THREE PHASE CIRCUITS

1. *Three Phase Voltages and Systems*

For reasons which are beyond the scope of this course, it is neither economical to ship electrical power down two wire lines nor sensible to drive large ac motors using the single phase power of a two wire line. Instead one employs what is known as a three-phase system in which there are three sinusoidally varying voltage sources of identical frequency which bear fixed phase relationships to one another. The arcana of this topic are extensive, and the student who has not taken EE 327 is referred to the texts by Dover (1947) and Edminister (1965) for elementary details. This section will be confined to the three-phase wye-connected source shown in Fig. 1.2A; the *neutral point* N is here taken to be the potential reference and is presumed to be tied to ground.

First, the voltage specified when speaking of a three-phase

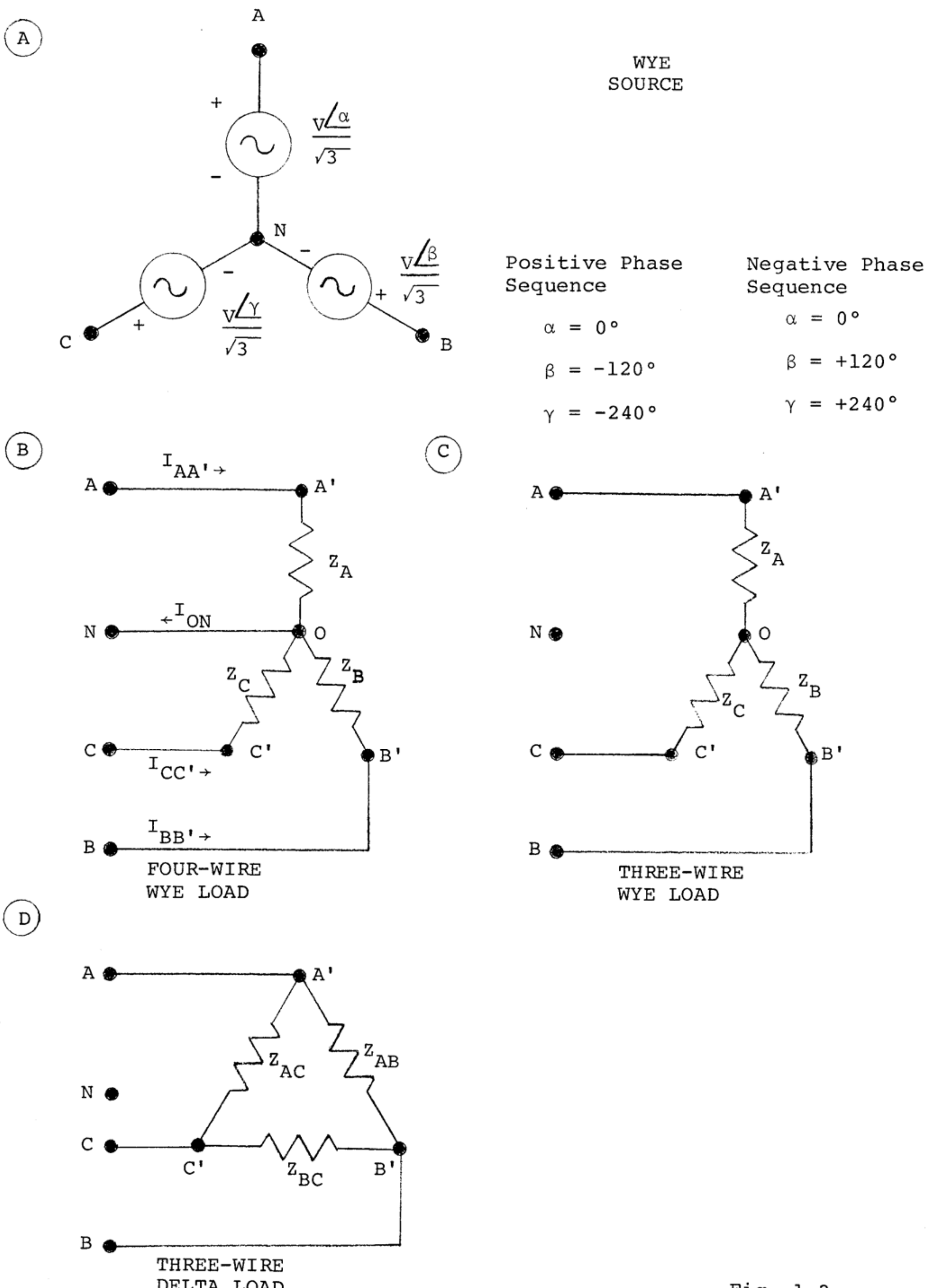
A
WYE SOURCE
N
C
B
Positive Phase Sequence
$\alpha = 0°$
$\beta = -120°$
$\gamma = -240°$
Negative Phase Sequence
$\alpha = 0°$
$\beta = +120°$
$\gamma = +240°$
$\frac{V\angle\alpha}{\sqrt{3}}$
$\frac{V\angle\beta}{\sqrt{3}}$
$\frac{V\angle\gamma}{\sqrt{3}}$
B
$I_{AA'}$
A'
Z_A
I_{ON}
O
Z_C
Z_B
C'
B'
$I_{CC'}$
$I_{BB'}$
FOUR-WIRE WYE LOAD
C
THREE-WIRE WYE LOAD
D
Z_{AC}
Z_{AB}
Z_{BC}
THREE-WIRE DELTA LOAD

