

Appendix **G** An Abbreviated Table of Integrals

1. $\int x e^{ax} dx = \frac{e^{ax}}{a^2}(ax - 1)$
2. $\int x^2 e^{ax} dx = \frac{e^{ax}}{a^3}(a^2 x^2 - 2ax + 2)$
3. $\int x \sin ax dx = \frac{1}{a^2} \sin ax - \frac{x}{a} \cos ax$
4. $\int x \cos ax dx = \frac{1}{a^2} \cos ax + \frac{x}{a} \sin ax$
5. $\int e^{ax} \sin bx dx = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx)$
6. $\int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx)$
7. $\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a}$
8. $\int \frac{dx}{(x^2 + a^2)^2} = \frac{1}{2a^2} \left(\frac{x}{x^2 + a^2} + \frac{1}{a} \tan^{-1} \frac{x}{a} \right)$
9. $\int \sin ax \sin bx dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)}, \quad a^2 \neq b^2$
10. $\int \cos ax \cos bx dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)}, \quad a^2 \neq b^2$
11. $\int \sin ax \cos bx dx = -\frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)}, \quad a^2 \neq b^2$
12. $\int \sin^2 ax dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$
13. $\int \cos^2 ax dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$
14. $\int_0^{\infty} \frac{a dx}{a^2 + x^2} = \begin{cases} \frac{\pi}{2}, & a > 0; \\ 0, & a = 0; \\ -\frac{\pi}{2}, & a < 0 \end{cases}$
15. $\int_0^{\infty} \frac{\sin ax}{x} dx = \begin{cases} \frac{\pi}{2}, & a > 0; \\ -\frac{\pi}{2}, & a < 0 \end{cases}$
16. $\int x^2 \sin ax dx = \frac{2x}{a^2} \sin ax - \frac{a^2 x^2 - 2}{a^3} \cos ax$
17. $\int x^2 \cos ax dx = \frac{2x}{a^2} \cos ax + \frac{a^2 x^2 - 2}{a^3} \sin ax$

$$18. \int e^{ax} \sin^2 bx \, dx = \frac{e^{ax}}{a^2 + 4b^2} \left[(a \sin bx - 2b \cos bx) \sin bx + \frac{2b^2}{a} \right]$$

$$19. \int e^{ax} \cos^2 bx \, dx = \frac{e^{ax}}{a^2 + 4b^2} \left[(a \cos bx + 2b \sin bx) \cos bx + \frac{2b^2}{a} \right]$$