CIRCUITS LABORATORY

EXPERIMENT 1

DC Circuits – Measurement and Analysis

1.1 Introduction

In today's high technology world, the electrical engineer is faced with the design and analysis of an increasingly wide variety of circuits and systems. However, underlying all of these systems at a fundamental level is the operation of DC circuits. Indeed, the ability to analyze and simplify such circuits is central to the understanding and design of more complicated circuits. Furthermore, the measurement of DC circuit quantities, i.e., voltage, current and resistance, are the most basic and fundamental measurements an electrical engineer can make. In this experiment, the student will become acquainted with the use and limitations of a modern digital multimeter, as well as experimentally verify the validity of Thevenin's theorem, one of the key concepts in circuit theory.

1.2 Objectives

At the end of this experiment, the student will be able to:

(1) Assemble simple DC circuits containing resistors and voltage sources,

(2) Use a digital multimeter to measure voltage, current, and resistance,

(3) Predict the loading effect caused by the use of a DC voltmeter and/or a DC ammeter,

(4) Measure current by using only a voltmeter and an additional resistor, and

(5) Experimentally determine the Thevenin equivalent of a given circuit.

1.3 Theory

The digital multimeter (DMM) is a versatile instrument that can be used to make a variety of electrical measurements. The laboratory instrumentation rack at each station contains one DMM: the Tektronix DMM4050. As the name would suggest, these meters have a digital (liquid crystal) display. In this experiment, you will use the DMMs to measure DC voltage, DC current, and resistance. In future experiments, you will learn how to use the DMM to measure AC voltage and AC current.

1.3.1 Use and Limitations of DC Voltmeters

The first use of the DMM that we will consider is the measurement of DC voltage, that is, the use of the DMM as a voltmeter. To illustrate this, consider

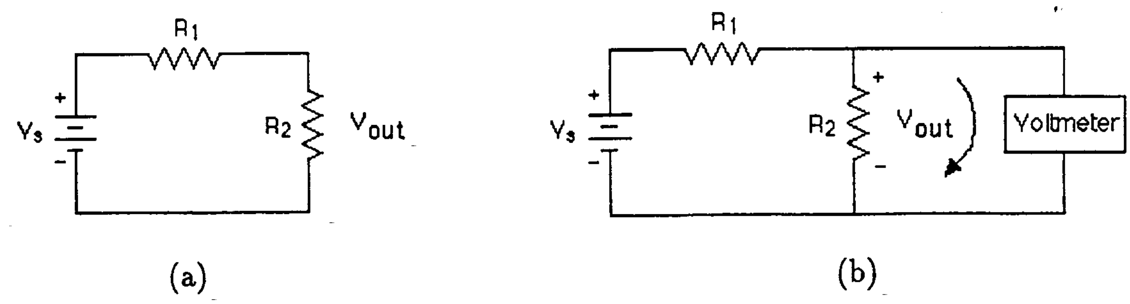


Figure 1.1: The Use of a Voltmeter to Measure Voltage (a) Voltage Divider Circuit (b) Voltage Divider Circuit with Voltmeter Used to Measure Vout

the simple voltage divider circuit shown in Figure 1.1 (a) and suppose that we wish to measure the voltage across R_2, denoted by V_{out}. To do so, we must place the voltmeter in parallel across R_2, as shown in Figure 1.1 (b). This illustrates a general rule: **To measure the voltage drop across a circuit component, the voltmeter must be placed in parallel across that component in question**.

In the ideal case, the insertion of the voltmeter as in Figure 1.1 (b) would not affect the operation of the circuit in Figure 1.1 (a), and the voltage reading obtained by our voltmeter would be the true value of V_{out}. However, life is not so simple. In general, any instrument used to make physical measurements extracts energy from the system in question while making measurements and the DMMs in the lab are no exception. The effect of this extraction of energy is to change the quantity being measured. Certainly, one of the main goals in designing a "good" instrument is to minimize this extraction of energy so as to not disturb the system in question. While this may be possible to do under certain "normal" conditions, there will always be situations in which the extraction of energy is large, leading to a large measurement error. It is important for the engineer to understand the reasons for this effect, known as the loading effect of a meter, so that the limitations of the capabilities of the meter are understood.

To illustrate this effect with a voltmeter, let us consider the loading effect of the voltmeter on the circuit in Figure 1.1. Using the voltage divider rule, one can clearly see that the voltage V_{out} the circuit of Figure 1.1 (a) is given by

$$V_{out} = \frac{R_2}{R_1 + R_2} V_s \tag{1.1}$$

Now, to examine the loading effect of the voltmeter in Figure 1.1 (b), we must develop an equivalent circuit model for the voltmeter. Without going into the details of the voltmeter operation, it is sufficient to say the voltmeter can be represented by an equivalent resistance, known as R_{vm}. Thus, with the voltmeter inserted into our circuit, the equivalent circuit is given in Figure 1.2. Again, using the voltage

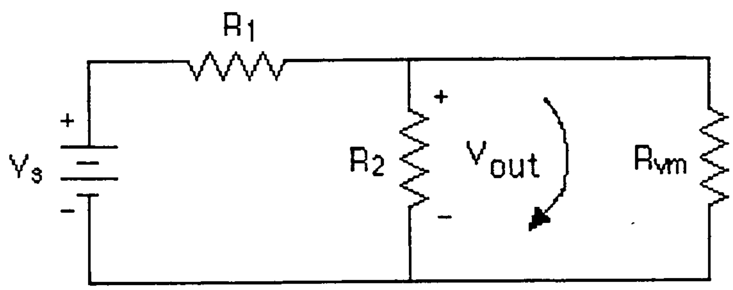


Figure 1.2: Equivalent circuit obtained when a voltmeter is used in the simple voltage divider circuit.