Fig. 1.2

system is never that from line-to-neutral but is always that from line-to-line. Thus, for the positive sequence,

$$V_{BA} = \frac{V\angle\beta}{\sqrt{3}} - \frac{V\angle\alpha}{\sqrt{3}} = V\angle 210° \quad (1.21a)$$

$$V_{CB} = V\angle 90° \quad (1.21b)$$

$$V_{AC} = V\angle -30° \; ; \quad (1.21c)$$

while, for the negative sequence,

$$V_{BA} = V\angle 150° \quad (1.22a)$$

$$V_{CB} = V\angle 270° \quad (1.22b)$$

$$V_{AC} = V\angle 30° \quad (1.22c)$$

Clearly,

$$(\text{rms line-line voltage}) = \sqrt{3}\ (\text{rms line-neutral voltage}). \quad (1.23)$$

Second, as illustrated in Figs. 1.2B, 1.2C and 1.2D, there is a variety of ways in which loads can be hitched to this source, even neglecting the complications which arise when the wires which connect source to load are of nonzero impedance. In Fig. 1.2B, it is clear that the load (or *phase*) currents equal the line currents, that

$$I_{ON} = \frac{V}{\sqrt{3}}\left[Y_A + Y_B e^{j\beta} + Y_C e^{j\gamma}\right] , \quad (1.24)$$

and that* $I_{ON} = 0$ when $Y_A = Y_B = Y_C$ (*balanced load*). In Fig. 1.2C, the phase currents still equal the line currents, but the phase voltages of the load do not necessarily equal those of the source since§

$$V_{ON} = \frac{V_{AN}Y_A + V_{BN}Y_B + V_{CN}Y_C}{Y_A + Y_B + Y_C} \qquad (1.25)$$

and $V_{ON} = 0$ only if the load is balanced. In Fig. 1.2D, the line-line phase voltages clearly equal the line-line source voltages; in general, one can say nothing about the phase currents except that, if the load be balanced, they will be given by†

$$\text{(rms line current)} = \sqrt{3}\ \text{(rms phase current)}. \qquad (1.26)$$

Third, for a balanced load, the complex power is

$$S_{3\phi} = \sqrt{3}\ V_{\text{line-line}}\ I^*_{\text{line}} \qquad (1.27a)$$

† Let the reader test his grasp of polyphase by demonstrating this.

§ This is yet another celebrated polyphase theorem which belongs to a class of results labelled *displacement neutral*. It is also familiar to neurophysiologists.

* This is a special case of a celebrated polyphase theorem

$$0 = \sum_{n=1}^{N} e^{j(n-1)\frac{2\pi}{N}} ,$$

which follows at once from simple manipulations with geometric series.

with

$$P_{3\phi} = \sqrt{3}\, V_{rms} I_{rms} \cos\theta \ , \tag{1.27b}$$

where V_{rms} always denotes the line-line voltage and I_{rms} always denotes the line current.

2. *The Determination of Phase Sequence*

Given four terminals at a wall outlet, it is easy enough to determine which is supposed to be neutral since its opening will be shape coded. However, ordering the other three for the desired phase sequence is only slightly more difficult, and a variety of techniques are known (Dover, 1947; Kinnard, 1962).

