OVERVIEW OF BEAMFORMING

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Outline

- Introduction
- Spatial and temporal filtering
- Frequency response
- Design methods
- Examples of applications
- Performance analysis
- Conclusion

Introduction

- Beamforming: A method used in array signal processing mainly for the following two goals:
  - Finding the direction of a desired signal.
  - Enhancing the desired signal.
- Beamforming is a general filtering operation that combines temporal and spatial filtering.
- It uses a weighted sum of the outputs of multiple sensors at certain time instants.
- In other words, beamforming is a linear combination of the temporal outputs of multiple sensors.

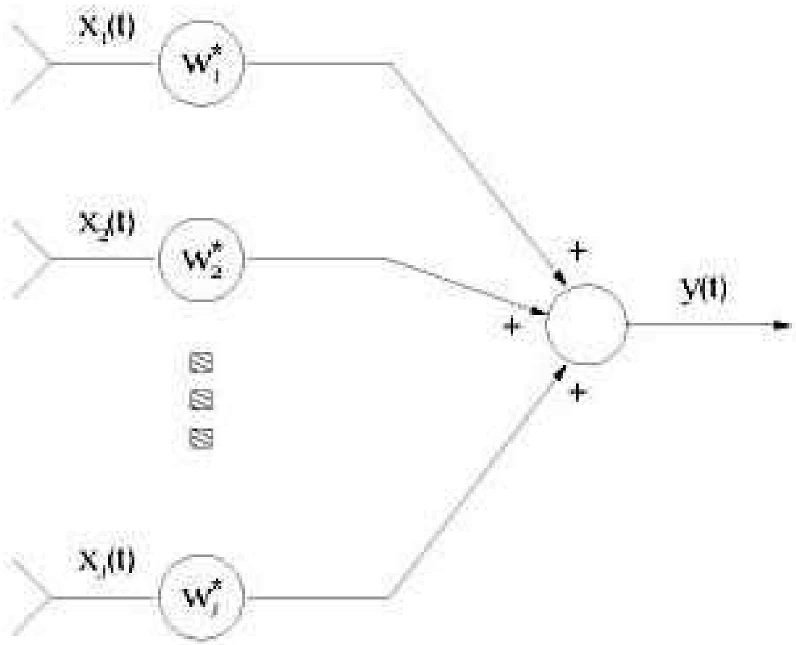
Applications

- Radar.
- Sonar.
- Biomedicine.
- Communications.
- Imaging.
- Geophysics.
- Astrophysics.

Spatial vs. Temporal Filtering

- Spatial filtering allows resolving signals sharing the same temporal frequency but spatially separated.
- Temporal filtering can only resolve signals only having different temporal frequency content.
- Spatial filtering weights outputs of multiple sensors located at different physical locations.
- Temporal filtering weights the delayed versions of the output of a single sensor.

Spatial Filtering



Spatial Beamformer.

A spatial beamformer weights the outputs of the sensors at a certain time.

Spatial Filtering (Cont.)

Output of the filter is:

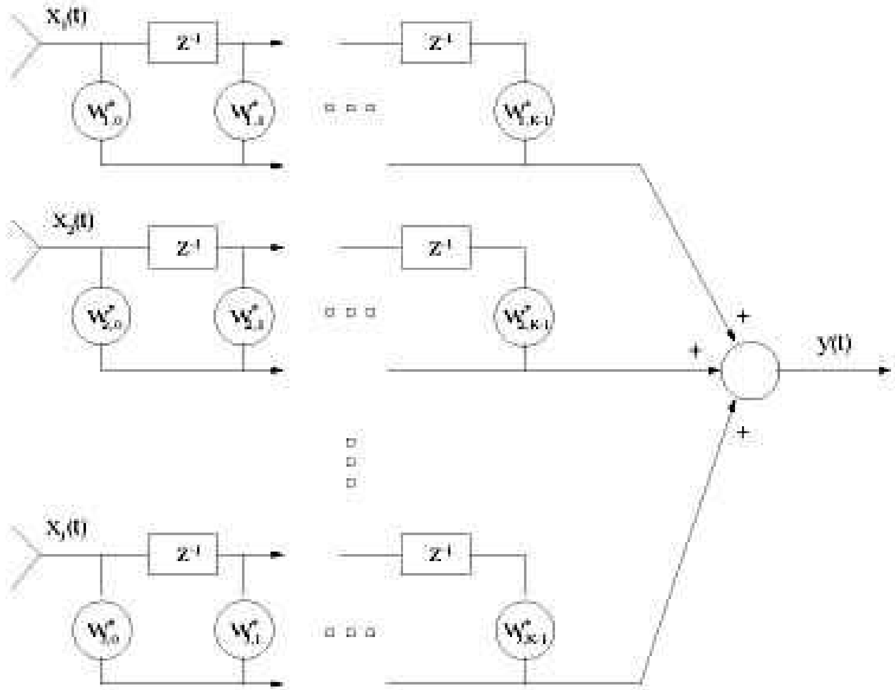
$$y(t) = \sum_{i=1}^{J} w_i^* x_i(t),$$

where J denotes the number of sensors, $x_i(t)$'s are the signals arriving at these sensors at time t, and w_i's are the spatial weights. In vector form:

$$y(t) = \boldsymbol{w}^{\mathrm{H}} \boldsymbol{x}(t),$$

where $\boldsymbol{x}(t) = [x_1(t), x_2(t), \ldots, x_{_J}(t)]^{\mathrm{T}}$, $\boldsymbol{w} = [w_1, w_2, \ldots, w_{_J}]^{\mathrm{H}}$, and we have used "T" for transpose and "H" for Hermitian conjugate.

Spatial Filtering Combined With Temporal Filtering



Space-time beamformer.

Here, we weight the outputs of the sensors and their delayed versions.

Spatial Filtering Combined With Temporal Filtering (Cont.)

$$y(t) = \sum_{i=1}^{J} \sum_{p=0}^{K-1} w_{i,p}^{*} x_i(t - pT),$$

where $K-1$ is the number of delays in each of the sensor channels and T is the duration of a single delay.
In vector form:

$$y(t) = \boldsymbol{w}^{\mathrm{H}} \boldsymbol{x}(t),$$

where

$$\begin{aligned} \boldsymbol{x}(t) &= [x_1(t),\, x_1(t-T), \ldots, \\ &\quad x_1(t-(K-1)T), \ldots, x_2(t), \ldots, x_{J}(t-(K-1)T)]^{\mathrm{T}}, \\ \boldsymbol{w} &= \left[w_{1,0},\, w_{1,1}, \ldots, w_{1,(K-1)}, \ldots, w_{2,0}, \ldots, w_{J,(K-1)}\right]^{\mathrm{H}}. \end{aligned}$$

Frequency Response

- We now analyze the filters in the frequency domain.
- Frequency response of a filter $= \frac{\text{Response to an exponential}}{\text{Exponential}}$
- Frequency response of a spatial filter: For an exponential input $e^{j\omega t}$, where ω represents the frequency of the exponential, the output is:

$$y(t) = \sum_{i=1}^{J} w_i^* e^{j\omega(t-\Delta_i(\theta))},$$

  where $\Delta_i(\theta)$ is the delay due to propagation at each sensor and θ is the direction of arrival of the signal.

Frequency Response (Cont.)

- Therefore,

$$r(\theta, \omega) = \frac{y(t)}{e^{j\omega t}} = \sum_{i=1}^{J} w_i^* e^{j\omega \Delta_i(\theta)}.$$

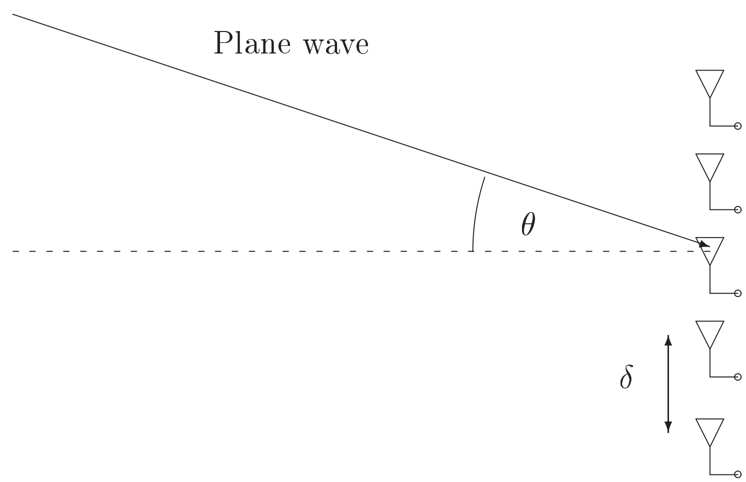
- In vector form:

$$r(\theta, \omega) = \boldsymbol{w}^{\mathrm{H}} \boldsymbol{d}(\theta, \omega),$$

  where $\boldsymbol{d}(\theta, \omega) = [1,\ e^{-j\omega \Delta_2(\theta)}, \ldots, e^{-j\omega \Delta_J(\theta)}]^{\mathrm{T}}$.

- $\boldsymbol{d}(\theta, \omega)$ gives a complete idea of the array spatial response. It is often called steering vector, array response vector, or direction vector.

