Experiment 7: Frequency Modulation and Phase Locked Loops

Frequency Modulation Background

Normally, we consider a voltage wave form with a fixed frequency of the form

\[ v(t) = V \sin(\omega_c t + \theta) , \]  

where \( \omega_c \) is the fixed angular frequency and \( \theta \) is the phase. When we have a voltage wave form with a variable frequency, this has the form

\[ v(t) = V \sin \phi(t) \]  

where \( \phi(t) \) is the total angular displacement at time \( t \). Consistent with this viewpoint, the instantaneous frequency in radians/sec is

\[ \omega_i = 2\pi f_i(t) = \frac{d\phi(t)}{dt} \]  

For example, a constant frequency, \( \omega_c \), implies

\[ \phi(t) = \omega_c t + \theta \]  

in which case

\[ \omega_i = \frac{d(\phi(t))}{dt} = \omega_c \]  

In this manner, a frequency modulated wave with sinusoidal modulation has an instantaneous frequency

\[ f_i(t) = f_c + \Delta f \cos \left(2\pi f_m t\right) \]  

where \( f_c \) is the average frequency of the carrier wave, and \( \Delta f \) is the maximum deviation of the instantaneous frequency from the average frequency. Notice that \( \Delta f \) is proportional to the peak amplitude of the modulating signal and is independent of the modulating frequency, \( f_m \). From equation (3) and equation (6) we see that

\[ \frac{d\phi(t)}{dt} = \omega_c + 2\pi \Delta f \cos \omega_m t \]  

\[ \phi(t) = \omega_c t + \frac{2\pi \Delta f}{\omega_m} \sin \omega_m t + \theta \]  

Neglecting \( \theta \) for simplicity, we then have the frequency modulated signal from equation (2),

\[ v(t) = V \sin \left( \omega_c t + \frac{2\pi \Delta f}{\omega_m} \sin \omega_m t \right) \]
Equation (9) is usually written in the form

$$v(t) = V \sin (\omega_c t + m_f \sin \omega_m t)$$  \hspace{1cm} (10)

where

$$m_f = \frac{2\pi \Delta f}{2\pi f_m} = \frac{\Delta f}{f_m}$$  \hspace{1cm} (11)

the quantity $m_f$ is known as the modulation index.

As can be seen in equation (6), $\Delta f$ is proportional to the maximum amplitude of the modulation signal, $V_m$, and

$$\Delta f = (\text{VCO Sensitivity}) \ V_m$$  \hspace{1cm} (12)

As will be discussed later, frequency modulation is implemented with a voltage controlled oscillator (VCO) and the VCO sensitivity is the transfer characteristic of this device in Hz/V.

Making use of the trigonometric identity

$$\sin (A + B) = \sin A \cos B + \cos A \sin B$$  \hspace{1cm} (13)

equation (10) becomes

$$v(t) = V \sin \omega_c t \cos (m_f \sin \omega_m t) + V \cos \omega_c t \sin (m_f \sin \omega_m t).$$  \hspace{1cm} (14)

We now use the identities:

$$\cos (m_f \sin \omega_m t) = J_0(m_f) + 2J_2(m_f) \cos 2\omega_m t + 2J_4(m_f) \cos 4\omega_m t + \ldots.$$  \hspace{1cm} (15)

And

$$\sin (m_f \sin \omega_m t) = 2J_1(m_f) \sin \omega_m t + 2J_3(m_f) \sin 3\omega_m t + \ldots.$$  \hspace{1cm} (16)

Equation (14) can now be expanded in terms of Bessel functions of the first kind of order $n$ and argument $m_f$

$$v(t) = V \sin \omega_c t [J_0(m_f) + 2J_2(m_f) \cos 2\omega_m t + \ldots]$$

$$+ V \cos \omega_c t [2J_1(m_f) \sin \omega_m t + 2J_3(m_f) \sin 3\omega_m t + \ldots]$$  \hspace{1cm} (17)

Using the trigonometric identities for sum and difference angles

$$v(t)/V = J_0(m_f) \sin \omega_c t$$
$$+ J_1(m_f) [\sin (\omega_c + \omega_m)t - \sin (\omega_c - \omega_m)t]$$
$$+ J_2(m_f) [\sin (\omega_c + 2\omega_m)t + \sin (\omega_c - 2\omega_m)t]$$
$$+ J_3(m_f) [\sin (\omega_c + 3\omega_m)t - \sin (\omega_c - 3\omega_m)t]$$
$$+ \ldots.$$  \hspace{1cm} (18)
So, a frequency modulated signal consists of a carrier and a large number of upper and lower side bands with amplitudes given by Bessel functions of the first kind with the modulation index, $m_f$, as its argument. Some of these values are shown in the Table 1 below:

<table>
<thead>
<tr>
<th>$m_f$</th>
<th>$J_0$</th>
<th>$J_1$</th>
<th>$J_2$</th>
<th>$J_3$</th>
<th>$J_4$</th>
<th>$J_5$</th>
<th>$J_6$</th>
<th>$J_7$</th>
<th>$J_8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.25</td>
<td>0.98</td>
<td>0.12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.50</td>
<td>0.94</td>
<td>0.24</td>
<td>0.03</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.00</td>
<td>0.77</td>
<td>0.44</td>
<td>0.11</td>
<td>0.02</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.50</td>
<td>0.51</td>
<td>0.56</td>
<td>0.23</td>
<td>0.06</td>
<td>0.01</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.00</td>
<td>0.22</td>
<td>0.58</td>
<td>0.35</td>
<td>0.13</td>
<td>0.03</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.40</td>
<td>0.00</td>
<td>0.52</td>
<td>0.43</td>
<td>0.20</td>
<td>0.06</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.50</td>
<td>-0.05</td>
<td>0.50</td>
<td>0.45</td>
<td>0.22</td>
<td>0.07</td>
<td>0.02</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.00</td>
<td>-0.26</td>
<td>0.34</td>
<td>0.49</td>
<td>0.31</td>
<td>0.13</td>
<td>0.04</td>
<td>0.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.00</td>
<td>-0.40</td>
<td>-0.07</td>
<td>0.36</td>
<td>0.43</td>
<td>0.28</td>
<td>0.13</td>
<td>0.05</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>5.00</td>
<td>-0.18</td>
<td>-0.33</td>
<td>0.05</td>
<td>0.36</td>
<td>0.39</td>
<td>0.26</td>
<td>0.13</td>
<td>0.05</td>
<td>0.02</td>
</tr>
<tr>
<td>5.50</td>
<td>0.00</td>
<td>-0.34</td>
<td>-0.12</td>
<td>0.26</td>
<td>0.40</td>
<td>0.32</td>
<td>0.19</td>
<td>0.09</td>
<td>0.03</td>
</tr>
<tr>
<td>6.00</td>
<td>0.15</td>
<td>-0.28</td>
<td>-0.24</td>
<td>0.11</td>
<td>0.36</td>
<td>0.36</td>
<td>0.25</td>
<td>0.13</td>
<td>0.06</td>
</tr>
<tr>
<td>7.00</td>
<td>0.30</td>
<td>0.00</td>
<td>-0.30</td>
<td>-0.17</td>
<td>0.16</td>
<td>0.35</td>
<td>0.34</td>
<td>0.23</td>
<td>0.13</td>
</tr>
<tr>
<td>8.00</td>
<td>0.17</td>
<td>0.23</td>
<td>-0.11</td>
<td>-0.29</td>
<td>-0.10</td>
<td>0.19</td>
<td>0.34</td>
<td>0.32</td>
<td>0.22</td>
</tr>
</tbody>
</table>

Table 1 – Bessel functions specifying carrier ($J_0$) and side band values ($J_1$ to $J_8$)

Notice that these Bessel functions have a damped oscillatory behavior.

**Voltage Controlled Oscillator**

![Figure 1 – Voltage controlled oscillator](image)

Frequency modulation can be performed using a voltage-controlled oscillator (VCO). Figure 1 shows a relatively simple but very practical VCO that consists of a multiplier, an integrator, a comparator with hysteresis (or Schmitt trigger circuit), and Zener diodes to limit the output voltage swing. As we will see, the frequency of the output voltage is determined by the RC time constant of the integrator and the control voltage input, $V_c$. 

![Figure 1 – Voltage controlled oscillator](image)
Schmitt Trigger Circuit

First, look at the Schmitt trigger circuit on the right side of Figure 1. Because of the positive feedback voltage divider, the differential voltage into the operational amplifier is

\[ V_+ = \frac{V_0 + V_R}{2} \]  

(19)

where \( V_R \) is the integrator output. This circuit always operates in the non-linear mode. The output, \( V_0 \), is always saturated at either \(+V_L\) or \(-V_L\). If the output is at \(+V_L\), then the input, \( V_R \), must fall below \(-V_L\) before the output will change. When the input drops below \(-V_L\), the output will go to \(-V_L\) as quickly as the amp can slew because of the positive feedback through \( R' \). In order for the output to go to \(+V_L\) again \( V_R \) must now rise above \(+V_L\). This produces the hysteresis in the transfer characteristic shown in Figure 2.