divider rule, one can show that with the voltmeter in the circuit, the voltage Vout is given by

$$V_{out} = \frac{R_2 \parallel R_{vm}}{R_1 + R_2 \parallel R_{vm}} V_s \tag{1.2}$$

In comparing Equation (1.1) and Equation (1.2), one can see that the voltmeter will introduce a small measurement error when R_{vm} is large relative to R_2. In fact, as R_{vm} approaches infinity, one can see that $R_2 \parallel R_{vm}$ will approach R_2, which means that Equations (1.1) and (1.2) will become equal, i.e., no measurement error will be introduced. As R_{vm} approaches the same order of magnitude as R_2, the error can become significant. Thus, in order to design a voltmeter that minimizes the measurement error, one must make R_{vm} as large as possible. The voltmeters in the lab have an **R_{vm} of 10 MΩ**. Thus, when measuring voltages across components that have a resistance more than about 10 kΩ, one must be concerned about the potential measurement error introduced from the loading effect.

In order to get a quantitative feel for how large the errors introduced by the voltmeters in our lab can be, we shall calculate the percent error (% error) between the ideal value of V_{out}, i.e., with no voltmeter attached, and the actual value of V_{out}, i.e., with the voltmeter attached. This is a calculation that will frequently be employed in this course to quantify the difference between an ideal (theoretical) and an actual (measured) value. In general, the % error between two quantities is given by

$$\%\ error = \left(\frac{Actual\ Value - Ideal\ Value}{Ideal\ Value} \right) x 100\%. \qquad (1.3)$$

In our case, the % error becomes

$$\%\ error = \left(\frac{\frac{R_2 \parallel R_{vm}}{R_1 + R_2 \parallel R_{vm}} V_s - \frac{R_2}{R_1 + R_2} V_s}{\frac{R_2}{R_1 + R_2} V_s} \right) x\, 100\%. \qquad (1.4)$$

After some algebraic manipulations, this reduces to

$$\% \; error = \left(\frac{-R_1 R_2}{R_1 R_2 + (R_1 + R_2) R_{vm}} \right) x \, 100\% = \left(\frac{-1}{1 + \dfrac{R_{vm}}{R_1 \parallel R_2}} \right) x \, 100\%. \qquad (1\text{-}5)$$

Using this last formula, and recalling that R_{vm} is 10 MΩ for the voltmeters in our lab, we can calculate the % error as a function of $R_1 \| R_2$. A few such values have been tabulated in Table 1.1. Notice that the % error in this case is always negative.

$R_1 \parallel R_2$	*% error*
10 MΩ (100% of R_{vm})	-50.0%
5 MΩ (50% of R_{vm})	-33.3%
1 MΩ (10% of R_{vm})	-9.09%
100 kΩ (1% of R_{vm})	-0.99%
10 kΩ (0.1% of R_{vm})	-0.099%

Table 1.1: The % error in the actual voltmeter reading as a function of $R_1 \parallel R_2$.

This means that the actual voltmeter reading is less than the value that would be obtained with no voltmeter present. This is to be expected since the voltmeter will place a load on the circuit under test, drawing current, and consequently reducing the voltage level.

1.3.2 Use and Limitations of DC Ammeters

Another important use of the DMM that we will consider is its use to measure DC current, that is, when it is used as an ammeter. To illustrate this, consider the simple resistive circuit shown in Figure 1.3 (a) and suppose that we wish to measure the current I_s. To do so, we must place the ammeter in series with the resistor R_3, as shown in Figure 1.3 (b). This illustrates another general measurement rule:

To measure current in a circuit, the ammeter must be placed in series with the current in question.

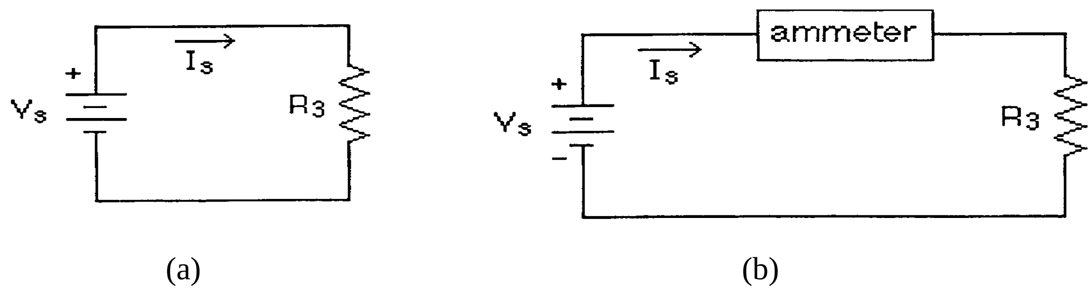


Figure 1.3: The Use of an Ammeter to Measure Current: (a) Simple Resistive Circuit (b) Simple Resistive Circuit with Ammeter Used to Measure I_s