Frequency Response (Cont.)



$$\boldsymbol{d}(\theta) = \begin{bmatrix} 1 \\ \exp[-j\omega\delta\sin(\theta)] \\ \exp[-j2\omega\delta\sin(\theta)] \\ \exp[-j3\omega\delta\sin(\theta)] \\ \exp[-j4\omega\delta\sin(\theta)] \end{bmatrix}$$

Example: Uniform Linear Array where $\delta = d/v$ with d the distance between the sensors and v is the waveform speed of propagation.

Frequency Response (Cont.)

- Frequency response of spatial-temporal filter:
  In this case, we also use the delayed versions of the exponential, so the output is:

$$y(t) = \sum_{i=1}^{J} \sum_{p=0}^{K-1} w_{i,p}^{*} e^{-j\omega(\Delta_i(\theta)+pT-t)}.$$

Therefore,

$$r(\theta, \omega) = \sum_{i=1}^{J} \sum_{p=0}^{K-1} w_{i,p}^{*} e^{-jw(\Delta_i(\theta)+pT)}.$$

In vector form:

$$\begin{aligned} r(\theta, \omega) &= \boldsymbol{w}^{\mathrm{H}} \boldsymbol{d}(\theta, \omega), \\ \boldsymbol{d}(\theta, \omega) &= [1,\, e^{T_{1,1}(\theta)}, \ldots, e^{T_{J,K}(\theta)}], \\ T_{i,p}(\theta) &= -j\omega(\Delta_i(\theta) + pT). \end{aligned}$$

Design Methods

- When designing a beamformer, we first set our objective, which may be one of: signal enhancement, direction finding, interference suppression, and possibly others.
- Then, we choose a mathematical criterion to help us achieve this objective.
- The weights are selected to meet the criteria as close as possible.
- There are two main criteria for designing beamformers:
  - Classical beamforming (data independent) and
  - Adaptive beamforming.

Design Methods: Data Independent

Data-independent methods assume apriori knowledge of second-order statistics of the data.

1. Least-squares error optimization:

   - In this method, we choose our filter coefficients to approximate a desired frequency response.
   - We want to minimize the error between $r_{\mathrm{d}}(\theta, \omega)$, the desired response, and $r(\theta, \omega)$ that we obtain as a result of our design at P points.

Design Methods: Data Independent (Cont.)

- Mathematically, the optimum weights are given by:

$$\min_{w} |A^{\mathrm{H}}\boldsymbol{w} - \boldsymbol{r}_{\mathrm{d}}|^2,$$

where

$$A = [d(\theta_1, \omega_1),\ d(\theta_2, \omega_2), \ldots, d(\theta_P, \omega_P)],$$

and

$$\boldsymbol{r}_{\mathrm{d}} = [r_{\mathrm{d}}(\theta_1, \omega_1),\ r_{\mathrm{d}}(\theta_2, \omega_2), \ldots, \mathrm{r}_{\mathrm{d}}(\theta_P, \omega_P)]^{\mathrm{H}}.$$

- Solution to this problem is the familiar pseudo-inverse:

$$\boldsymbol{w}_{\mathrm{opt}} = (AA^{\mathrm{H}})^{-1}A\boldsymbol{r}_{\mathrm{d}}.$$

Design Methods: Data Independent (Cont.)

- How to choose the desired response? We can choose it to be unity in the direction of the signal, and zero in the directions of interferences.
- The disadvantage of this method is the requirement of knowing the desired signal and interference directions.
- Also we face the classical problem of the inverses, amplification of the noise when the matrix A is ill-conditioned.

Design Methods: Data Independent (Cont.)

2. Maximum signal-to-noise ratio (SNR):

- In this case we design our beamformer to improve the SNR for a source with a given direction and frequency.
- We maximize the ratio of the SNR's at the output and input.
- Assuming stationarity and a narrow-band random signal, the output power is:

$$\mathrm{E}\{|y|(t)^2\} = |\boldsymbol{w}^{\mathrm{H}}\boldsymbol{d}(\theta_0, \omega_0)|^2\sigma_{\mathrm{s}}^2 + \boldsymbol{w}^{\mathrm{H}}\Sigma\boldsymbol{w},$$

  where σ_{s}^2 is the variance of the signal with direction θ_0 and frequency ω_0, and Σ is the covariance matrix of the interferences and noise.

Design Methods: Data Independent (Cont.)

