Lucas–Kanade method Description

The Lucas-Kanade method is a widely used differential method for optical flow estimation developed by Bruce D. Lucas and Takeo Kanade. It has 3 assumptions: ①brightness constancy: the value of pixel is same for a point I(x,y,t) and I(x',y',t+1) , ②small motion: the motion vector for a point is small, ③small region move together: the pixel's neighbors have the same (u,v).

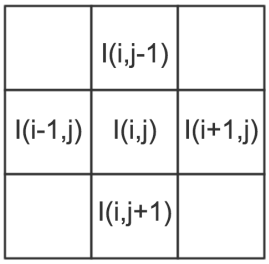
Algorithm steps:

Input: I1, I2, assuming image size = 128×128

(I1 is the image at time t, I2 is the image at time t+1)

Output: motion vector V = (Vx, Vy)

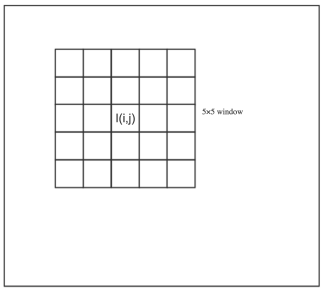
1) Calculate the partial derivatives of the image I1 (Ix, Iy, It) with respect to position x, y and time t. At the edge of the image I1, Ix, Iy and It can be set as 0, because I1(i-1, j), I1(i+1, j), I1(i, j-1), I2(i, j+1) might not exis.



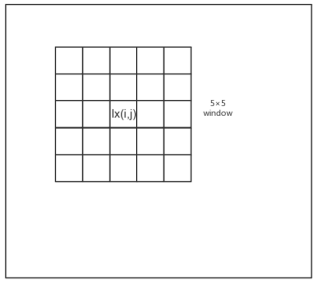
Ix(i, j) = I1(i+1, j) – I1(i-1, j)
Iy(i, j) = I1(i, j+1) – I2(i, j-1)
It(i, j) = I2(i, j) – I1(i, j)

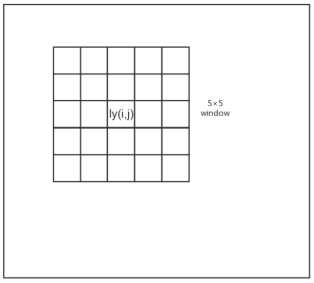
2) For every pixel in I1, take location (i, j) as an example:

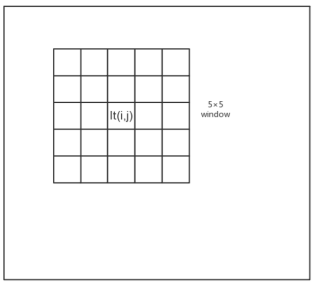
Within a window centered at I1(i, j), assuming the window size is 5×5



We have the corresponding 5×5 windows in Ix, Iy, It matrixes.







For pixel q1=I1(i-2,j-2), q2= I1(i-2,j-1), …, q25= I1(i+2,j+2) in the 5×5 window, we have the equation:

$$I_x(q_1)V_x + I_y(q_1)V_y = -I_t(q_1)$$
$$I_x(q_2)V_x + I_y(q_2)V_y = -I_t(q_2)$$
$$\vdots$$
$$I_x(q_n)V_x + I_y(q_n)V_y = -I_t(q_n)$$

These equations can be written in matrix form Av=b, where

$$A = \begin{bmatrix} I_x(q_1) & I_y(q_1) \\ I_x(q_2) & I_y(q_2) \\ \vdots & \vdots \\ I_x(q_n) & I_y(q_n) \end{bmatrix}, \quad v = \begin{bmatrix} V_x \\ V_y \end{bmatrix}, \quad \text{and} \quad b = \begin{bmatrix} -I_t(q_1) \\ -I_t(q_2) \\ \vdots \\ -I_t(q_n) \end{bmatrix}$$

3) The motion vector V = (Vx, Vy) = $(A^TA)^{-1}A^Tb$

$$\begin{bmatrix} V_x \\ V_y \end{bmatrix} = \begin{bmatrix} \sum_i I_x(q_i)^2 & \sum_i I_x(q_i)I_y(q_i) \\ \sum_i I_y(q_i)I_x(q_i) & \sum_i I_y(q_i)^2 \end{bmatrix}^{-1} \begin{bmatrix} -\sum_i I_x(q_i)I_t(q_i) \\ -\sum_i I_y(q_i)I_t(q_i) \end{bmatrix}$$

The inverse of a matrix