As in the case of the voltmeter, the insertion of the ammeter into our circuit may also disturb the current we are trying to measure. To examine this loading effect, let us first examine the circuit in Figure 1.3 (a). Using Ohm's law, it is clear that the current I_s, is given by

$$I_s = \frac{V_s}{R_3} \quad . \tag{1.6}$$

Again, to examine the loading effect of the ammeter in Figure 1.3 (b), we must develop an equivalent circuit model for our ammeter. As in the case of the voltmeter, the ammeter can be represented by its equivalent resistance, R_{am}. Thus, with the ammeter inserted into our circuit, the equivalent circuit is given in Figure 1.4. Again using Ohm's law, one can show that with the ammeter in the circuit, the current I_s is given by

$$I_s = \frac{V_s}{R_3 + R_{am}} \tag{1.7}$$

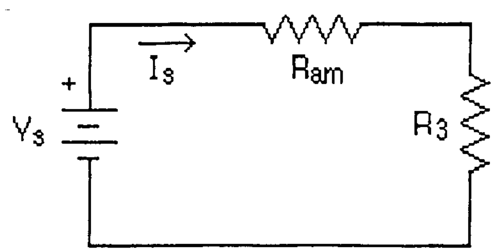


Figure 1.4: Equivalent circuit obtained when an ammeter is used in the simple resistive circuit.

In comparing Equation (1.6) and Equation (1.7), one can see that the ammeter will introduce a small measurement error when R_{am} is small relative to R_3. In fact, as R_{am} approaches zero, one can see that Equations (1.6) and (1.7) will become equal, i.e., no measurement error will be introduced. As R_{am} approaches the same order of magnitude as R_3, the error can become significant. Thus, in order to design an ammeter that minimizes the measurement error, one must make R_{am} as small as possible while retaining sufficient decimal accuracy. The ammeter in the lab has a R_{am} that depends on the current range selected as given in Table 1.2.

Full-scale Current	Maximum Value of R_{am}	Display Readout
100 µA	100 Ω	±XXX.DDDD
1 mA	100 Ω	±X.DDDDDD
10 mA	1 Ω	±XX.DDDDD
100 mA	1 Ω	±XXX.DDDD
400 mA	1 Ω	±XXX.DDDD
1 A	0.01 Ω	±X.DDDDDD
3 A	0.01 Ω	±X.DDDDDD
10 A	0.01 Ω	±XX.DDDDD

Table 1.2: Ammeter Resistance R_{am} as a Function of Current Range

As indicated, the specification given here is the maximum value of R_{am} for each Current range. Thus, if your DMM is configured as an ammeter with a full-scale Current of 1 mA, then the value of R_{am} will not exceed 100 Ω, although it may be considerably less. In order to insure that loading effects will not be a problem, one must make sure that value of R_{am} for the current range that is needed is small relative to the resistance of the branch in which current is being measured.

As in the case of the voltmeter, one would like to get a quantitative feel for how large the measurement error introduced by an ammeter can be. To do this, we shall calculate the % error between the ideal value of I_s (i.e., with no ammeter inserted) and the actual value of I_s (i.e. with the ammeter inserted). In this case, the % error becomes

$$\%\, error = \left(\frac{\dfrac{V_s}{R_{am} + R_3} - \dfrac{V_s}{R_3}}{\dfrac{V_s}{R_3}} \right) x\, 100\%. \tag{1.8}$$

After some algebraic manipulations, this reduces to

$$\%\, error = \left(\frac{-R_{am}}{R_{am} + R_3} \right) x\, 100\% = \left(\frac{-1}{1 + \dfrac{R_3}{R_{am}}} \right) x\, 100\%. \tag{1.9}$$

Using this last formula, we can calculate the % error in our ammeter readings as a function of the ammeter resistance R_{am}. The results of a few of these calculations are shown in Table 1.3.

R_{am} / R_3	% error
1	-50.0%
0.5	-33.3%
0.1	-9.09%
0.01	-0.99%
0.001	-0.099%

Table 1.3: The % error in the actual ammeter reading as a function of R_{am}/R_3.

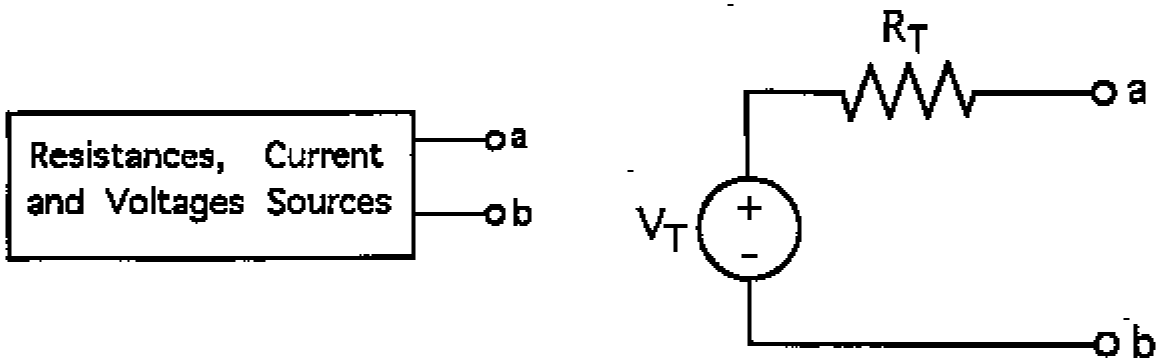
Note that, as in the case of the voltmeter, the % error is always negative in this case. This means that the actual current measured is less than the current that would exist if no ammeter were present. This is again as expected, because the ammeter will introduce an additional series resistance, which will decrease the current in the circuit under test.

