JCE 4600
Fundamentals of Traffic Engineering

Horizontal and Vertical Curves

Agenda
- Horizontal Curves
- Vertical Curves
- Passing Sight Distance
Roadway Design Motivations

- Vehicle performance
  - Acceleration and deceleration
  - Turning radius
- Driver performance
  - Stopping-sight distance
  - Comfort
- Constraints
  - Economic
  - Social/Environmental
- Traffic Operations

Alignment

- Roadways are three dimensional
Alignment

- Design process separates into 2-dimensional problems

  **Plan view (horizontal alignment)**

  **Profile view (vertical alignment)**

Alignment

- **Horizontal alignment**
  - Plan view or Aerial photo view
  - Measures distance along the roadway in stations
  - Each station is 100 ft (1000 m for metric)
  - Notation: 4250 ft is written 42+50, for 42 stations and 50 ft

- **Vertical alignment**
  - Profile view
  - Elevation above a reference line
JCE 4600
Fundamentals of
Traffic Engineering

Horizontal Curves

Horizontal Alignment
**Example Plan Sheet**

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### Horizontal Alignment

- Connect two straight sections of roadway
- Two objectives
  - **Cornering performance**
    \[ R_{\text{min}} = \frac{V^2}{15(e \pm f)} \]
  - **Stopping sight distance**
    \[ SSD = (1.47 \times t_p \times v_o) + \frac{v_o^2 - v^2}{30(f \pm G)} \]

- **Formulas**
  - \( R_{\text{min}} = \) min. radius (ft)
  - \( V = \) design speed (mph)
  - \( e = \) superelevation (ft/ft)
  - \( f = \) side friction factor
  - \( SSD = \) Stopping Sight Distance (ft)
  - \( t_p = \) perception/reaction time (2.5 sec)
  - \( v = \) final velocity (mph)
  - \( v_o = \) initial velocity (mph)
  - \( f = \) friction coefficient
  - \( G = \% \) Grade/100
**Horizontal Curves**

- $R$ – radius (road centerline)
- $D$ – central angle of curve
- $PC$ – beginning Point of Curve
- $PI$ – Point of Tangent Intersection
- $PT$ – Point of ending Tangent
- $T$ – tangent length
- $E$ – external distance
- $M$ – middle ordinate
- $L$ – length of curve

- $\sin x = \frac{\text{side opposite } x}{\text{hypotenuse}}$
- $\cos x = \frac{\text{side adjacent } x}{\text{hypotenuse}}$
- $\tan x = \frac{\text{side opposite } x}{\text{side adjacent } x}$

### Curve Equations

<table>
<thead>
<tr>
<th>Variables</th>
<th>U.S. Units</th>
<th>Metric Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius</td>
<td>$R = \sqrt{\frac{15}{127}} (e+f)$</td>
<td>$R = \sqrt{\frac{127}{127}} (e+f)$</td>
</tr>
<tr>
<td>External Distance</td>
<td>$E = T \tan \left(\frac{\Delta}{4}\right)$</td>
<td>$E = T \tan \left(\frac{\Delta}{4}\right)$</td>
</tr>
<tr>
<td>Middle Ordinate</td>
<td>$M = E \cos \left(\frac{\Delta}{2}\right)$</td>
<td>$M = E \cos \left(\frac{\Delta}{2}\right)$</td>
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<tr>
<td>Tangent Length</td>
<td>$T = R \tan \left(\frac{\Delta}{2}\right)$</td>
<td>$T = R \tan \left(\frac{\Delta}{2}\right)$</td>
</tr>
<tr>
<td>Length of Curve</td>
<td>$L = R \Delta (\Delta \text{ in radians})$</td>
<td>$L = R \Delta (\Delta \text{ in radians})$</td>
</tr>
<tr>
<td>Degree of Curve</td>
<td>$D = \frac{5729.578}{R}$</td>
<td>Not Used</td>
</tr>
<tr>
<td>Chord Length</td>
<td>$LC = 2R \sin \left(\frac{\Delta}{2}\right)$</td>
<td>$LC = 2R \sin \left(\frac{\Delta}{2}\right)$</td>
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</table>
Types of Curves

- Fixed radius circular curves are the most simple
- Circular curves can be connected together as compound curves
- Reverse curves shift traffic laterally
- Spiral curves have a continuously changing radius
  - More difficult to design and build
  - Uses:
    - Railroads
    - Sharp, high-speed curves,
    - Transition to a superelevation

Cornering Performance

\[ R_{\text{min}} = \frac{V^2}{15(e \pm f)} \]