![Schmitt circuit transfer characteristic](image)

Assume for the moment that \( V_c \) is a constant (DC) voltage. Because of the charging capacitor in the integrator, the reference voltage, \( V_R \), changes from \(+V_L\) to \(-V_L\) in a half period of oscillation. This behavior and the transfer characteristic shown in Figure 2 produce the waveforms shown in Figure 3.

![Ideal wave forms for the reference voltage, \( V_R \), and output voltage, \( V_0 \).](image)

**Integrator**

The integrator in Figure 1 performs the function

\[ V_{out}(t) - V_{out}(0) = -\frac{1}{RC} \int_0^t V_{in}(t) dt \]  

(20)
In terms of the parameters of our circuit equation (20) becomes

\[ V_R(t) - V_R(0) = -\frac{1}{RC} \int_0^t V_c V_o(t) dt \]  

(21)

Evaluating equation (21) over the half period from 0 to T/2 gives

\[ V_R \left( \frac{T}{2} \right) - V_R(0) = -\frac{1}{RC} \int_0^{T/2} V_c (+V_L) dt \]  

(22)

Or

\[ -V_L - V_L = -\frac{V_L V_L T}{RC} \]  

(23)

giving

\[ f = 1/T = \frac{V_C}{4RC} \]  

(24)

Notice that the control voltage, \( V_c \), must be multiplied instead of added. Otherwise we would just have a change in duty cycle with \( V_c \), and the period would be

\[ T = \frac{2V_L RC}{V_L + V_c} + \frac{2V_L RC}{V_L - V_c} \]  

(25)

**Phase Locked Loop**

![Figure 4 – Schematic diagram of the LM565 phase locked loop](image)

Although useful in many applications, phase locked loops perform FM demodulation in a
straightforward manner. Referring to Figure 4 a frequency modulated input signal, \( \sin(\omega t + \theta) \), is applied to a phase sensitive detector, which is essentially a multiplier. From equations (10) and (11) the phase angle, indicated by \( \theta \) in Figure 4, is

\[
\theta(t) = \frac{\Delta f}{f_m} \sin(\omega_m t)
\]

(26)

where we have assumed sinusoidal, single frequency modulation. In the multiplier this input signal is multiplied by the output signal from the voltage controlled oscillator, \( \omega_0 t \), to produce a sinusoidal signal with sum and difference frequencies. This signal containing the sum and difference frequencies is amplified and low pass filtered to obtain only a signal with the difference frequency as the output. When \( \omega_0 \) equals \( \omega_c \), this output is just the phase angle, \( \theta \), which is fed back into the VCO to maintain the VCO output frequency, \( \omega_v \), exactly equal to \( \omega_c \). The system is said to be “in lock” when \( \omega_v = \omega_c \). Since the input phase, \( \theta \), varies with time, the output will also vary in time the same way. If we compute the frequency of the output we find, as we expect, that

\[
f_{out} = \frac{1}{2\pi} \frac{d\theta}{dt} = \Delta f \cos(\omega_m t)
\]

(27)

So, with an FM modulated input signal, the output, except for a scale factor, is just the original modulating signal.

**Preliminary Preparation**

Before you come to the laboratory, determine the values for \( R_0 \) and \( C_0 \) that will give a voltage controlled oscillator (VCO) center frequency (\( f_0 \)) of 30 kHz. (See page 8 of Reference 1 posted on the Electronics Laboratory Website for the necessary equations. Note that \( R_0 \) and \( C_0 \) in this note are the same as \( R \) and \( C \) in Equation (24).) \( R_0 \) should be in the range from 2 k to 20 k\( \Omega \) with an optimum value between 4 k\( \Omega \) and 5 k\( \Omega \), so assume that \( R_0 = 4.7 \) k\( \Omega \). The filter bandwidth (the capture range of the loop) is determined by the internal resistance \( R_{12} \) (\( R_1 \) in Ref 1) of 3600\( \Omega \) and \( C_1 \). Determine the value of \( C_1 \) that will give a capture range (\( f_n \)) of 8 kHz. You will also need an additional low pass filter to eliminate the carrier frequency in the demodulated output. Determine appropriate resistance and capacitance values for this filter.
Experiment 7 Procedures

Equipment List

1  Integrated Circuit LM 565 Phase Locked Loop
1  Solderless Wiring Fixture
1  AFG 2021 Arbitrary Function Generator
   Assorted Resistors and Capacitors

Procedure

1. Frequency Modulation. The AFG 2021 arbitrary function generator can be frequency modulated using the internal FM source. To set the carrier frequency, select the sine wave output and set its frequency to be the carrier frequency. Set the modulation shape, modulation frequency $f_m$, and frequency deviation $\Delta f$, using the modulation push button / option choices on the AFG 2021 arbitrary function generator.

