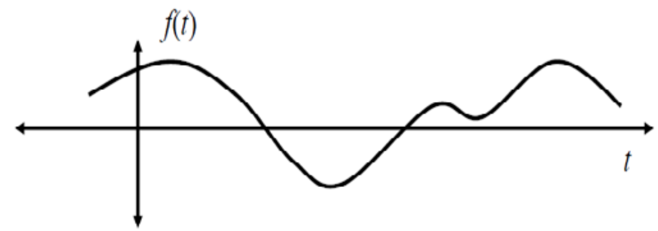
A very

Brief Introduction to Signals & Systems

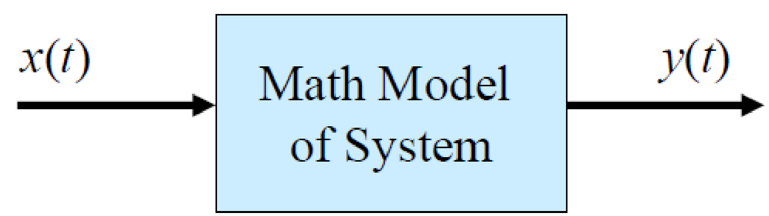
Outline

- Signals & Systems
- Continuous and discrete time signals
- Properties of Systems
- Input- Output relation : **Convolution**
- **Frequency domain representation of signals & systems**
- Analog to digital Conversion
- Sampling – **Nyquist Sampling Theorem**
- Basic Filter Theory
- Types of filters
- **Designing practical filters in Labview and Matlab**

- What is a signal?
  - A signal is a function defined on the continuum of time values

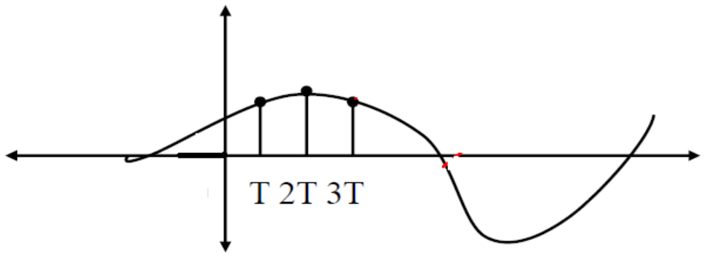


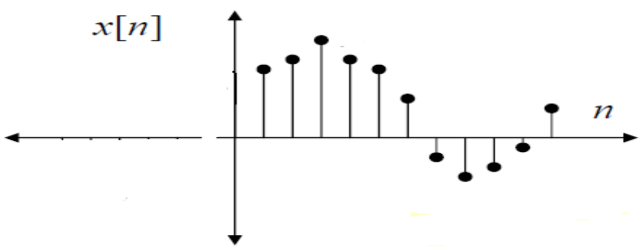
- What is a system ?
  - a system is a black box that “takes in” one or more input signals and “produces” one or more output signals



Continuous time Vs Discrete time Signals

- Most of the modern day systems are discrete time systems. E.g., A computer.
- A computer can't directly process a continuous time signal but instead it needs a stream of numbers, which is a **discrete time signal.**



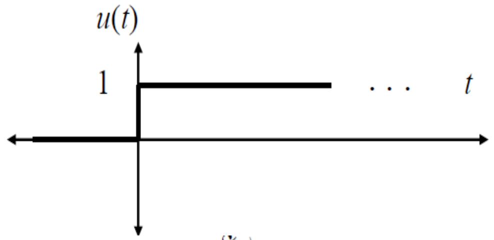


- Discrete time signals are obtain by sampling the continuous time signals
- How fast should we sample the signal?