1.3.3 Thevenin and Norton Equivalent Circuits

There are times in DC circuit analysis when we wish to determine what happens at a specific pair of terminals. The use of either Thevenin's or Norton's theorem enables us to replace an entire linear circuit made up of voltage and current sources and resistors, seen at a pair of terminals, by an equivalent circuit made up of a single resistor and a single source. Therefore, we can determine the voltage and current for a single element in a relatively complex circuit by (i) replacing the rest of the circuit with an equivalent resistance and source, and (ii) then analyzing the resulting circuit. It follows that Thevenin and Norton equivalent circuits provide a very important technique for analyzing complex circuits.

In general, any two terminals of a linear network made up of sources (both independent and dependent) and resistors can be reduced to a Thevenin equivalent circuit with an equivalent voltage, V_T, and an equivalent series resistance, R_T,. This is illustrated in Figure 1.5.

In its most elementary form, the Thevenin theorem states that for an arbitrary external circuit attached to its terminals, the Thevenin equivalent circuit will result in the same voltage and current as when the external circuit is attached to the actual network. This equivalence will hold for all possible values of load resistance. In order to represent the original circuit by its Thevenin equivalent, we must determine the Thevenin equivalent voltage, V_T, and the Thevenin equivalent resistance, R_T.



(a) (b)

Figure 1.5: A Thevenin Equivalent Circuit: (a) Actual Network (b) Thevenin Equivalent Circuit

These two parameters of the Thevenin equivalent can be found as follows. First, we note that if the load resistance is infinitely large in the Thevenin equivalent circuit shown in Figure 1.5 (b) above, we have an open-circuit condition. It follows that the open-circuit voltage between terminals a and b under this condition will be V_T. By hypothesis, this must be the same as the open-circuit voltage between terminals a and b in the actual original circuit. Therefore, to obtain the Thevenin voltage V_T, we simply calculate or measure the open-circuit voltage of the original circuit.

If the load resistance is reduced to zero, we have a short-circuit condition. Now, if we place a short-circuit across terminals a and b of the Thevenin equivalent circuit in Figure 1.5 (b), the short-circuit current directed from a to b is

$$I_{SC} = \frac{V_T}{R_T} \tag{1.10}$$

Again, by hypothesis, this short-circuit current must be identical to the short-circuit current in the original network. It follows from Equation (1.10) that

$$R_T = \frac{V_T}{I_{SC}} \quad . \tag{1.11}$$

Thus, the Thevenin resistance is the ratio of the open-circuit voltage to the short-circuit current.

The Norton equivalent circuit consists of an independent current source in parallel with the Norton equivalent resistance as shown in Figure 1.6. It can be derived

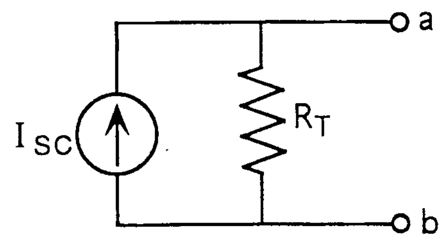


Figure 1.6: Norton Equivalent Circuit

from the Thevenin equivalent circuit by simply making a source transformation. Thus, the Norton current equals the short-circuit current at the terminals of interest, and the Norton resistance is identical to the Thevenin resistance.

Another useful method to determine R_T, other than the one defined above, is applicable if the network *only contains independent sources*. To calculate R_T for such a network, we first deactivate all independent sources and then calculate the resistance seen looking into the network at the designated terminal pair. A voltage source is deactivated by setting its voltage to zero, i.e., replacing it with a short-circuit. A current source is deactivated by setting its current to zero, i.e., replacing it with an open-circuit.

1.3.4 Measurement of Current via a Series Resistor

It is often desirable to measure current without the use of a current meter. This situation is most often encountered when one wants to examine a current waveform on the oscilloscope. The problem, which arises here, is that one cannot measure current directly with an oscilloscope (which will become apparent in future experiments), so a method must be devised to measure current indirectly.

Consider the circuit of Figure 1.7 (a). The box labeled "circuit" is some collection