An especially simple sequencing circuit is shown in Fig. 1.3

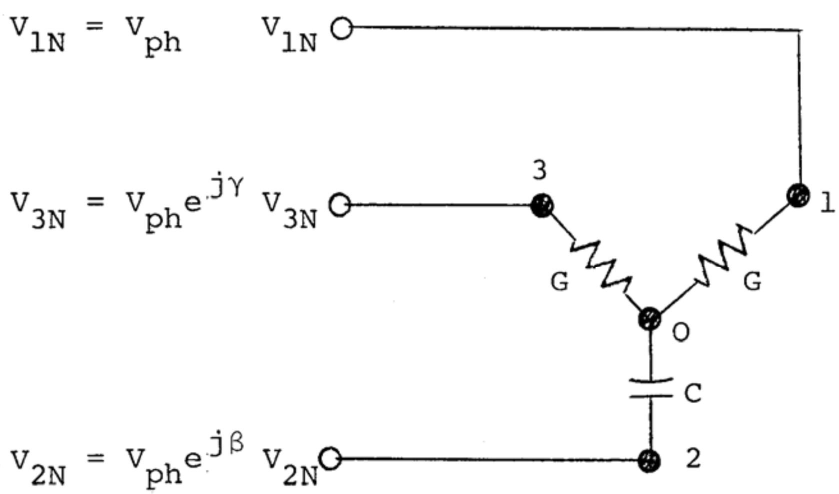

Fig. 1.3

By Eq. (1.25) it follows that

$$V_{1O} = V_{1N} - V_{ON} = V_{ph} \frac{G[1-e^{j\gamma}] + j\omega C[1-e^{j\beta}]}{2G+j\omega C} \tag{1.28a}$$

$$V_{30} = V_{ph} \frac{G[e^{j\gamma}-1]+j\omega C[e^{j\gamma}-e^{j\beta}]}{2G+j\omega C} \tag{1.28b}$$

If now we adjust the design parameters to give $G = \omega C$,

$$\left|\frac{V_{30}}{V_{10}}\right| = \frac{[\cos\gamma-\sin\gamma+\sin\beta-1]+j[\cos\gamma+\sin\gamma-\cos\beta]}{[-\cos\gamma+\sin\beta+1]+j[-\sin\gamma-\cos\beta+1]} \tag{1.29}$$

For the positive phase sequence, $\left|V_{30}/V_{10}\right| = 3.732 \ldots$.

For the negative phase sequence, $\left|V_{30}/V_{10}\right| = 0.267 \ldots$.

Thus, if the two conductances be similar incandescent lamps, we have the rule:

Lamp No. 1 brighter implies negative phase sequence.

Lamp No. 3 brighter implies positive phase sequence.

3 *Blondel's Theorem and Its Consequences*

THEOREM (A. Blondel) If energy is supplied to a load through N wires, the total power delivered to that load is the algebraic sum of the readings of N wattmeters so arranged that (i) each of the N wires is observed by one wattmeter current sensor and (ii) each wattmeter voltage sensor is connected between the current sensed wire and a common point on the load.

To demonstrate this, let the N wires be denoted by 1, 2, ... , n, ... , N , the current in a wire by $i_n(t)$, the voltage of a wire with respect to some arbitrary reference point $\mathscr{R}$ by $v_n(t)$, and the voltage of the wattmeters' common point by $v_0(t)$. By Kirchhoff's current law $v_0 \sum_{n=1}^{n} i_n = 0$. Then if point $\mathscr{R}$ is taken to be on a virtual return path for each of the N currents it follows from Eq. (1.1) and this relation that the instantaneous load power is

$$p_{load} = \sum_{n=1}^{N} v_n(t) i_n(t) = \sum_{n=1}^{N} i_n(t)[v_n(t)-v_0(t)] \ .$$

Thus

$$P_{load} = \lim_{T\to\infty} \frac{1}{T} \sum_{n=1}^{N} \int_0^T i_n(t)[v_n(t)-v_0(t)] \ . \qquad (1.34)$$

But the individual integrals comprising P_{load} are just the readings of the N wattmeters and the theorem is thereby proven. Surprisingly, since this result seems far from "deep", the theorem was not proven until 1893.

COROLLARY. If the common point for attaching the N wattmeter voltage sensors is taken on one of the N wires, then only $(N-1)$ wattmeters are required and the total average power is their algebraic sum.