- Hence the optimum weights are given by:

$$\max_{\boldsymbol{w}} \frac{\mathrm{SNR}_{\mathrm{out}}}{\mathrm{SNR}_{\mathrm{in}}} = \max_{\boldsymbol{w}} \frac{|\boldsymbol{w}^H \boldsymbol{d}(\theta_0, \omega_0)|^2}{\boldsymbol{w}^H \Sigma \boldsymbol{w}} \frac{\mathrm{trace}(\Sigma)}{J}.$$

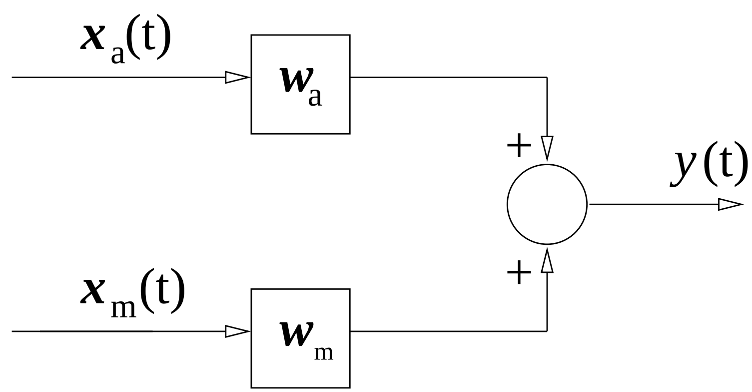
- Solution to this problem is similar to a matched filter:

$$\boldsymbol{w}_{\mathrm{opt}} \, \alpha \, \Sigma^{-1} \boldsymbol{d}(\theta_0, \omega_0).$$

- This method can be used to enhance of the desired signal.
- Disadvantage of this method is the requirement of knowing the frequency and direction of the signal as well as the covariance of interferences and noise.

Design Methods: Data Independent (Cont.)

3. Generalized sidelobe canceller:



- We extract the main signal component from the signals arriving at the sensors using a set of coefficients denoted by $\boldsymbol{w}_{\mathrm{m}}$ in the figure, and eliminate the auxiliary components using a set of coefficients denoted by $\boldsymbol{w}_{\mathrm{a}}$ in the figure.

Design Methods: Data Independent (Cont.)

- To completely cancel the interferences is sometimes impossible and usually results in large white-noise gain.
- Weights are chosen to trade-off interference suppression for white-noise gain.
- Especially useful when the desired signal is weak compared with interference.

Design Methods: Data Independent (Cont.)

4. Linearly-constrained minimum variance beamformer (LCMV):
   - In this method, we design our beamformer so that the output power is minimum subject to a constraint.
   - The constraint allows us to have a certain phase and amplitude of the response at certain directions.
   - Minimizing the output power is necessary to attenuate the contributions from the interferences and noise.

Design Methods: Data Independent (Cont.)

- Mathematically, we can write:

$$\boldsymbol{w}_{\text{opt}} = \underset{\boldsymbol{w}}{\operatorname{argmin}}\,(\boldsymbol{w}^{\mathrm{H}} R \boldsymbol{w}),$$

  subject to

$$C^{\mathrm{H}}(\boldsymbol{\theta}, \omega)\boldsymbol{w} = \boldsymbol{f},$$

  where R is the covariance matrix of the measurements at the sensors, $C(\boldsymbol{\theta}, \omega)$ is a matrix formed by the steering vectors of directions that we want to put constraints on, and $\boldsymbol{f}$ is a pre-determined complex vector chosen to give the desired amplitude and phase response at the given directions.

Design Methods: Data Independent (Cont.)

- The solution to this minimization problem can be found using Lagrange multipliers, thus:

$$\boldsymbol{w}_{\mathrm{opt}} = R^{-1}[C^{\mathrm{H}}R^{-1}C]^{-1}\boldsymbol{f}.$$

Design Methods: Data Independent (Cont.)

5. Minimum variance distortionless response (MVDR) or Capon method:

- This is a special case of LCMV with $\boldsymbol{f} = 1$ and $C = \boldsymbol{d}^{\mathrm{H}}(\theta, \omega)$.
- It deserves special attention since it is very effective, robust and commonly used.
- Optimum weights are given by:

$$\boldsymbol{w}_{\mathrm{opt}} = \frac{R^{-1}\boldsymbol{d}(\theta, \omega)}{\boldsymbol{d}^{\mathrm{H}}(\theta, \omega)R^{-1}\boldsymbol{d}(\theta, \omega)}.$$

- With these optimum weights, the minimum output power is:

$$\mathrm{E}\{y(t)^2\} = \boldsymbol{w}_{\mathrm{opt}}^{\mathrm{H}} R \boldsymbol{w}_{\mathrm{opt}} = \frac{1}{\boldsymbol{d}^{\mathrm{H}}(\theta, \omega)R^{-1}\boldsymbol{d}(\theta, \omega)}.$$

Design Methods: Adaptive

- All of the above methods require the knowledge of second-order statistics of the data.
- If we consider that these are known to the designer and we do not need to estimate them, optimum weights are independent of data, hence the name.
- But if we need to estimate the second-order statistics, our optimum weights are now dependent on the data.