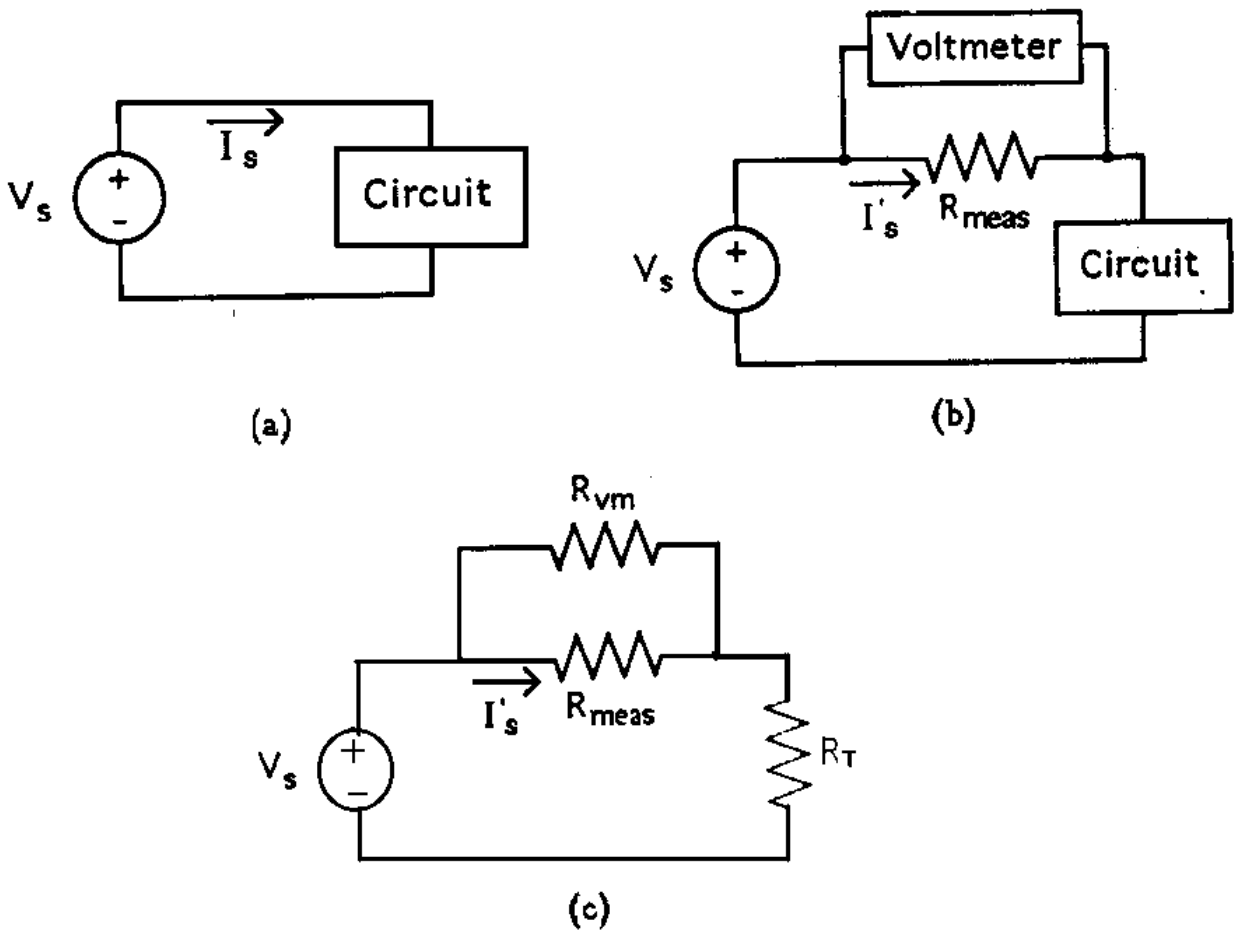


Figure 1.7: The Use of a Resistor to Measure Current: (a) Original Circuit indicating current I_S, (b) Resulting Circuit after the Insertion of a Resistor R_{meas} to measure I_S using a Voltmeter, and (c) Equivalent Circuit with Voltmeter resistance R_{VM} shown.

of circuit elements. Suppose that we wish to measure the current I_S without the use of an ammeter. One simple way to do this is with the circuit of Figure 1.7 (b), where a resistor R_{meas} has been added in series between V_s and the Circuit and a voltmeter is used to measure the voltage across R_{meas}. It follows that we can easily determine I_s' (that is, the current through R_{meas}) through the use of Ohm's law. However, note that a loading effect similar to what occurs with an ammeter can occur here. This is clearly illustrated by the equivalent circuit shown in Figure 1.7 (c), where the Thevenin equivalent resistance R_T of the circuit and the resistance of the voltmeter R_{vm} replace the network and the voltmeter, respectively. It follows that if the resistor R_{meas} is not small compared to R_T, then the addition of R_{meas} into the circuit will make the measured current I_s' significantly smaller than the actual current I_s. Thus, when using this method, care must be taken to choose an appropriate value of R_{meas} such that $R_{meas} << R_T$ and $R_{vm} >> R_{meas}$.

1.4 Advanced Preparation

The following advanced preparation is required before coming to the laboratory:

(a) Thoroughly read and understand the theory and procedures.

(b) Perform a PSpice Bias Point simulation for each of the circuits shown in Figures 1.8, 1.9, 1.10, and 1.11 using the "a" and "b" resistor values in the handout. Use 10 MΩ for the voltmeter resistance. For Figure 1.9, first assume that $R_{am} = 0\ \Omega$, then repeat the simulation assuming that the ammeter is on the 20 mA current scale.

(c) Determine the Thevenin and Norton equivalent circuits for the circuit shown in Figure 1.11 based on PSpice results.

1.5 Experimental Procedure

In this experiment and in some of those to follow, in the section labeled "Experimental Procedure", a number of the values that you will need to perform the experiment are represented only with symbols (such as adjust the voltage source to V_s). Your instructor will inform you of the correct values for you to use.

You should recognize that it is difficult (and expensive) to manufacture large quantities of resistors with a given value of resistance. Thus, resistor values are given with a tolerance, typically five to ten percent for the resistors in our lab. What this means is that a "2 kΩ" resistor with a five percent tolerance may have a resistance anywhere between 1.9 kΩ and 2.1 kΩ. For this reason, in order for your calculations to agree with the measurements that you take in the lab, ***you must measure the values of all of the resistors that you use*** with the ohmmeter provided (this is one of the many uses of the DMM). Furthermore, recognize that the digital readout on the power supplies should only be used as a guide; to get the true value, ***you must measure the exact value*** with the DMM. ***You are required to use the values of resistance and voltage measured by the DMM in your report*****.**

1.5.1 Use and Limitations of DC Voltmeters

Using one of the DMMs provided, adjust one of the DC power supplies until it indicates the exact value given by your instructor for V_{s1}. Now construct the circuit shown in Figure 1.8, using the "a" values given to you by your instructor for R_1 and R_2. Using the second DMM, measure the voltage V_{out}, as shown in the figure. Next, use the "b" values given to you by your instructor for R_1 and R_2 and repeat the adjustment of V_{s1} and your measurement of V_{out}.