Examples

- Signals
  - Unit Step function

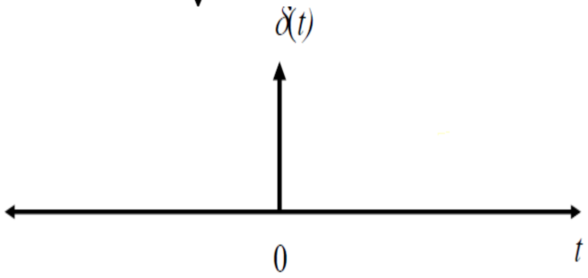
$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



  - Continuous time impulse function

$$\delta(t) = 0, \quad \text{for any } t \neq 0$$

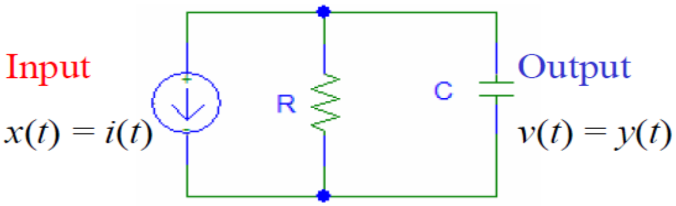
$$\int_{-\varepsilon}^{\varepsilon} \delta(t)dt = 1, \quad \text{for any } \varepsilon > 0$$



  - Discrete time

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

- Systems
  - A simple circuit



$$\sum_{n=-\infty}^{\infty} \delta[n] = 1$$

Compare

$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

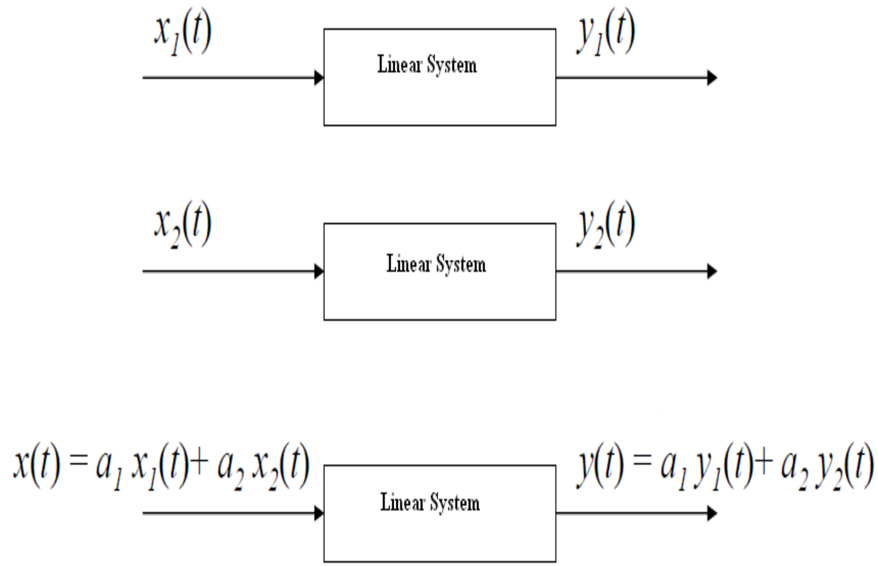
$$\sum_{n=-\infty}^{\infty} x[n]\delta[n-n_0] = x[n_0]$$

Compare

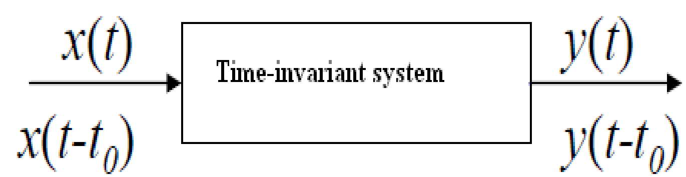
$$\int_{-\infty}^{\infty} x(t)\delta(t-t_0)dt = x(t_0)$$

Basic System Properties

- Linearity
  – System is linear if the principle of superposition holds



- Time- Invariance
  – The system does not change with time



Convolution

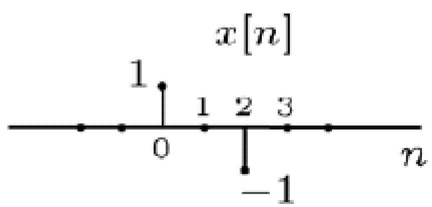
- Linear & Time invariant (LTI) sytems are characterized by their impulse response
- Impulse response is the output of the system when the input to the system is an impulse function
- For Continuous time signals $y(t) = \int_{-\infty}^{\infty} x(s)\, h(t-s)\, ds.$
- For Discrete time signals $y[n] = \sum_{i=-\infty}^{\infty} x[i]h[n-i]$