Design Methods: Adaptive (Cont.)

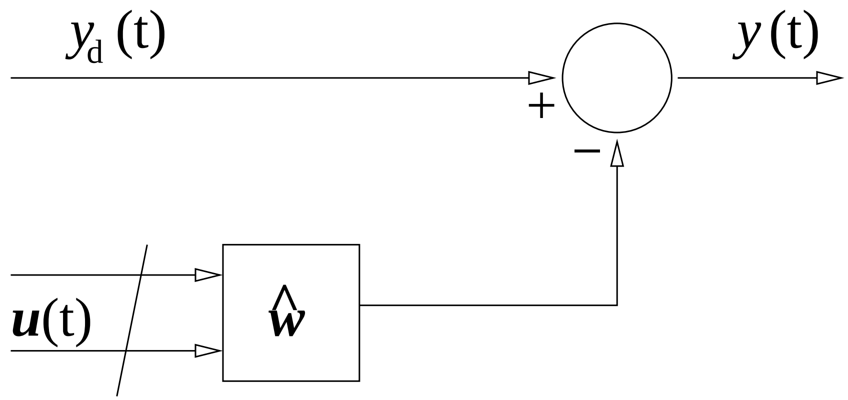
- There are two approaches to estimating second-order statistics:
  - Block adaptation: Statistics are estimated from a temporal block of array data, and used in calculating the optimum weights.
  - Continuous adaptation: Estimator for the statistics is adaptive, hence updated as new data arrives. In this case, optimum weights are updated with the new data and converges to the optimum solution.
- An example of block adaptation is the sample covariance matrix:

$$\hat{R} = \frac{1}{L}\sum_{k=0}^{L} \boldsymbol{x}(k)\boldsymbol{x}^H(k),$$

where L is the total number of samples we have.

Design Methods: Adaptive (Cont.)

Example of continuous adaptation:



- Here, we use the new arriving data to update our estimate of the second-order statistics.
- We select weights to minimize the mean-squared error:

$$
\begin{aligned}
\mathrm{E}\{|y_\mathrm{d}(t) - \boldsymbol{w}^\mathrm{H}\boldsymbol{u}(t)|^2\} \quad = \quad & \mathrm{E}\{|y_\mathrm{d}|^2(t)\} - \boldsymbol{w}^\mathrm{H}\mathrm{E}\{\boldsymbol{u}(t)y_\mathrm{d}^*(t)\} - \\
& \mathrm{E}\{\boldsymbol{u}^\mathrm{H}(t)y_\mathrm{d}^*(t)\}\boldsymbol{w} + \boldsymbol{w}^\mathrm{H}R\boldsymbol{w},
\end{aligned}
$$

where $\boldsymbol{u}(t)$ is the input and $y_\mathrm{d}(t)$ the desired output.

Design Methods: Adaptive (Cont.)

- Assuming stationarity, solving this minimization problem we obtain the optimum weights as:

$$\boldsymbol{w}_{\mathrm{opt}} = R^{-1}\mathrm{E}\{\boldsymbol{u}(t)y_{\mathrm{d}}(t)\}.$$

- We can use sample covariance for R and sample average for $\mathrm{E}\{\boldsymbol{u}(t)y_{\mathrm{d}}(t)\}$, using the arriving data at the sensors, and possibly adapting the new data to get better estimates.
- We can see that this method is similar to multiple sidelobe canceller if we treat the main channel component as $y_{\mathrm{d}}(t)$ and the auxiliary components as $\boldsymbol{u}(t)$.
- Also this method is equivalent to the least-squares error problem if we treat the desired signal as $y_{\mathrm{d}}(t)$ and signals arriving at the sensors as $\boldsymbol{u}(t)$.