- \( R_{\text{min}} \) = min. radius (ft)
- \( V \) = design speed (mph)
- \( e \) = superelevation (ft/ft)
- \( f \) = side friction factor

![Graph showing cornering performance](chart.png)
Simple Relationships

Degree of Curve

- Measures the sharpness of a curve
- The angle subtended by a 100 ft arc along the horizontal curve

- $100/2\pi R = \frac{D}{360}$
- $D = \frac{5730}{R}$

<table>
<thead>
<tr>
<th>METRIC</th>
<th>US Customary</th>
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<tr>
<td>Design Speed (mph)</td>
<td>Maximum $\varepsilon$ (%)</td>
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<td>15</td>
<td>4.0</td>
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<tr>
<td>20</td>
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<tr>
<td>150</td>
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</table>
Stopping-Sight Distance

- Stopping-sight distance is important for horizontal curves
- Finding it is complicated by off road visual obstructions, i.e. curve around a house
- The road must be an appropriate distance from such obstructions

Horizontal Curves and Stopping-Sight Distance

- $R$ Distance from obstruction to center of inside lane
- Highway centerline
- Center of inside lane
- SSD is the length of the arc along the center of the inside lane, marked by the triangle
Horizontal Curves and Stopping-Sight Distance

- $M_s$ (the middle ordinate) is the distance of obstruction from the center of the inside lane required to provide adequate stopping sight distance
- $m = R[1 - \cos(\Delta/2)]$

Horizontal Curve Example Problem

- Consider a horizontal curve on a two lane rural highway
  - Radius of the highway's centerline = 100 ft
  - $f_{\text{max}} = 0.5$;
  - $e = 0.08$
  - $G = 0\%$
  - Lanes are each 12 feet wide
  - Buildings are 10 feet from the edge of the highway
  - What speed limit would insure MSSD?
Vertical Alignment

- Specifies the elevation of points along the roadway
- Determined by the lay of the land, need for drainage of rainfall, safety
- Two different elevations of roadway must be connected with a vertical curve

Vertical Curves

- Crest vertical curve
  - Over a hill
- Sag vertical curve
  - Down and up
**Vertical Curves**

- $G_1$ – initial roadway grade in percent or ft/ft (m/m), slope of the initial tangent line
- $G_2$ – final roadway grade
- $A$ – absolute value of grade difference $|G_1 - G_2|$ (usually percent)
- PVC – Point of the Vertical Curve (starting point)
- PVI – Point of Vertical Intersection (intersection of initial and final grades, tangents)
- PVT – Point of Vertical Tangent (stopping point)
- $L$ – Length of curve in stations measured in a constant-elevation horizontal plane (along the road)

**Curve Equations**

- $A = g_2 - g_1$
- $E = \frac{(G_1 - G_2)L}{8} = \frac{AL}{800}$
- $E_x = E_{PC} + G_1x + \frac{(G_2 - G_1)x^2}{2L}$
- $y = \frac{(G_2 - G_1)x^2}{2L}$

For any point $p$ on curve,

For high (low) point on curve,

$K = \frac{L}{A}$

$K$ = Distance needed to change 1% grade

$G = \text{Grade (actual)}$

$g = \text{grade (%) }$
## Stopping-Sight Distance

- To save on construction costs we want vertical curves as short as possible
- From braking we learned that there is a stopping distance
- The stopping-sight distance sets limits on the minimum crest vertical curve length
  - Drainage can also impact vertical curve design
- We must develop a design stopping-sight distance that incorporates margins of safety

## Crest Vertical Curve and Sight Distance

\[ \text{Minimum curve length} \]

- \( H_1 \): Driver eye height
- \( H_2 \): Height of object to avoid
Crest Vertical Curve and Sight Distance

- Longer curves, \( L \), provide longer sight distance, \( S \)
- \( H_1 \) is the driver eye height in ft (3.5 ft)
- \( H_2 \) is the minimum height (ft or m) of object to be avoided (2 ft)

Case 1: \( S > L \)

\[
L = 2S \left( \frac{200 \sqrt{H_1 + H_2}}{A} \right) \quad \text{for} \quad S > L
\]

- \( L \) = Length of Curve
- \( S \) = Sight Distance
- Assumes \( H_1 = 3.5 \) feet; \( H_2 = 2 \) Feet
Case 2: S<L

\[ L = \frac{AS^2}{100 \sqrt{2H_1 + \sqrt{2H_2}}} \]

\[ L = \frac{AS^2}{2158} \quad \text{for } S < L \]

L = Length of Curve
S = Sight Distance
Assumes H_1 = 3.5 feet; H_2 = 2 Feet

Design Controls for CVC
Sag Vertical Curves

- The stopping-sight distance is only a concern in nighttime conditions.
- You can see across the curve during daytime.
- The height of the headlights and the illuminated distance (affected by headlight angle) become the limiting factors.
- Absolute minimum length = 3 times velocity (in mph).

\[ \beta \] – inclined angle of headlight beam in degrees.