This corollary's proof, which follows at once from Eq. (1.34), is left as an exercise for the reader. Its application to three-phase three-wire polyphase systems (*cf.* Laws, 1938) will be illustrated here; its application to an ideal three-phase four-wire polyphase system is trivial. Fig. 1.2 (parts C and D) illustrates the two possible cases. Suppose (a) that $V_{BA} = V$, $V_{CB} = Ve^{-j\frac{2\pi}{3}}$, and $V_{AC} = Ve^{-j\frac{4\pi}{3}}$ and (b) that the current sensors are placed in lines AA' and BB'. Then, clearly,

$$P = \mathrm{RE}\ \{V_{AC} I^*_{AA'}\} + \mathrm{RE}\{-V_{CB} I^*_{BB'}\}\ . \tag{1.35}$$

But simple network analysis suffices to show* that, for the wye load of Fig. 1.2C and $Z_A = Z_B = Z_C = Z = R + jX$,

$$P = \frac{V^2}{2ZZ^*}[R + \frac{\sqrt{3}}{3}X] + \frac{V^2}{2ZZ^*}[R - \frac{\sqrt{3}}{3}X]. \tag{1.36}$$

Similarly, for Fig. 1.2D with $Z_{AB} = Z_{BC} = Z_{AC} = Z$,

$$P = \frac{3V^2}{2ZZ^*}\ [R + \frac{\sqrt{3}}{3}X] + \frac{3V^2}{2ZZ^*}[R - \frac{\sqrt{3}}{3}X]\ . \tag{1.37}$$

Hence, in a balanced three-phase three-wire system, inequality of wattmeter reading is a sign of non-unity power factor. Note also that either of the meters can be driven backwards if current and voltage are sufficiently far out of phase!

9. *References* 4

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B. *EXPERIMENT - THREE PHASE CIRCUITS*

1. Equipment List

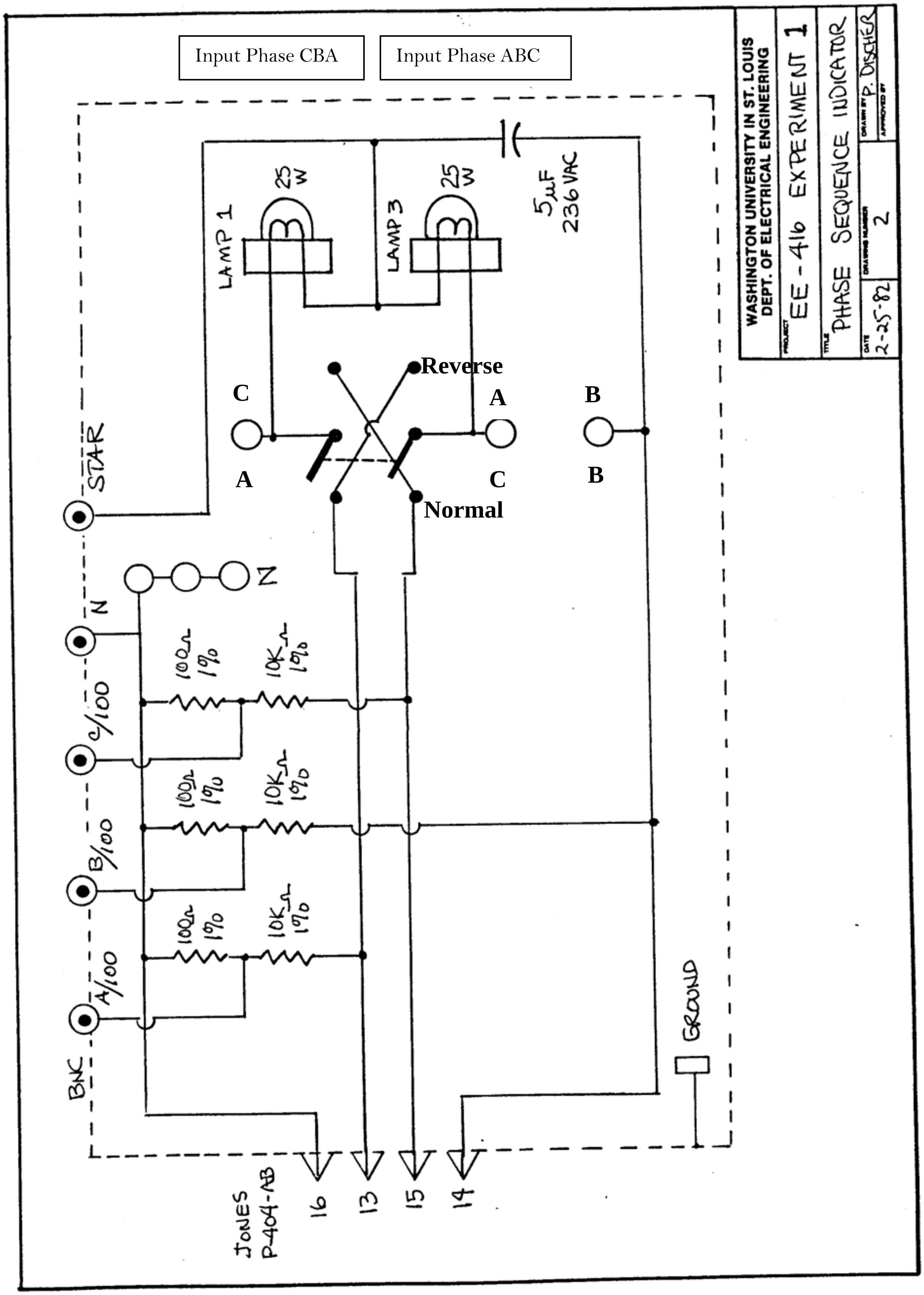
a. The standard instrument rack found at each station.
b. Various current shunts and probes.
c. One single-phase wattmeter.
d. One three-phase transformer.
e. One Phase Sequence Indicator (PSI).
f. One rheostat module with two 50-Ω rheostats rated at 4.5 Amp each.
g. Various power cords.