Practical Applications: Numerical Examples

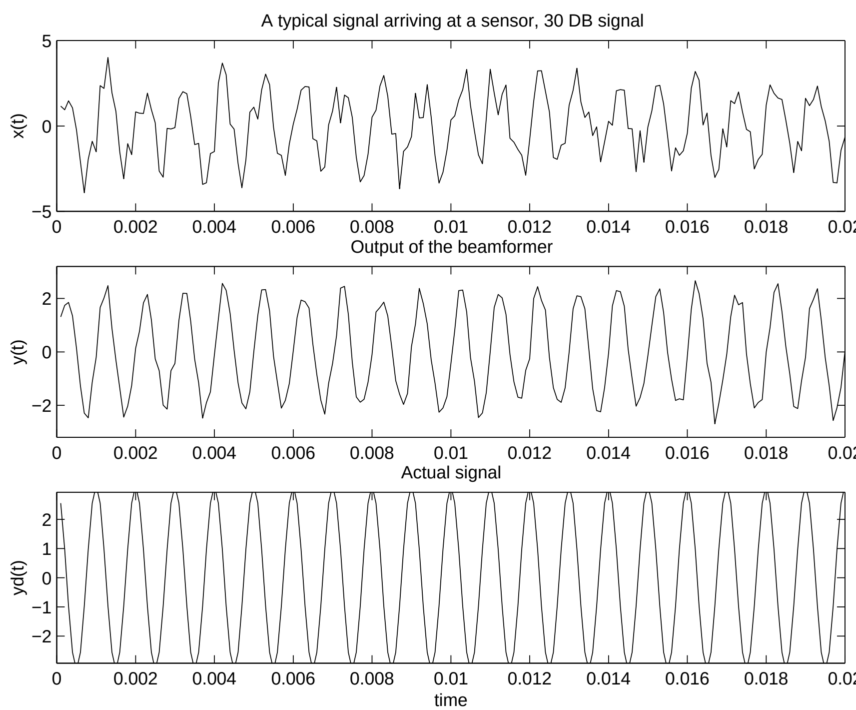
- Now, we demonstrate how beamforming can be used in practical applications by giving a number of numerical examples for:
  - Signal enhancement.
  - Direction finding.
  - Interference suppression.

Signal Enhancement

- We want to eliminate noise to improve performance.
- We can use any of the methods discussed to enhance the desired signal. Here, we give an example of the Capon method for signal enhancement.
- We can enhance the desired signal using Capon beamformer, assuming that we know the direction of the signal, and the covariance matrix of the data arriving at the sensors.
- However, we can use time averages to obtain the covariance matrix assuming ergodicity, when the true covariance is not available.

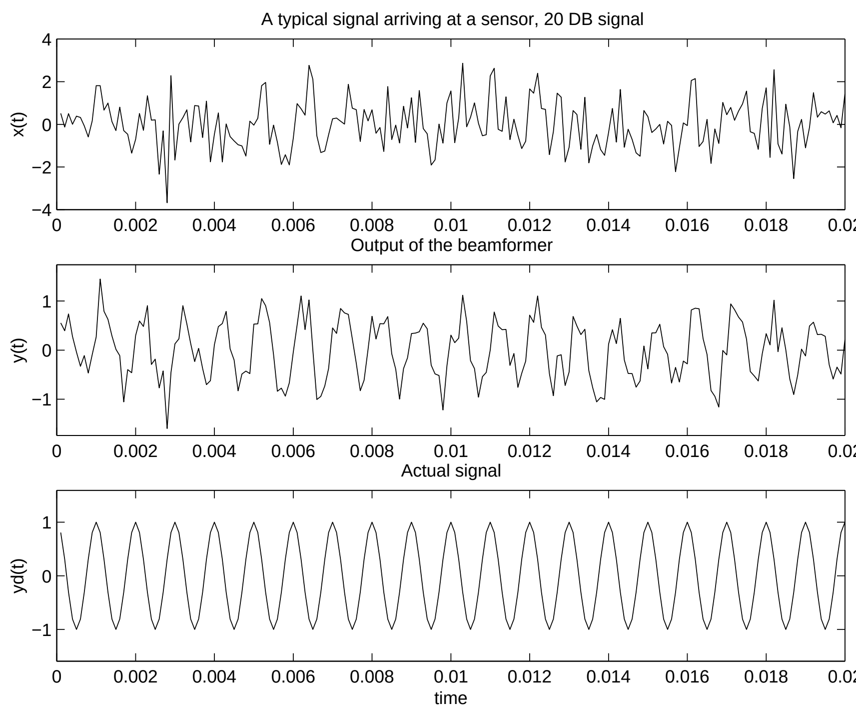
Signal Enhancement: Example-1

- 30 dB signal, additive white Gaussian noise, at each sensor.
- Uniform linear array formed by 6 sensors separated by $\lambda/2$, where λ is the wavelength.
- Azimuth of arrival: $\theta = \pi/6$.



Signal Enhancement: Experiment-2

- 20 dB signal, additive white Gaussian noise, at each sensor.
- Uniform linear array formed by 6 sensors separated by $\lambda/2$.
- Azimuth of arrival: $\theta = \pi/6$.