Visualizing the calculation of $y[n] = x[n] * h[n]$

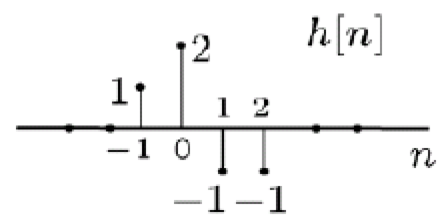
Choose value of n and consider it fixed

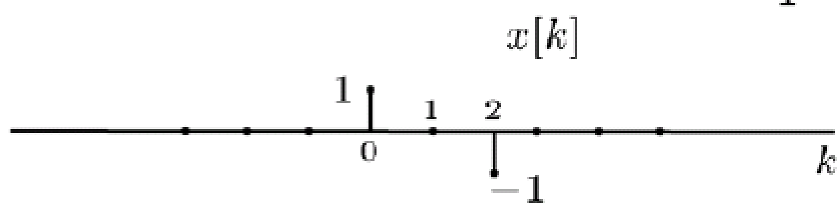
$$y[n] = \sum_{k=\infty}^{\infty} x[k]h[n-k]$$

View as functions of k with n fixed

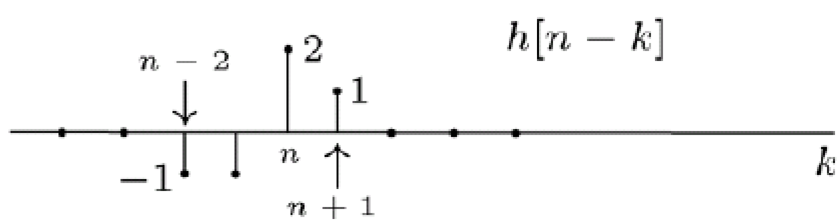


$\Downarrow$



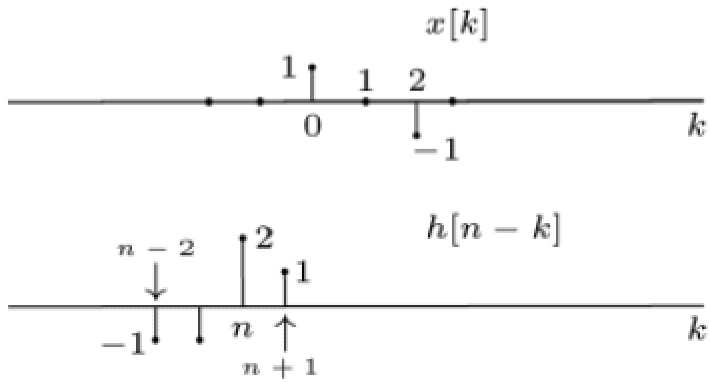


$y[0]$ = Σ prod of overlap for $n = 0$



$y[1]$ = Σ prod of overlap for $n = 1$

Calculating Successive Values: Shift, Multiply, Sum



$$
\begin{aligned}
y[n] &= 0 \quad \text{for } n < -1 \\
y[-1] &= 1 \times 1 = 1 \\
y[0] &= 0 \times 1 + 1 \times 2 = 2 \\
y[1] &= (-1) \times 1 + 0 \times 2 + 1 \times (-1) = -2 \\
y[2] &= (-1) \times 2 + 0 \times (-1) + 1 \times (-1) = -3 \\
y[3] &= (-1) \times (-1) + 0 \times (-1) = 1 \\
y[4] &= (-1) \times (-1) = 1 \\
y[n] &= 0 \quad \text{for } n > 4
\end{aligned}
$$