2. *Three Phase Power Supply*

a. A schematic of the Phase Sequence Indicator (PSI) is provided on the next page. Plug this device into the Three Phase (3ϕ) wall power source using the 3-Phase Cord Set and determine the phase sequence (ABC or CBA) using the PSI. Be sure the switch is in the **NORMAL** position. Also, observe the Phase A and Phase B waveforms on the oscilloscope using two 10X Probes or the A/100 and B/100 test points with coaxial cables. Determine their phase relationship and the phase angle between them using the scope. Copy this display. Now repeat the above step for Phase A and Phase C and record the display. Next reverse the phase sequence by flipping the switch to **REVERSE** and observe the lamp pattern. Be sure and return the switch to the **NORMAL** position for the rest of the experiment. Finally, observe the voltage waveform for the Star Point (STAR) on the Phase Sequence Indicator on the scope and copy this display.

b. Being fully aware of the potential for an electric shock, and therefore **careful to shut off the 3-phase breaker each time the DMM connections are moved**, accurately measure the rms values for the three line-to-line voltages and the three line-to-neutral voltages at your station using three DMMs.

c. The "green" neutral wire on the 3-phase supply provides a return to ground. It is well and thoroughly grounded and will remain at Mother Earth potential as long as there is no IR voltage drop, i.e, as long as the neutral return current is zero. This is the case for perfectly balanced three phase voltage sources and perfectly


Input Phase CBA
Input Phase ABC
WASHINGTON UNIVERSITY IN ST. LOUIS
DEPT. OF ELECTRICAL ENGINEERING
EE-416 EXPERIMENT 1
PHASE SEQUENCE INDICATOR
P. DISCHER
2
2-25-82
LAMP 1
LAMP 3
25 W
25 W
5 μF
236 VAC
Reverse
Normal
C
A
B
A
C
B
STAR
N
C/100
B/100
A/100
BNC
100Ω 1%
10KΩ 1%
GROUND
JONES
P-404-AB
16
13
15
14

balanced loads since no neutral current will exist to produce an IR voltage drop. Using a coaxial cable, determine the existence of any neutral point offsets by observing on the scope the voltage waveform between the Neutral Point (N) on the Phase Sequence Indicator and the "green" ground on its chassis and copy your observation. Note that the scope is already referenced to "green" ground.

3. *Balanced Three Phase Resistive Loads*

a. Using a DMM to measure the left and right side resistances of the left rheostat, adjust it to make 2 equal resistances of approximately 25 Ω each. Record the 2 resistance values. Now adjust the right rheostat until its left side resistance is equal to the two left rheostat resistances and record its value. Now connect these three equal resistances to form a balanced Three Phase (3ϕ) Wye load by connecting the yellow receptacles on the rheostats to form a load resistance Common Point (O). Connect the left side of the left rheostat to the Phase A output of the Phase Sequence Indicator (PSI) and the right side to the Phase B output of the PSI. Connect the left side of the right rheostat to the Phase C output of the PSI. Connect the rheostat Common Point (O) to the Neutral Point (N) of the PSI to form a Neutral Return line. Show your set-up to the instructor.