Direction Finding

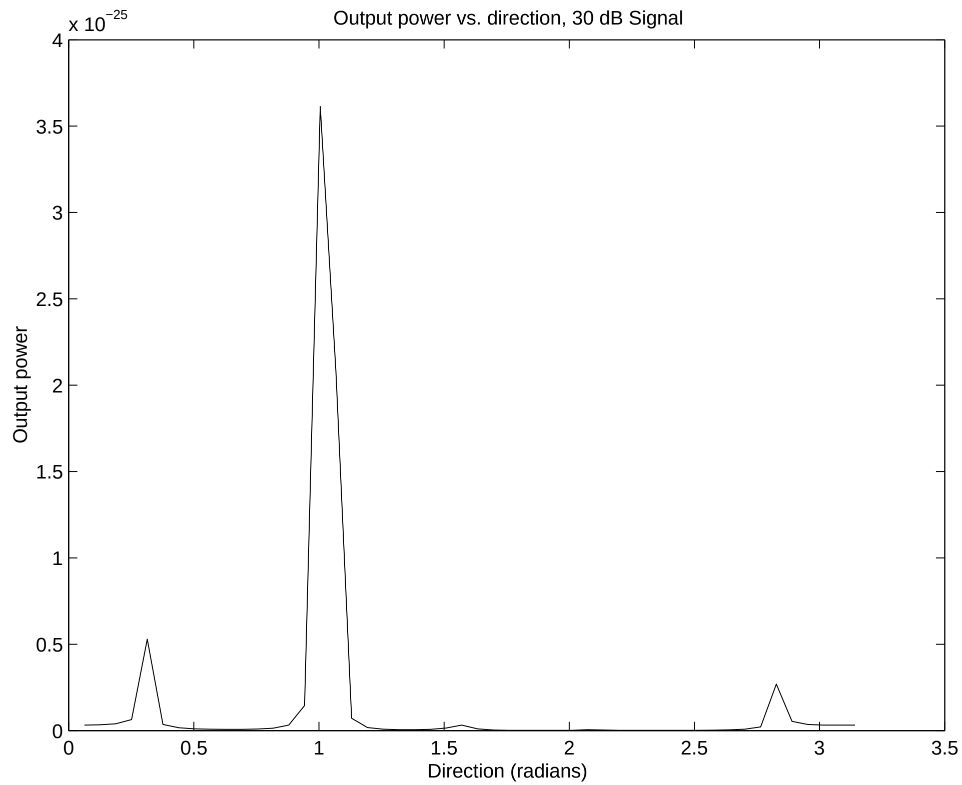
- Direction of arrival (DOA) estimation is a major subject in array signal processing, since it has plenty of applications in radar, communications, and many other areas.
- To enhance a signal using Capon beamformer, we need to know its direction.
- However, we can use Capon beamformer to estimate the direction of the signal, assuming that the covariance matrix of the array measurements are known.
- We scan all possible directions. Estimated direction of the signal is the one that maximizes the output power:

$$\frac{1}{\boldsymbol{d}^{\mathrm{H}}(\theta,\omega)R^{-1}\boldsymbol{d}(\theta,\omega)}.$$

Direction Finding: Example

- 30 dB signal, additive white noise, at each sensor.
- Uniform linear array formed by 6 sensors separated by $\lambda/2$.
- Azimuth of arrival: $\theta = \pi/3$.

Direction Finding: Example (Cont.)



- DOA is successfully estimated as $\pi/3$.

Interference Suppression

- The LCMV beamformer can be used, for interference suppression as follows.
- Form C using the steering vectors of the signal direction θ_0 and interference direction ϕ. Use $\boldsymbol{f}$ with unity for the signal, and zero for the interference.
- We can summarize this mathematically as follows:

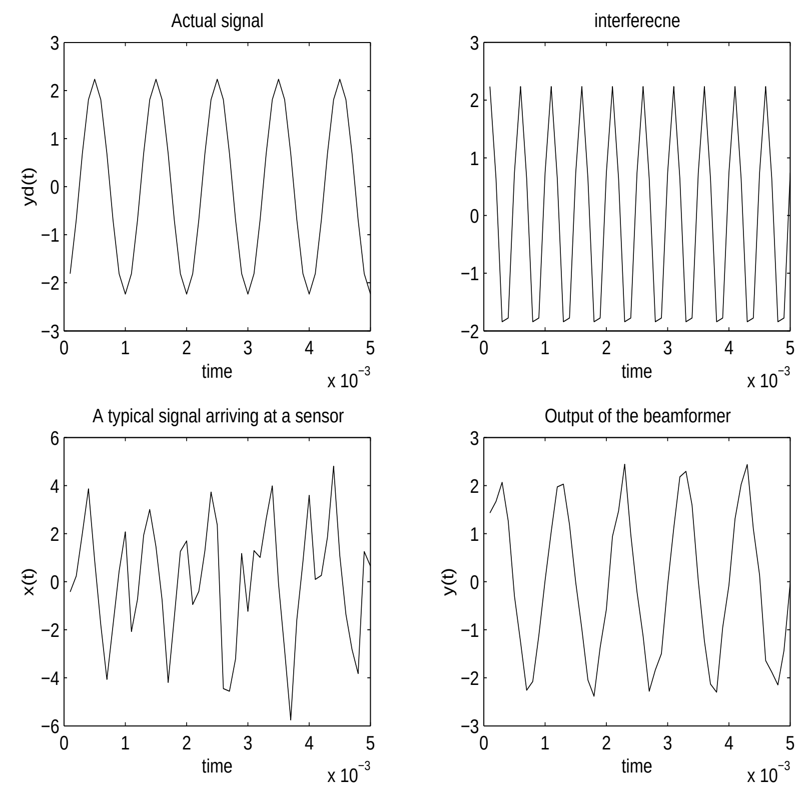
$$C = [\boldsymbol{d}(\theta_0, \omega),\, \boldsymbol{d}(\phi, \omega)],$$