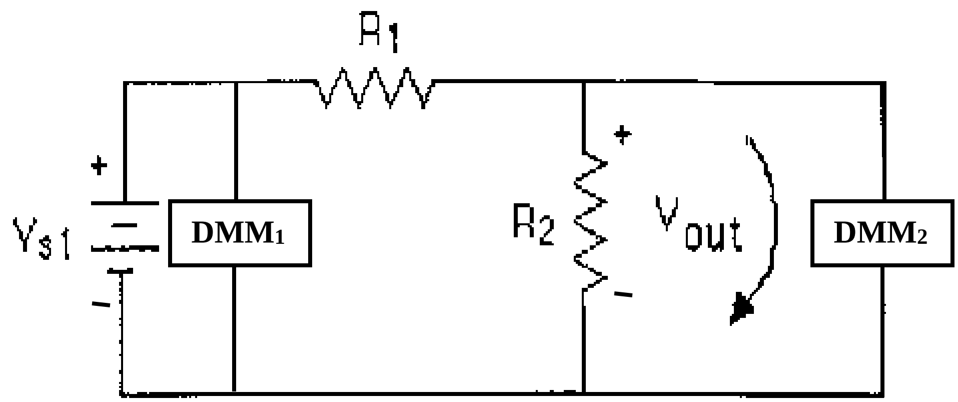


Figure 1.8: Circuit to be used to examine the limitations of DC Voltmeters

1.5.2 Current Measurement Via Series Resistance or a DC Ammeter

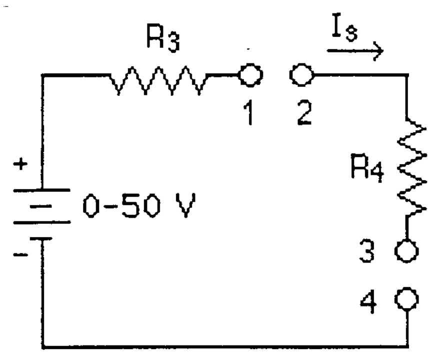


Figure 1.9: Circuit to be used to examine current measurement via a DC Ammeter and a series resistor in shunt with a DC voltmeter.

Construct the circuit of Figure 1.9 using the "a" values given to you by your instructor for R_3 and R_4. Connect a DC Voltmeter (DMM_1) across the power supply terminals and set it to the 100 volt DC scale. Connect the 1 kΩ Shunt Resistor provided by the instructor between terminals 1 and 2 and connect a DC Voltmeter (DMM_2) set to the 100 volt DC scale across it. Be sure to measure the actual resistance of this "1 kΩ" resistor before inserting it in the circuit. Note that a 1 volt reading on this DMM is approximately equal to 1 milliamp. Connect a DC Ammeter (DMM_3) between terminals 3 and 4 and set it on the 10 mA DC scale. Now advance the power supply from zero until the DC Ammeter (DMM_3) reads I_S. Record both DMM voltages and the ammeter current. Now cycle the DC Ammeter (DMM_3) through all of its current ranges and record its current as well as the corresponding two DMM voltages. If you are unable to obtain an ammeter reading for any range because the current is over-range, note this fact.

Repeat this procedure using the "b" values for R_3 and R_4.

1.5.3 Measurement of Current in a Current Divider Circuit

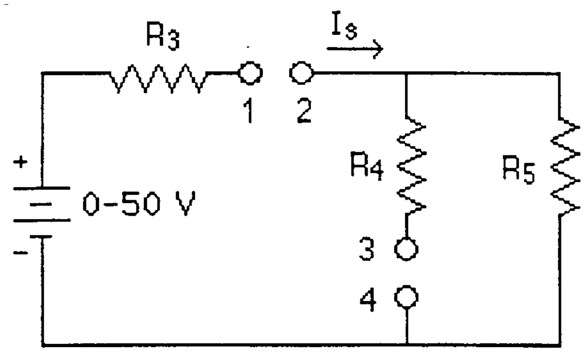


Figure 1.10: Circuit to be used to measure current in a Current Divider circuit

Modify the circuit of Figure 1.9 by adding resistor R_5 to obtain the current divider circuit shown in Figure 1.10 above. Note that you are now using the "b" values given to you by your instructor for R_3, R_4, and R_5. Note also that the ammeter is now measuring the current through the R_4 leg of the R_4-R_5 current divider. Set the ammeter back to the 10 mA scale. Adjust the power supply until the DC voltmeter (DMM_2) across the 1 kΩ resistor indicates I_S. Measure and record the ammeter current and the both voltmeter readings.

Repeat this procedure using the "a" values for R_3, R_4, and R_5.

1.5.4 Thevenin and Norton Equivalent Circuits

Build the circuit shown in Figure 1.11 on the next page using the parameter values specified by your instructor. Prior to construction, adjust V_{in} on the DC supply and measure the actual value of its voltage using a DMM. Next, construct the circuit

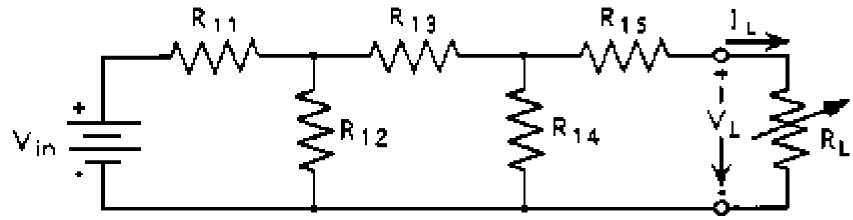


Figure 1.11: Test Circuit for Thevenin and Norton Circuits

such that one DMM is wired in series with R_L in order to measure load current I_L and the other DMM is wired in parallel with R_L to measure the load voltage, V_L. A decade resistor box should be used for R_L.

