{V_o}{V_i} = \frac{1}{1 + j\omega RC}. \quad (3.25) $$

Now $A(\omega)$ can be expressed in terms of its magnitude and phase angle, where the magnitude is

$$ |A(\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}}, \quad (3.26) $$

while the phase angle is

$$ \theta(\omega) = -\tan^{-1}(\omega RC). \quad (3.27) $$

If we define the critical frequency $\omega_1$ to be

$$ \omega_1 = 1/RC \quad (3.28) $$

then, upon substitution in Equation (3.26) and Equation (3.27), we find

$$ |A(\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_1}\right)^2}}, \quad (3.29) $$

and

$$ \theta(\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_1}\right). \quad (3.30) $$

Now, the amplitude of the voltage ratio is normally specified in decibels (dB), and is defined as

$$ A_{dB} = 20 \log_{10} |A(\omega)| \text{ dB}. \quad (3.31) $$
Bode plots relate the magnitude and phase angle of the voltage ratio of a network with frequency as the independent variable. Usually, frequency in Hertz or radians per second is used as the abscissa on a logarithmic scale and the amplitude of the voltage ratio in dB on one ordinate and the phase angle on a linear scale as another ordinate. An example for the low pass-filter is shown in Figure 3.6 where we have plotted $A_{dB}$ versus log $\omega$ and $\theta$ versus log $\omega$. Note that Bode plots are shown for both the straight line approximation (solid) and the actual (dashed) values.

Figure 3.6: A Bode Plot for a Low-Pass Filter with Critical Frequency $\omega_1$. 

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As indicated in the figure, the Bode plot shows the frequency response of the filter. The $-3$ dB point is marked at $\omega_1$, and the $-20$ dB/decade slope is observed for frequencies above $\omega_1$. The phase angle $\theta$ is shown to decrease linearly with log frequency.
We can obtain values for $|A(\omega)|$ and $\theta(\omega)$ for the low-pass filter using Equation (3.29) and Equation (3.30). For $\omega \ll \omega_1$, we have from Equation (3.29),

$$|A(\omega)| \approx \frac{1}{\sqrt{1}} = 1.$$  \hspace{1cm} (3.32)

We can determine the phase angle from Equation (3.30), namely,

$$\theta \equiv \tan^{-1}(0) = 0^\circ.$$  \hspace{1cm} (3.33)

In decibels, we see that $A_{dB} = 0$ since $20 \log_{10}(1) = 0$.

Now, when $\omega$ is an order of magnitude below $\omega_1$ (i.e., $\omega = 0.1\omega_1$), we find that

$$\theta = \tan^{-1}(0.1) = -5.710^\circ \approx -6^\circ.$$  \hspace{1cm} (3.34)

These relationships between $A(\omega)$ and $\theta(\omega)$ are shown plotted in Figure 3.6.

For $\omega = \omega_1$, we can compute the magnitude of the voltage amplitude ratio from Equation (3.29) as

$$|A(\omega_1)| = \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}} = 0.707.$$  \hspace{1cm} (3.35)

Thus, $|V_0| = 0.707 |V_i|$ at the critical frequency $\omega_1$. In decibels, we can express the magnitude of the voltage ratio as

$$A_{dB} = 20 \log_{10} \left(1/\sqrt{2}\right) = -3.0 \text{ dB}.$$  \hspace{1cm} (3.36)

It follows, then, that at $\omega = \omega_1$ the magnitude of the voltage ratio is reduced by 3 dB. As a result, the critical frequency is often called the -3 dB point. Also, since the power-transfer ratio is proportional to the square of the voltage ratio, i.e.,
(1/\sqrt{2})^2$, the critical frequency $\omega_n$ (or $f_1$ when expressed in Hertz) is also known as the half-power frequency. The phase angle for $\omega = \omega_n$ is calculated by substituting into Equation (3.30) and is given by

$$\theta = -\tan^{-1}(1) = -45^\circ . \quad (3.37)$$

For $\omega \gg \omega_n$, we determine from Equation (3.29) that

$$|A(\omega)| \approx \omega_n / \omega . \quad (3.38)$$

Hence, in this region, $|A(\omega)|$ varies inversely with frequency. The rate of change can be determined by considering a decade of frequency. Let

$$X = |A(\omega_n)| \text{ and } X_{dB} = 20 \log_{10} X , \quad (3.39)$$

where $X$ is the magnitude of the voltage ratio function at some frequency $\omega_n$, and let

$$Y = |A(10\omega_n)| \text{ and } Y_{dB} = 20 \log_{10} Y , \quad (3.40)$$

where $Y$ is the magnitude of the voltage ratio function at a frequency 10 times $\omega_n$. Then, using Equation (3.38), the difference in voltage ratio (in dB) over a decade of frequency is given by

$$X' - Y' = 20 \left[ \log_{10} \left( \frac{\omega_1}{\omega_n} \right) - \log_{10} \left( \frac{\omega_1}{10\omega_n} \right) \right] , \quad (3.41)$$

which simplifies to

$$X' - Y' = 20 \log_{10} (10) = 20 \text{ dB} . \quad (3.42)$$

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Thus, the gain at $10\omega_n$ is 20 dB less than it was at $\omega_n$. It follows then, that the rate of fall of the amplitude ratio, where $\omega \gg \omega_n$, is -20 dB/decade.