b. Now instrument the set-up, using 3 DMMs and 1 Wattmetter, to measure Phase A line current (I_A), Phase A load voltage (V_{AO}), Phase A load power (P_{AO}), and Neutral Return line current (I_{ON}). Use current shunts as required. Use a 10x probe to observe Phase A rms voltage (V_{AG}) and a coaxial cable to observe the rheostat Common Point (O) rms voltage (V_{OG}). Show your set-up to the instructor before turning on 3ϕ power. Once your set-up is approved by the instructor, turn on 3ϕ power and record all measurements and copy the scope display. With power on, carefully move the 10X probe to also measure Phase B (V_{BG}) and Phase C (V_{CG}) rms voltages. Separately record these rms measurements. Turn off 3ϕ power when finished recording all measurements.

c. Now remove the "O" to "N" Neutral Return wire and change the DMM being used to measure Neutral Return current to now measure rheostat Common Point (O) rms voltage (V_{ON}) relative to the Neutral Point (N) on the PSI. Turn on 3ϕ power and record all measurements. Turn off 3ϕ power when finished.

4. *Unbalanced Three Phase Resistive Loads*

a. Now disconnect the Phase A and Phase B leads from the left rheostat. Connect a DMM to the left side of the left rheostat and adjust this rheostat until a resistance of 15 Ω is obtained. Measure and record both of the left rheostat resistances. Now reconnect the circuit as it was in Part B.3.c. above and show it to the instructor. When approved by the instructor, turn on 3ϕ power and record all DMM and wattmeter measurements. Turn off 3ϕ power when finished.

b. Obtain a second Wattmeter and reconfigure the instrumentation to measure load power using the 2 wattmeter method. Configure the DMMs to measure the three load voltages relative to the rheostat Common Point (O). Show this set-up to the instructor. When approved, turn on 3ϕ power and record the measured powers and load voltages. Be sure that both Wattmeters read upscale. Turn off 3ϕ power when finished.

c. Remove both Wattmeters from the circuit and reconnect the Neutral Return wire between the rheostat Common Point and the Neutral Point of the PSI. Reconfigure the DMMs to measure Phase A line current, Phase A load voltage, and Neutral Return line current. Turn on 3ϕ power and record all DMM measurements. Turn off 3ϕ power when finished.

5. *The Three Phase Transformer*

> *Because this portion involves exposed power points, it is electrically the most dangerous procedure of the course.* <u>You will therefore power up only under the direct and continuous supervision of one of the faculty.</u> NO EXCEPTIONS !!

a. Using DMM voltage measurements, determine how to use the three individual transformers in the *step-down* direction and calculate the three step-down ratios. Parts b and c below operate in the step-down direction.

b. Connect the high voltage primaries of the transformers to form a wye. The secondaries of the transformers are to be connected as a delta, but **before closing the delta, you must determine that phasing is correct by measuring the voltage across the open delta to see that it is approximately zero.** With power off, close the delta through a rheostat set to 50 Ω. Turn the power back on and measure the voltage across the rheostat. If it still measures approximately zero, adjust the rheostat to minimum resistance and measure the 3 secondary line-to-line delta voltages.

c. With the power off and the primaries still wye connected, open the delta and reverse the connections for one of the secondaries, **but leave the delta open**. Now turn power back on and measure and record the voltage across the open terminals in the delta. Note that it is large. With the power off, **close the delta through a rheostat set to 50 Ω.** Now measure the "ring" current in the 50 Ω rheostat by measuring the voltage across it.

C. *REPORT - THREE PHASE CIRCUITS*

1. *Three Phase Power Supply*

a. What is the phase sequence provided by the 3-phase wall supply in the laboratory. Assuming that the phase A voltage is the reference (0° phase shift), what are the measured phase angles for the phase B and C voltages? Did the Phase Sequence Indicator provide the same information that you deduced from oscilloscope observation? Also, discuss and explain the STAR point waveform you observed and recorded in part B.2.a.

b. Make a table showing the three line-to-line voltages, their average, and their deviation from the average value. Make a second table showing the three line-to-neutral (phase) voltages, their average, and their deviation from average value. What is the ratio these two averages? Is it in agreement with theory? Considering the voltage deviations from average shown in the tables, do you think the laboratory supply is adequate for 3 phase circuit experiments?

c. Comment on the Neutral Point (N) voltage you observed on scope. Does the Neutral Point appear to be well grounded to Mother Earth?