Absolute minimum length = 3v.
Case 1: S>L

\[ L = 2S - \frac{400 + 3.55S}{A} \]

L = Length of Curve
S = Light beam distance (assumes 1 degree downward deflection)
Assumes H1, H2 = 2 Feet

Case 2: S<L

\[ L = \frac{AS^2}{400 + 3.55S} \]

L = Length of Curve
S = Light beam distance (assumes 1 degree downward deflection)
Assumes H1, H2 = 2 Feet
Grades

- Maximum grades controlled by vehicle operating characteristics
  - Typically 5% for high speed design.
  - 7 to 12% acceptable for low speed design
- Minimum grades controlled by drainage considerations
  - Typically 0.5% desirable minimum, 0.3% absolute minimum

Example Problem

- Given: \( f_{\text{max}} = 0.4 \) and assume the worst case value of G (i.e. \( G = -0.06 \)) for all parts of this problem.

  a) What is the maximum safe speed to insure MSSD for a 900 foot crest vertical curve connecting a 6% grade and a -3% grade. What is the K value of this curve?

  b) If the PVC of a 900 ft crest vertical curve connecting a 6% grade and a -3% grade is at station 100+00 and elevation 1000 ft, what is the station and elevation of the midpoint of the curve? What is the station and elevation of the PVT?
**Passing Sight Distance**

- Important design consideration on 2-lane roads
- Provide frequent, regularly spaced passing zones
- PSD made up of 4 components
  - Distance traveled during perception/reaction time
  - Distance traveled while passing in left lane
  - Clear distance between passing vehicle and opposing vehicle
  - Distance traveled by opposing vehicle during 2/3 of time passing vehicle is in opposing lane
- PSD measured 3.5-ft eye height 3.5-ft target
AASHTO Passing Sight Distance

- $d_p$: Distance traversed during perception and reaction time and during the initial acceleration to the point of encroachment on the left lane.
- $d_t$: Distance traveled while the passing vehicle occupies the left lane.
- $d_f$: Distance between the passing vehicle at the end of its maneuver and the opposing vehicle.
- $d_o$: Distance traversed by an opposing vehicle for two-thirds of the time the passing vehicle occupies the left lane, or $\frac{2}{3}d_f$ above.

Exhibit 3-4. Elements of Passing Sight Distance for Two-Lane Highways

Exhibit 3-6. Total Passing Sight Distance and Its Components—Two-Lane Highways
MUTCD Passing Sight Distance

Table 3B-1. Minimum Passing Sight Distances

<table>
<thead>
<tr>
<th>85th-Percentile or Posted or Statutory Speed Limit (km/h)</th>
<th>Minimum Passing Sight Distance (meters)</th>
<th>85th-Percentile or Posted or Statutory Speed Limit (mph)</th>
<th>Minimum Passing Sight Distance (feet)</th>
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<tr>
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Homework (±3 hours)
Due: Next Class

1. What is the maximum allowable degree of curvature, D, assuming e=6% for a 30 mph curve? Assume a value of “f” allowing for driver comfort. Justify your “f” value.

2. Consider a horizontal curve on a two lane rural highway
   - Radius of the highway’s centerline = 500 ft; f\text{max} = 0.5; e = 0.04; G = 0%; lanes are each 12 feet wide; buildings are 15 feet from the edge of the highway
   - What speed limit would insure MSSD?

3. Given: f\text{max} =0.3 and assume the worst case value of G (i.e. G = - 5%) for all parts of this problem.
   - a) What is the maximum safe speed to insure MSSD for a 450 foot crest vertical curve connecting a 5% grade and a -3% grade. What is the K value of this curve?
   - b.) If the PVC of a 450 ft crest vertical curve connecting a 5% grade and a -3% grade is at station 200+50 and elevation 800 ft, what is the station and elevation of the midpoint of the curve? What is the station and midpoint of the PVT?

4. A -4% grade and a +1% grade meet at station 24+40 and elevation 2420 (PVI). They are joined with a 800’ vertical curve.
   - The curve passes under an overpass at station 25+00. The lowest elevation of the overpass is 2480’. What is the available clearance?