Finally, from the expression for the phase angle in Equation (3.30), we observe that as $\omega \to \infty$, $\theta \to -90^\circ$, but at $\omega = 10\omega_n$,

$$\theta = -\tan^{-1}(10) = -84.289 \approx -84^\circ . \tag{3.43}$$

As indicated previously, Figure 3.6 is a Bode plot for a low-pass filter with critical frequency $\omega_1$. Notice that the approximate Bode plot makes use of the straight-line segments to approximate the actual response (shown as a dotted line around the critical frequency). Also, note that the maximum amplitude rolloff is -20 dB/decade and the maximum phase shift is - 90°.

Equally important to the low-pass filter just considered is a network in which the positions of the capacitor and the resistor are interchanged as shown in Figure 3.7. In this case, since the capacitance is in series with the input voltage source and the output terminals of the network, the voltage $v_0$ becomes very small as the frequency is reduced. For zero frequency (i.e., "dc"), the output voltage is zero since a capacitor blocks dc voltages. At very high frequencies, the capacitor's impedance approaches
a short-circuit resulting in an output voltage that tends to be very nearly equal to the input voltage. Thus, this circuit configuration is classified as a high-pass filter since it attenuates low frequency signal components and passes high frequency components. When a capacitance is used to couple time-varying signal components from one part of an electronic circuit to another, this type of equivalent circuit often results. Also, this type of circuit may be intentionally introduced in signal processing systems to eliminate undesirable low-frequency (including "dc") components.

The analysis of this circuit is similar to that of a low-pass filter. The output voltage is given by

\[ V_o = RI = R \left( \frac{V_i}{R + \frac{1}{j\omega C}} \right) = \frac{V_i}{1 + \frac{1}{j\omega RC}}, \]  

and, if we let \( A \) again be defined as the voltage ratio, it follows that

\[ A(\omega) = \frac{1}{1 + \frac{1}{j\omega RC}}. \]

The magnitude and phase angle are

\[ |A(\omega)| = \frac{1}{\sqrt{1 + \left(\frac{1}{\omega RC}\right)^2}}, \]

and

\[ \theta(\omega) = + \tan^{-1} \left(\frac{1}{\omega RC}\right). \]
Again, in terms of the critical frequency $\omega_1 = 1/RC$, we have

$$|A(\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega_1}{\omega}\right)^2}},$$  

(3.48)

and

$$\theta(\omega) = \tan^{-1}\left(\frac{\omega_1}{\omega}\right).$$  

(3.49)

A Bode plot for the high-pass filter is shown in Figure 3.8. Notice in this case that

![Bode plot](image)

Figure 3.8: A Bode plot for a High-Pass Filter with Critical Frequency $\omega_1$. 

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the magnitude and phase angle of the voltage ratio is again 3 dB down and +45°, respectively, at the critical frequency. The rate of increase for \( \omega \ll \omega_1 \), is 20 dB/decade and as \( \omega \to 0, \theta \to + 90° \), while at \( \omega = 0.1\omega_1, \theta = + 84° \). For \( \omega \gg \omega_1 \), the magnitude of the voltage ratio is 1 (0 dB) and the phase angle is 0°. Obtaining values for \( A(\omega) \) and \( \theta(\omega) \) from Equation (3.48) and Equation (3.49), as shown previously for the low-pass filter, is left to the student.

### 3.4 Advanced Preparation

The following advanced preparation is required before coming to the laboratory:

1. Thoroughly read and understand the theory and procedures.
2. Solve for the steady-state voltage across \( C_2 \) in the circuit shown in Figure 3.9 at both frequencies given to you by your instructor.
3. Perform a PSpice Transient simulation to find the steady-state voltage across \( C_2 \) in the circuit shown in Figure 3.9. Note that the simulation time must be long enough for the transient response to reach steady-state. Make a copy of a segment of the steady-state voltage. Do the simulation for both frequencies.
4. Solve for the steady-state voltage across \( R_4 \) in the circuit shown in Figure 3.10 at both frequencies given to you by your instructor.
5. Perform a PSpice Transient simulation to find the steady-state voltage across \( R_4 \) in the circuit shown in Figure 3.10. Note that the simulation time must be long enough for the transient response to reach steady-state. Make a copy of a segment of the steady-state voltage. Do the simulation for both frequencies.
6. Calculate the critical frequency in Hertz (using the values given to you by your instructor) for the low-pass and high-pass filter test circuits shown in Figure 3.11 and Figure 3.12, respectively.
3.5 Experimental Procedure