2. *Balanced Three Phase Resistive Loads*

a. Based on data from part B.3.a, calculate the average value of load resistance and make a table showing the load resistances and their deviation from the average value. Did you present a nearly balanced load to the 3ϕ power supply?

The measured values of the load resistors inherently include the resistances of the banana plug cords if the measurements were made using the DMM as an ohmmeter. Comment on any effect these added resistances may have on the experiments of parts B.3.b and B.3.c.

b. Make a table comparing expected and measured values of I_A, V_{AO}, P_{AO}, and I_{ON} for the results of part B.3.b. In calculating the expected values of I_A, P_{AO}, and I_{ON}, use the 3 resistances measured in part B.3.a and the 3 phase voltages measured in part B.2.b and assume that the phase shift between the voltages is exactly 120° in the proper sequence (CBA or ABC). Comment on the deviations observed. Why is the assumption that total power (P_T) is equal to 3 P_{AO} (P_T = 3 P_{AO}) a reasonable assumption for a balance load with a neutral return line?

c. Make a table comparing expected and measured values of I_A, V_{AO}, P_{AO}, and V_{ON} for the results of part B.3.c. In calculating the expected values of I_A, P_{AO}, and V_{ON}, use the 3 resistances measured in part B.3.a and 3 phase voltages measured in part B.2.b and assume that the phase shift between the phase voltages is exactly 120° in the proper sequence (CBA). Comment on the deviations observed. Why is the assumption that P_T = 3 P_{AO} a reasonable assumption for a balance load with no neutral return line?

d. Make a table comparing measured values of I_A, V_{AO}, and P_{AO} made in part B.3.b and B.3.c. How should these values compare considering that the neutral return line was not used in part B.3.c?

3. *Unbalanced Three Phase Resistive Loads*

a. Based on data from part B.4.a, calculate the average value of load resistance and make a table showing the load resistances and their deviation from the average value. Did you present an unbalanced load to the 3ϕ power supply?

b. Make a table comparing expected and measured values of I_A, V_{AO}, P_{AO}, and V_{ON} for the results of part B.4.b. In calculating the expected values of I_A, V_{AO}, P_{AO}, and V_{ON}, use the 3 resistances measured in part B.4.a and the 3 phase voltages measured in part B.2.b and assume that the phase shift between the phase voltages is exactly 120° in the proper sequence (CBA or ABC). Comment on the deviations observed. Now calculate the expected values of V_{BO} and V_{CO} and the corresponding expected values of P_{BO}, and P_{CO}. Obtain the total power P_T as the sum of these powers. Compare this total power to the sum of the powers measured using 2 Wattmeters. Why is the assumption that $P_T = 3\ P_{AO}$ not reasonable for an unbalanced load with no neutral return line? For this circuit, it is clear that 3 Wattmeters could have been used to measure total power. Explain why total power can be measured using only 2 Wattmeters.

c. Make a table comparing expected (calculated) and measured values of I_A, V_{AO}, P_{AO}, and I_{ON} for the results of part B.4.b. In calculating the expected values of I_A, V_{AO}, P_{AO}, and I_{ON}, use the resistances measured in part B.4.a and the phase voltages measured in part B.2.b and assume that the phase shift between the phase voltages is exactly 120° in the proper sequence (CBA or ABC). Comment on the deviations between measured and calculated values shown in the table. Calculate P_{AO}, P_{BO}, and P_{CO} and obtain the total power P_T as the sum of these powers. Why is the assumption that $P_T = 3\ P_{AO}$ not reasonable for an unbalanced load with a neutral return line? Obviously, 3 Wattmeters could be used to measure total power. Explain why total power cannot be measured using only 2 Wattmeters.

d. Make a table comparing measured values of I_A and V_{AO} made in parts B.4.b and B.4.c. Also include the calculated values of P_{AO} in this table. Include in the table the calculated values of P_T for the unbalanced load with and without the Neutral Return line. Comment on the results shown in the table.

4. *Three Phase Transformer*

a. In part B.5.a, what is the step-down ratio for each transformer?

b In part B.5.b, what are the secondary line-to-line voltages?

c In part B.5.c, what are the open delta voltage and “ring” current with one secondary phase wired backwards? Explain the last result with the aid of phasor algebra and a phasor diagram.