Now perform the following steps.

(a) Given the polarity and direction for V_L and I_L as shown in Figure 1.11, record V_L and I_L for the following five different values of R_L: (i) infinite (open-circuit), (ii) R_{16} , (iii) R_{17} , (iv) R_{18} , and (v) zero (short-circuit).

(b) Calculate the Thevenin equivalent circuit from the above experimental data.

(c) Adjust the decade resistance R_L until V_L is equal to half of the value of V_L*(open-circuit)* measured in (a) (i) above. Record the setting for R_L on the decade resistor box. Note that this is also the value of R_T. Turn off the power, remove the decade resistor box, and measure and record its actual resistance.

(d) Zero V_{in} by disconnecting V_{in} and connecting a short between the open end of R_{11} and ground. Measure the resistance of the network "looking into" the two output terminals using a DMM as an ohmmeter. Compare this measured Thevenin resistance to the Thevenin resistance calculated in part (b) above.

(e) Next, build the Thevenin equivalent circuit found in part (b) above using the power supply and a second decade resistor box for R_T. Measure I_L and V_L for this network using the same load resistances as in part (a).

1.6 Report

1.6.1 Use and Limitations of DC Voltmeters

1.6.1.1 Attach your PSpice Bias Point simulations for the circuit in Figure 1.8.

1.6.1.2 For both the "a" and the "b" values, calculate the expected value for V_{out} when no voltmeter is attached. Make a table comparing these results to those obtained using PSpice.

1.6.1.3 For both the "a" and the "b" values, determine the % error in your measured value compared to the value calculated in step 1.6.1.2 above.

1.6.1.4 Using the equivalent resistance of the voltmeter, calculate the value that you would expect the meter to indicate for both the "a" and the "b" values. Determine the % error in your measured value compared to this value.

1.6.2 Current Measurement Via Series Resistance or a DC Ammeter

1.6.2.1 Attach your PSpice Bias Point simulations for the circuit in Figure 1.9.

1.6.2.2 For both the "a" and the "b" values, calculate the values of the power supply voltage needed to make the ammeter placed between terminals 3 and 4 indicate I_s when on the 10 mA scale. Explain why these values make sense. Make a table comparing these results to those obtained using PSpice.

1.6.2.3 For both the "a" and the "b" values, construct a table that summarizes the current measurements you made using the 1 kΩ series resistor and the DC Ammeter on all 8 ammeter ranges.

1.6.2.4 For both "a" and "b" values, calculate the expected values of current for each DC ammeter (DMM_3) scale. Compare these to the measured values. Include % error calculations for this comparison.

1.6.2.5 Using the measured power supply voltage (DMM_1) and the current measurements made using the 1 kΩ resistor (DMM_2), determine for the "b" circuit values the equivalent resistance R_{am} of the ammeter for each of its eight ranges. What trend is indicated for R_{am} as the scale is changed?

1.6.2.6 Assuming the expected current readings calculated in Step 1.6.2.4 are correct, determine the % error in the measurement of the current for both the "a" and "b" sets of values. Calculate the % error for the measurements taken for both sets on all possible ranges of the ammeter. Considering also the decimal places shown in the ammeter readout, which range selection provides the most accurate reading for the "a" and the "b" sets of values?

1.6.2.7 If the choice for measuring current in a circuit is either that of inserting a 1 kΩ resistor or a DC Ammeter, which would you choose? Explain your answer.

1.6.2.8 If a resistor of known resistance is already part of the circuit, would your choice of current measurement technique change? Explain you answer.

1.6.3 Measurement of Current in a Current Divider Circuit

1.6.3.1 Attach your PSpice Bias Point simulations for the circuit in figure 1.10.

1.6.3.2 For both the "a" and the "b" values, calculate the expected value of the current assuming the ammeter is on the 10 mA scale.

1.6.3.3 Present the data that you took in for part 1.5.3. For both the "a" and the "b" values, calculate the current in R_5. Make a table showing all current values.

1.6.3.4 For both the "a" and the "b" values, calculate the % error between the expected value of I_4, the current in R_4, and the measured value.

1.6.4 Thevenin and Norton Equivalent Circuits

1.6.4.1 Attach your PSpice Bias Point simulations for the circuit in Figure 1.11.

1.6.4.2 Present the Thevenin and Norton equivalent circuits from part 1.4(c).

1.6.4.3 Show your calculations and draw the Thevenin and Norton equivalent circuits indicating the values of V_T, I_T, and R_T from part 1.5.4(b).

1.6.4.4 Calculate the theoretical values for V_L and I_L using the Thevenin equivalent circuit determined in part 1.6.4.3 above for each of the five load resistors used in part (a) of the experimental procedure in Section 1.5.4.

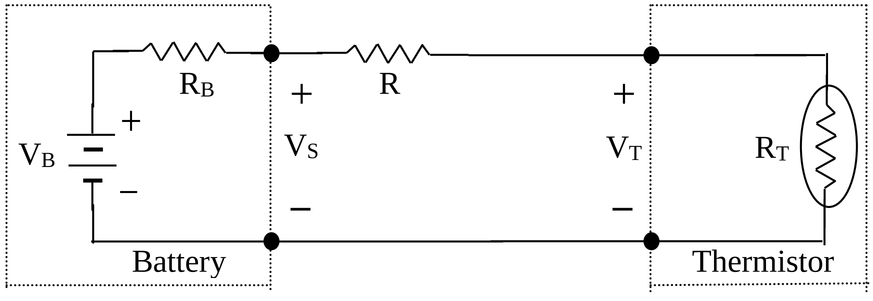
1.6.4.5 Now prepare a table presenting the values for V_L and I_L for each of the five resistor values using (i) PSpice results of part 1.4(b), (ii) measured values from the originally constructed circuit of part 1.5.4(a), (iii) measured values from the constructed Thevenin equivalent circuit of part 1.5.4(e), and (iv) computed values from part 1.6.4.4 above.