3.5.1 AC Analysis of a RC Circuit

Construct the circuit in Figure 3.9 using the HP33120A Function Generator output as

\[ v_T(t) \]

Connect Channel 1 of the scope across \( v_T(t) \) and Channel 2 across \( C_2 \) and trigger the scope on Channel 1. Set the output of the HP33120A to produce a sine wave, adjust its peak-to-peak voltage to \( V_1 \) volts (which is a peak voltage of \( V_1/2 \) volts) using the scope, and set its frequency to \( f_a \) Hz. Check using the frequency counter and digital oscilloscope. Now, measure the voltage across \( C_2 \) by measuring the peak amplitude of the voltage waveform on Channel 2 plus the phase shift of the waveform on Channel 2 with respect to the waveform on Channel 1. Next, set the output of the HP33120A to have a frequency of \( f_b \) Hz and repeat the above procedure.

3.5.2 AC Analysis of a RLC Circuit

In a manner similar to the way the circuit in Figure 3.9 was constructed, assemble the circuit shown in Figure 3.10. Set the output of the HP33120A to produce a sine wave, adjust its peak-to-peak voltage to \( V_1 \) volts (which is a peak voltage of \( V_1/2 \) volts) using the scope, and set its frequency to \( f_a \) Hz. Check using the frequency counter and digital oscilloscope. Now, measure the voltage across \( C_2 \) by measuring the peak amplitude of the voltage waveform on Channel 2 plus the phase shift of the waveform on Channel 2 with respect to the waveform on Channel 1. Next, set the output of the HP33120A to have a frequency of \( f_b \) Hz and repeat the above procedure.
wave, adjust its peak-to-peak voltage to $V_2$ volts (which is a peak voltage of $V_2/2$ volts) using the scope, and set its frequency to $f_c$ Hz. Check using the frequency counter and digital scope. Now, measure the voltage across $R_4$ by measuring the peak amplitude of the voltage waveform on Channel 2 plus the phase shift of the waveform on Channel 2 with respect to the waveform on Channel 1. Next, set the output of the HP33120A to have a frequency of $f_d$ Hz and repeat the above procedure.

### 3.5.3 RC Low-Pass Filter

In this part, you are to experimentally determine the amplitude and phase angle of the voltage ratio of the RC low-pass filter network shown in Figure 3.11. This should be done over a frequency range from two orders of magnitude below up to two orders of magnitude above the critical frequency $f_1$. Construct the circuit in Figure 3.11 using the HP33120A output $v_{T(t)}$ as $v_i(t)$. Connect Channel 1 of the scope to measure $v_i$ and Channel 2 to measure $v_0$ and trigger the scope on Channel 1. Next, set the output to be sinusoidal with a peak-to-peak voltage of $V_3$ volts. Use the following procedure to determine the critical frequency and to obtain frequency response data, i.e., values for the amplitude and phase angle of the voltage ratio as a function of frequency. Be sure
to use the digital scope or frequency counter to get accurate frequency values.

(a) Experimentally determine the critical frequency $f_1$ for the circuit. Since we know at the critical frequency that $v_0 = v_i/\sqrt{2} = 0.707v_i$, this can be accomplished by varying the frequency of the HP33120A until the output voltage on Channel 2 is equal to $V_3/\sqrt{2}$ volts peak-to-peak. Record this frequency as $f_1$. Also, observe both waveforms on the scope and determine their phase angle difference at this frequency. Make a hardcopy of this display. If the phase difference is not close to $45^\circ$, then either the measurement is in error or the circuit is not functioning properly and you must correct this problem before proceeding.

(b) Now at each of the following frequencies, (1) record the peak-to-peak values of $v_0$ and $v_i$ in a table and calculate the ratio of $v_0$ divided by $v_i$, and (2) measure and record the phase angle between $v_0$ and $v_i$. The following frequencies were chosen to obtain adequate data points around $f_1$.

- $f_1/100$ (two decades down)
- $f_1/10$ (one decade down)
- $f_1/4$ (two octaves down)
- $f_1/2$ (one octave down)
\( f_i \times 2 \) (one octave up)
\( f_i \times 4 \) (two octaves up)
\( f_i \times 10 \) (one decade up)
\( f_i \times 100 \) (two decades up)

Be sure you adjust the HP33120A output to \( V_3 \) volts peak-to-peak if it changes significantly when you vary the frequency.

(c) Finally, record any additional data points for the peak-to-peak voltages and time differences needed to calculate accurate voltage ratios and phase angles from one decade to two decades below \( f_i \) and from one decade to two decades above \( f_i \). Again, be sure you adjust the HP33120A output to \( V_3 \) volts peak-to-peak if it changes significantly when you vary the frequency.