Frequency domain representation of signals

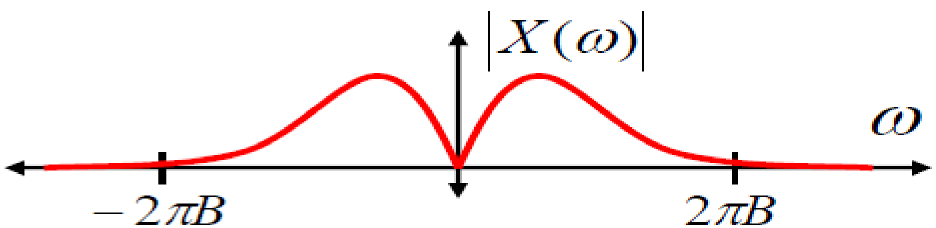
- In most of the real time applications it will be required to process the signals based on their frequencies
- In such cases, it is easier to represent the signals as a function of the frequency, rather than time
- A Fourier transform provides the mathematical representation

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \qquad x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$

Fourier Transform **Inverse Fourier Transform**

Bandwidth of the signal

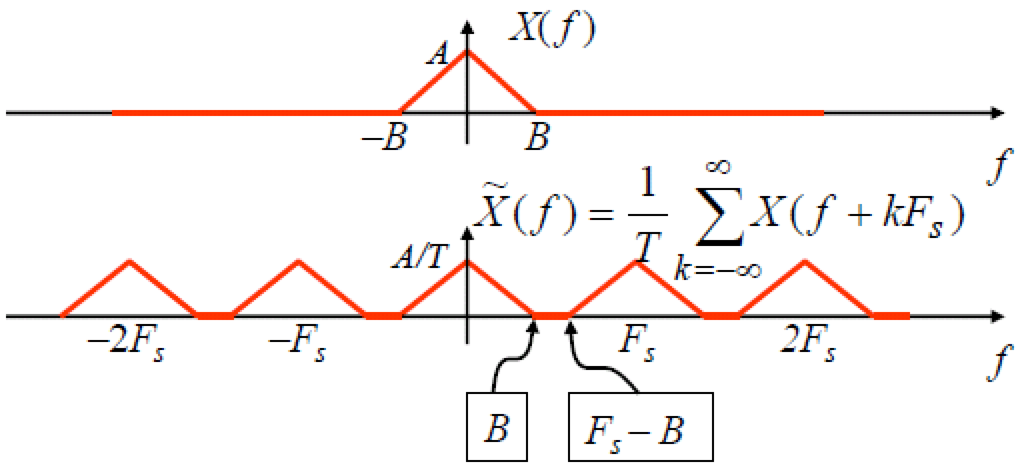
- For a lot of signals –like audio –they fill up the lower frequencies but then decay as ω gets large



- We say the signal's BW = *B in Hz if there is "negligible"content for* $|\omega| > 2\pi B$

Nyquist Sampling Theorem

- For band limited analog signals, sampling frequency should be at least twice the bandwidth to avoid aliasing.



Filters – Introduction

- Filtering is the most common signal processing procedure.
  - Used as echo cancellers, equalizers, front end processing in RF receivers
  - Used for modifying input signals by passing certain frequencies and attenuating others.
- Characterized by the impulse response like other Linear &Time Invariant systems.
- Both Analog and Digital Filters can be used.
- Analog
  - Uses analog electronic circuits made up of components like resistors and capacitors
  - Used widely for video enhancement in TV's
- Digital
  - Uses a general purpose processor for implementation
  - Used widely in many applications these days because of the flexibility they offer in design and implementation

Types of Filters

- High pass filter

–Attenuates the low frequency components of a signal and allows high frequency components