$$\boldsymbol{f} = [1, 0]^{\mathrm{T}}.$$

Interference Suppression: Example-1

- 30 dB signal at azimuth $\theta_0 = \pi/3$, additive white Gaussian noise, at each sensor.
- 30 dB interference at azimuth $\phi = \pi/6$ radians.
- Uniform linear array formed by 6 sensors separated by $\lambda/2$.

Interference Suppression: Example-1 (Cont.)

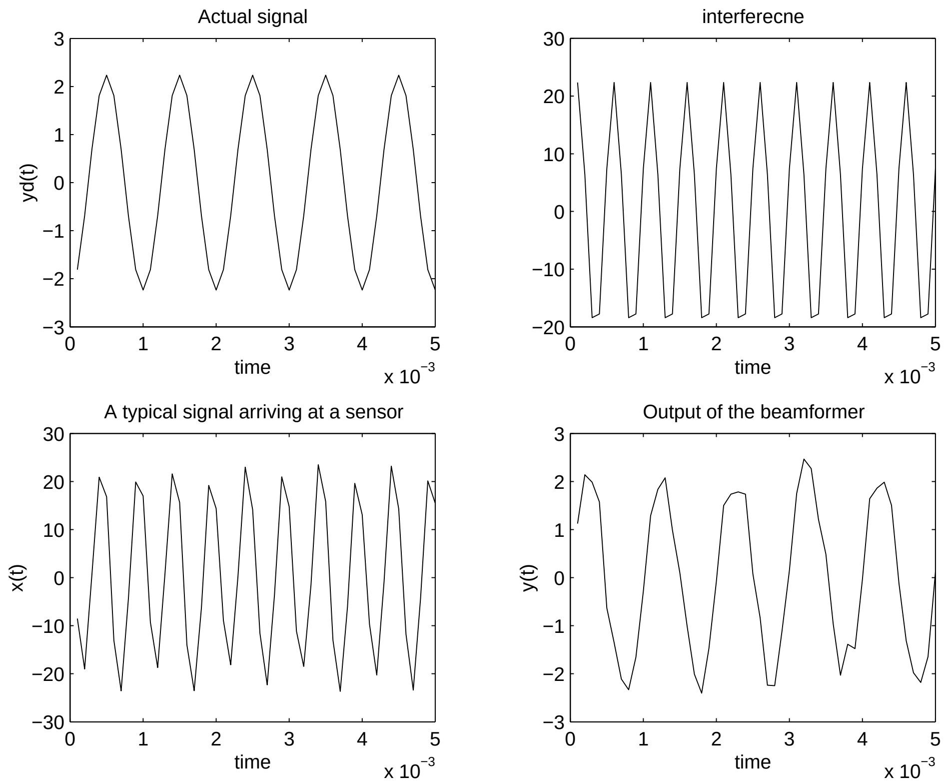


- Desired signal is approximately obtained at the output of the beamformer, and interference is eliminated.

Interference Suppression: Example-2

- 30 dB signal at azimuth $\theta_0 = \pi/3$, additive white Gaussian noise, at each sensor.
- 50 dB interference at azimuth $\phi = \pi/6$ radians.
- Uniform linear array formed by 6 sensors separated by $\lambda/2$.

Interference Suppression: Example-2 (Cont.)



- Desired signal is approximately obtained at the output of the beamformer, and interference is eliminated though it is much stronger than the desired signal.

Performance Analysis

When evaluating a beamformer's performance, we can use the cost functions we have mentioned as the criteria.
Some performance criteria can be listed as follows:

- Mean-squared error between the desired output and actual output.
- Output signal-to-noise ratio.

Computation time is also a factor.

Conclusion

- An overview of beamforming.
- Usefulness of beamforming.
- Various beamforming methods.
- Adaptive vs. data independent beamformers.
- Sample of practical applications.

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