3.5.4 RC High-Pass Filter

Now, experimentally determine the response of the high-pass filter shown in Figure 3.12 using the same procedure outlined 3.5.3.

Figure 3.12: High-Pass Filter test circuit.
3.6 Report

3.6.1 AC Analysis of a RC Circuit

3.6.1.1 Clearly show your analysis to solve for the steady-state voltage across the capacitor $C_2$ in Figure 3.9. In particular, draw the phasor-domain equivalent circuit, show all steps in determining the phasor voltage across $C_2$, and express your answer as a sinusoidal function in the time domain. This should be done for both frequencies used ($f_a$ and $f_b$).

3.6.1.2 Clearly explain the procedure you used to measure the peak voltage and phase angle of the voltage across the capacitor $C_2$ using the scope. Specifically, you should provide a hardcopy of $v_0$ compared to $v_s$ indicating the peak voltages in volts and phase difference in time. Also, show how you converted the phase difference from time to degrees.

3.6.1.3 Discuss the accuracy of your measured results compared to your solution in Problem 3.6.1.1 above and indicate the reasons for any errors between the measurements and the theory.

3.6.1.4 Attach a copy of the steady-state voltage across $C_2$ obtained per Section 3.4 (3) and comment on the experimental results versus PSpice simulation results.

3.6.2 AC Analysis of a RLC Circuit

3.6.2.1 Clearly show your analysis to solve for the steady-state voltage across the resistor $R_4$ in Figure 3.10. In particular, draw the phasor-domain equivalent circuit, show all steps in determining the phasor voltage across $R_4$, and express your answer as a sinusoidal function in the time domain. This should be done for both frequencies used ($f_c$ and $f_d$).
3.6.2.2 Discuss the accuracy of your measured results compared to your solution in Problem 3.6.2.1 above and indicate the reasons for any errors between the measurements and the theory.

3.6.2.3 Attach a copy of the steady-state voltage across \( R_4 \) obtained per Section 3.4 (5) and comment on the experimental results versus PSpice simulation results.

3.6.3 RC Low-Pass Filter

3.6.3.1 Prepare a table presenting your measurements for \( v_i, v_0, \) the amplitude of the voltage ratio \( |A| \), and the phase angle \( (\theta) \) as a function of frequency for the low-pass filter test circuit in Figure 3.11. Calculate \( A_{\text{dB}} \) for each of the frequencies and add these values to your table. Generate a Bode plot of \( A_{\text{dB}} \) and \( \theta \) as a function of frequency in Hertz on a semilog scale.

3.6.3.2 Calculate the theoretical critical frequency, \( f_1 \), in Hertz for the low-pass filter in Figure 3.11 and draw the straight line approximate Bode plot for the amplitude of the voltage ratio and phase angle on the same graph as the experimental data.

3.6.3.3 Discuss how accurate the straight line approximation is compared to the theory and your experimental data.

3.6.4 RC High-Pass Filter

3.6.4.1 Prepare a table presenting your measurements for \( v_i, v_0, \) the amplitude of the voltage ratio \( |A| \), and the phase angle \( (\theta) \) as a function of frequency for the high-pass filter test circuit in Figure 3.12. Calculate \( A_{\text{dB}} \) for each of the frequencies and add these values to your table. Generate a Bode plot of \( A_{\text{dB}} \) and \( \theta \) as a function of frequency in Hertz on a semilog scale.
3.6.4.2 Calculate the theoretical critical frequency, \( f_1 \), in Hertz for the high-pass filter in Figure 3.12 and draw the straight line approximate Bode plot for the amplitude of the voltage ratio and phase angle on the same graph as the experimental data.

3.6.4.3 Discuss how accurate the straight line approximation is compared to the theory and your experimental data.

3.6.5 **Design Problem**

A combination of low-pass and high-pass networks can give a frequency response that is similar to that of a band-pass filter. Consider the network below.

Design the network so as to have (1) \( |V_o/V_T| \approx 0.5 \) at \( f = 1 \) kHz, (2) \( |V_o/V_T| \leq 0.1 \) for \( f \leq 60 \) Hz, and (3) \( |V_o/V_T| \leq 0.1 \) for \( f \geq 16k \) Hz. The figure above indicates these specified values. Use the load resistance \( R_L \) specified by the instructor.

3.6.5.1 Identify the values selected for the capacitors (\( C_1 \) & \( C_2 \)) and resistor (\( R_1 \)).

3.6.5.2 Use PSpice to provide a plot of \( |V_o/V_T| \) showing the response of your circuit design. Use a log scale for frequency \( f \).

3.6.5.3 Verify that the design meets the requirements by doing a steady–state AC PSpice simulation of the circuit at the three frequencies indicated on the figure above. Present results in tabular form.
3.7 References