- Low pass filter

–Attenuates the high frequency component and allows low frequency component

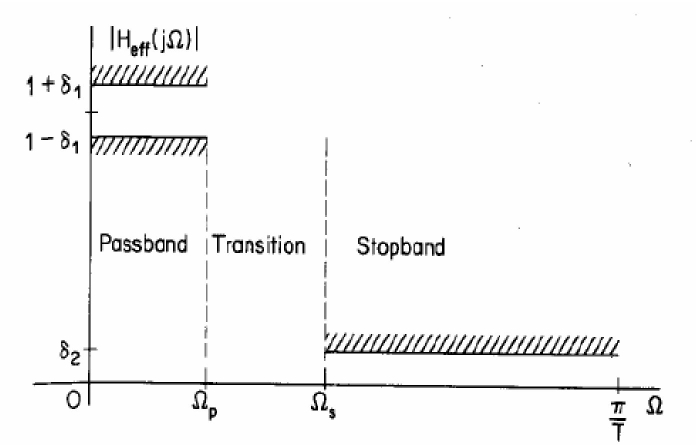
- Band pass filter

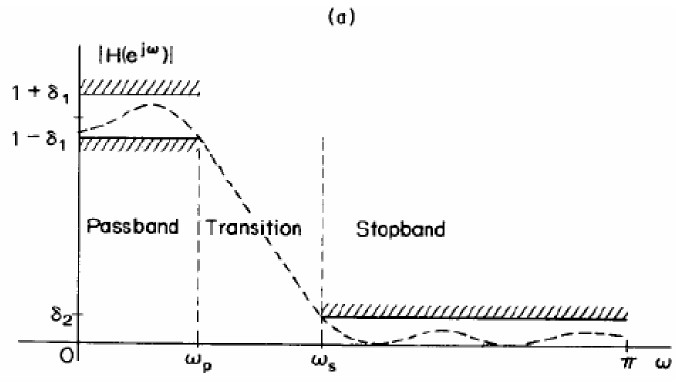
–Allows a particular frequency band and attenuates the rest of the frequency components.

- Band stop filter

–Attenuates the frequency components in a particular band and allows the other frequencies.

Filter Design





(From Discrete-Time Signal Processing, Oppenheim and Schafer)

- Ω_p is the Passband frequency
- Ω_s is the Stopband frequency
- δ_1 is the Passband Ripple
- δ_2 is the Stopband Attenuation

FIR Vs IIR Filters

- Several factors influence the choice of FIR / IIR filters like linear phase, stability, hardware required to build etc.

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] - \sum_{k=1}^{N} a_k y[n-k]$$ IIR filter equation

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$ FIR filter equation

- Several techniques for designing filters (both FIR & IIR)
- We don't learn the design techniques in this class. We use Matlabas a design tool
- IIR filter types
  - Butterworth : Maximally flat
  - Chebycheff : Equi-ripple in pass band (type 1) & stop band (type 2)
  - Elliptical : Sharp transition region

Some Matlab Commands

- plot
  - –PLOT(Y) plots the columns of Y versus their index. PLOT(X,Y) plots vector Y versus vector X.
- fir1
  - –B = FIR1(N,Wn) designs an Nth order lowpass FIR digital filter and returns the filter coefficients in length N+1 vector B. B = FIR1(N,Wn,'high') designs an Nth order highpass filter.
- butter
  - –[B,A] = BUTTER(N,Wn) designs an Nth order lowpass digital Butterworth filter and returns the filter coefficients in length N+1 vectors B (numerator) and A (denominator).
- cheby1
  - –[B,A] = CHEBY1(N,R,Wp) designs an Nth order lowpass digital Chebyshev filter with R decibels of peak-to-peak ripple in the passband. CHEBY1 returns the filter coefficients in length N+1 vectors B (numerator) and A (denominator). Use R=0.5 as a starting point, if you are unsure about choosing R
- See also cheby2 & ellip
- filter
  - –Y = FILTER(B,A,X) filters the data in vector X with the filter described by vectors A and B to create the filtered data Y where A and B are as in direct form II structure

Task

- Create a signal which is sum of two sinusoids with frequencies 5Hz and 15 Hz.
- Plot x(t) and X(f). Use time and frequency as x-axis while plotting, not the sample number.
- Create an FIR low pass filter with cutoff frequency 6Hz and plot the response of the filter. Change the order of filter and see how the frequency response changes.
- Pass the signal x(t) through the filter and plot the output.
- Create an FIR high pass filter with cutoff frequency 12 Hz and plot the response of the filter. Repeat for different orders.
- Pass the signal x(t) through the filter and plot the output.
- Repeat the experiment with an IIR filters of same order and see the performance difference