1(a). Set the carrier frequency to a 30 kHz, 1 Vp-p output, with a 500 Hz sine wave modulation having $\Delta f = 125$ Hz initially. Obtain the spectrum using procedures similar to those perfected in earlier lab 2. Determine the $\Delta f$ value required to obtain zero carrier in the spectrum, adjust the AFG to obtain this $\Delta f$ value and obtain the new spectrum.

1(b). With, successively, a sine, square, and triangular modulation waveform at 500 Hz, with $\Delta f = 4$ kHz and 1 V p-p from the AFG, record the spectra – include all pertinent data.

2. Phase Locked Loop

![Phase locked loop circuit diagram]

Figure 5 – Phase locked loop circuit
Construct the circuit shown in Figure 5 using your design parameters for a VCO center frequency of 30 kHz and a capture range of 8 kHz. Add an additional low pass R-C filter to the output of the demodulator to minimize the effect of the “sum” frequency. Set the AFG 2021 generator to provide a 30 kHz, 1V pp sinusoidal output with no offset and no FM and apply this signal to the oscilloscope and the modulation input of the phase locked loop. The oscilloscope should be synchronized with this signal. Apply the VCO output to the other oscilloscope channel and the demodulation output to the DMM. For proper operation the VCO output should be a square wave varying from about 0 to +12 V and the demodulation output should be a dc voltage of about +9 V.

2(a). Center Frequency. Adjust the generator frequency until the two oscilloscope signals are exactly 90 degrees out of phase. Record this frequency and the DC output voltage on the DMM. This frequency is the loop center frequency and the DC voltage is the nominal output voltage at zero signal input.

2(b). Upper Lock Range. Slowly increase the signal generator frequency above center until the phase has shifted by 90 degrees from its value at the loop center frequency. At this point the loop has dropped or is about to drop out of lock. If you have gone too far, return to the center and try again. Record the signal generator frequency and loop dc output voltage at this point. Beware of false locks.

2(c). Lower Lock Range. Next, slowly decrease the frequency below center until the phase has shifted by 90 degrees in the other direction. Again, the loop has dropped or is about to drop out of lock at this point. If you missed the values, return to the center and try again. Record the frequency and the loop dc output voltage.

2(d). Capture Range. Go up in frequency until the loop drops out of lock, then carefully come back down until it regains lock. Avoid early false locks. In false lock the two waveforms on the oscilloscope will be of different frequency, whereas in true lock they will be of the same frequency. A true lock will be at a lower frequency than a false lock. Record the true lock frequency. Repeat this procedure from the low frequency side of center frequency. Repeat this the capture range measurement with C0 equal to one-fourth its original value, four times its original value, and sixteen times its original value.

3. Demodulated Signal. Observe the low pass filtered output of the PLL FM demodulator for sine and square wave modulation. Set the AFG frequency to the center frequency of the PLL. Also set 1 Vpp output, sine FM modulation, and Δf = 1 kHz. Use C0 consistent with a capture range of 8 kHz. Observe the demodulated output on the scope. We may regard the system consisting of AFG 2021 FM modulator and carrier generator and PLL as a linear system and characterize it by its frequency response, much as we would an amplifier. Accordingly, for a sinusoidal modulation signal, record the magnitude and phase (relative to the sync output of the AFG) of the filtered demodulated output for a series of five modulation frequencies ranging from the lower limit to the upper limit of the capture range. Then also record the waveform of the filtered demodulated output for a square wave modulation with fm = 1 kHz.
Report

1(a) For the 500 Hz sine wave modulation data, compare the relative values of the frequency components in your recorded spectra with the table of Bessel functions given in the manual. Include the recorded spectra in this write up. Explain any discrepancies.

1(b) Refer to the recorded spectra of the sine, square, and triangle modulation with high modulation index. Include the spectra in this write up. Comment on the differences in the 3 different spectra. Why is the square wave spectrum so sharply peaked? Also, relate the spectra widths to the modulation frequency and index.

2(a) Compare and discuss any differences between your experimental and calculated PLL center frequencies.

2(b) Compare and discuss any differences between your experimental and calculated upper lock range.

2(c) Compare and discuss any differences between your experimental and calculated lower lock range.

2(d) Compare and discuss any differences between your experimental and calculated capture ranges.

3. Plot the demodulated amplitude and phase data that you recorded vs. the modulation frequency. Are the results reasonable? Why? Is the waveform of the demodulated output with square wave FM modulation consistent with the data? Why?

References and Suggested Reading