1.6.4.6 Based upon the entries in the table of part 1.6.4.5, are the Thevenin circuits equivalent to the original network?

1.6.5 Design Problem

The problem is to design a simple circuit using a thermistor to measure the temperature of a tank used to store volatile liquids. The measurement is to be accurate over a temperature range of 0°C to 50°C. The temperature must be measurable in an instrumentation room located a safe distance form the storage tank.

A thermistor is a temperature sensitive resistor that can be used in a voltage divider circuit to obtain an output voltage V_T that is functionally related to the thermistor temperature T as shown in the figure.



Thermistors with a negative temperature coefficient typically have a resistance versus temperature of the form

$$R_T = R_0 e^{-\alpha(T-T_0)} \tag{1.12}$$

An electronic supply catalog shows thermistors with the tabularized characteristics to be available. Note that T_0 = 25°C for these thermistors, i.e., $R_0 = R_T(25^oC)$.

Resistance Ratio (RR)	$R_T(\Omega)$ @ 25° C.
9.0	2.2K
9.0	6.8K
7.2	12K
7.2	22K
9.2	33K
9.4	47K
9.9	68K
9.9	100K
10.0	150K
12.0	470K

The Resistance Ratio (RR) shown in the table is (R_T at 0°C)/(R_T at 50°C), i.e.,

$$RR = R_T(0)/\ R_T(50). \tag{1.13}$$

Assume that a 47 KΩ thermistor from the above table is bonded to the storage tank in order to be measure tank temperature and that this bond has high thermal conductivity so that the thermistor temperature is the same as the tank temperature. Also, assume that electrical heating of the thermistor due to current is negligible and that the wires connecting the instrumentation room components to the thermistor are large enough that their resistance is negligible.

Assume that a Six Volt lead acid cell battery is available as a source and that the battery has a nominal terminal open circuit voltage (V_B) of 6.3 volts and an internal resistance (R_B) of 3 ohms. Additional constraints are that the battery has a 1 amp-hour rating, that the circuit must operate for 1 year without battery replacement, and that the series resistor R must be selected from standard values for 5% resistors shown in the Appendix I, Section 1.8.

Assume that a portable, battery powered, voltmeter with a 1MΩ input resistance is to be used to measure V_S and V_T. In your analysis, consider using a Thevenin equivalent circuit in order to incorporate the effect of this voltmeter on the circuit.

Design the circuit, i.e., select a series resistor R such that a plot of V_T versus T_T has a relatively small deviation from linearity over the temperature range of 0^o to 50^oC. Be sure that good sensitivity to temperature is obtained by the design, i.e., the range of voltage over the full range of temperature is sufficient to allow it to be accurately measured. A 50% voltage swing is considered adequate.

Once the design is established, find a linear equation for use in calculating tank temperature T_T from the voltage reading V_T. Note that in order to minimize errors due to any change in battery voltage, this equation should have the form

$$T_T = T_1 + K_V V_T/V_S. \tag{1.14}$$

Document your design as follows.

1.6.5.1 Clearly define the value selected for the series resistor R.

1.6.5.2 Clearly define T_1 and K_V used in the linear equation (1.14) to calculate T_T.

1.6.5.3 Briefly describe the process you used to arrive at a solution to the design problem. Give any additional assumptions made in arriving at the solution.

1.6.5.4 Provide a plot of V_T versus T_T using the nominal component values for your final design. Also provide a plot of the deviation in V_T from linearity. Use Excel, MATLAB, or any other mathematical computer program to generate these plots.

1.6.5.5 Identify the maximum deviation in the voltage reading V_T over the full temperature range and convert this to a maximum error in measured temperature.

1.7 References

7.1 Nilsson, J. W., **Electric Circuits**, (6^{th} ed.), Prentice Hall, New Jersey, 2001.

7.2 **8050A Digital Multimeter Manual**, John Fluke Co., 1979.

1.8 Appendix I - Standard Resistor Value Multipliers

These multiplier values in the table apply to all ± 5% tolerance resistors. Resistors with ± 10% tolerance are available only in values marked by a *. The multipliers are:

1.0*	1.5*	2.2*	3.3*	4.7*	6.8*
1.1	1.6	2.4	3.6	5.1	7.5
1.2*	1.8*	2.7*	3.9*	5.6*	8.2*
1.3	2.0	3.0	4.3	6.2	9.1

The multipliers in the table apply for nominal resistor values of $10^n\Omega$. For example, using the 1.1 multiplier, you can get standard 5% resistors with values of 11Ω, 110Ω, 1.1kΩ, 11kΩ, 110kΩ, 1.1MΩ, etc. Also, while you can choose 5% resistors with nominal values of 10Ω (1.0*10Ω) or 11Ω (1.1*10Ω) or 12Ω (1.2*10Ω), you cannot choose a 10% resistor with a nominal value of 11Ω (1.1*10Ω) since this value is within the tolerance limit of both the 10Ω and the 12Ω resistors.