Size Density Function p.d.f.

$n(R)\,dR =$ (# fraction of particles of size between R & $R+dR$) in

$p(R)\,dR =$ (mass fraction particles in the size range R to $R+dR$)

$\int_{R_m}^{R_M} n(R)\,dR = 1$ $\qquad \int p(R)\,dR = 1$

Assuming $\rho_s \neq \rho_s(R)$ i.e. constant density and spherical particles

$$\frac{4}{3}\pi R^3 \rho_s N n(R)\,dR = W\, p(R)\,dR$$

N = total number of particles

W = total mass of particles

$$(1)\quad n(R) = \frac{3W}{4\pi R^3 \rho_s N}\, p(R)$$

↑ usually required

Mass Average particle size $\bar{R}_w$ is the size of the particle whose mass ρ_s is the mean of all N particles in the mixture. $m_p \propto R^3$

$$\int_{R_{min}}^{R_{max}} R^3 n(R)\,dR = \bar{R}_w^{\,3}$$

$$\frac{3W}{4\pi\rho_s N}\int_{R_{min}}^{R_{max}} p(R)\,dR = \frac{3W}{4\pi\rho_s N} = \bar{R}_w^{\,3}$$

$W = \frac{4}{3}\pi\rho_s \bar{R}_w^{\,3} N$

Now from (1)

(1a) $$n(R) = \frac{\bar{R}_w^3}{R^3} \, n(R)$$

$$1 = \int_{R_m}^{R_M} n(R)\,dR = \bar{R}_w^3 \int_{R_m}^{R_M} \frac{n(R)}{R^3}\,dR$$

(2) $$\boxed{\bar{R}_w = \frac{1}{\left[\int_{R_m}^{R_M} \frac{n(R)}{R^3}\,dR\right]^{1/3}}}$$ $\bar{R}_w = \mu_{-3}^{-1/3}$

<u>Surface mean size $\bar{R}_s$</u> is the size of the particle whose surface to volume ratio equals that of all solids in the mixture (This average size is needed in ΔP and mass transfer calculations)

$$S_{ext} = N \int_{R_m}^{R_M} 4\pi R^2 \, n(R)\,dR$$

$$V_t = N \int_{R_m}^{R_M} \frac{4}{3}\pi R^3 \, n(R)\,dR$$

$$\left(\frac{S_{ex}}{V}\right)_t = \frac{3 \int_{R_m}^{R_M} R^2 \, n(R)\,dR}{\int_{R_m}^{R_M} R^3 \, n(R)\,dR} = \frac{3}{\bar{R}_s}$$

Substitute (1) or (1a)

$$\frac{3 \int_{R_m}^{R_M} \frac{n(R)}{R}\,dR}{\int_{R_m}^{R_M} n(R)\,dR} = \frac{3}{\bar{R}_s}$$

$$\bar{R}_s = \frac{1}{\int_{R_m}^{R_M} \frac{n(R)}{R}\,dR}$$ $\bar{R}_s = \mu_{-1}^{-1}$

Population Balance for Fluidized Beds

Interval dR (Steady state)

$$\begin{pmatrix}\text{number of particles}\\ \text{of size between } R \,\&\, R+dR\\ \text{entering the bed per unit time}\end{pmatrix} - \begin{pmatrix}\text{number of particles}\\ \text{of size between } R \,\&\, R+dR\\ \text{leaving the bed per}\\ \text{unit time}\end{pmatrix}$$

$$+ \begin{pmatrix}\text{number of particles}\\ \text{of size between } R \,\&\, R+dR\\ \text{generated in the bed}\\ \text{per unit time}\end{pmatrix} - \begin{pmatrix}\text{number of particles}\\ \text{of size between } R \,\&\, R+dR\\ \text{destroyed per unit}\\ \text{time}\end{pmatrix}$$

$$+ \begin{pmatrix}\text{number of particles}\\ \text{of size } R-dR \text{ growing}\\ \text{into size } R \text{ to } R+dR\\ \text{per unit time}\end{pmatrix} - \begin{pmatrix}\text{number of particles}\\ \text{of size } R \,\&\, R+dR\\ \text{outgrowing this size range}\\ \text{per unit time}\end{pmatrix} = 0$$

~~steady state $\dot{W}_{in} = \dot{W}_{out}$~~

$$\dot{N}_{in}(R) - \dot{N}_{out}(R) + \dot{B}(R)^{\,0} - \dot{D}(R)^{\,0} + \frac{N(R)\,\dot{R}}{dR} - \frac{N(R+dR)\,\dot{R}}{dR} = 0$$

$$\dot{N}_{in}\, n_{in}(R)\,dR - \dot{N}_{out}\, n_{out}(R)\,dR + N n(R)\,dR\,\frac{\dot{R}}{dR} - N n(R+dR)\,\frac{\dot{R}\,dR}{dR} = 0$$

CSTR assumption $n_{out}(R) = n(R)$

(1) $$\dot{N}_{in}\, n_{in}(R) - \dot{N}_{out}\, n(R) - N\frac{d}{dR}\left[n\dot{R}\right] = 0$$

$N n(R)\,dR = N(R)$ — number of particles of size $R - R+dR$

$W p(R)\,dR = W(R)$ — weight of particles of size R to $R+dR$

~~$N(R)$~~ $\quad N = \dfrac{W}{\frac{4}{3}\pi R^3 \rho}$ $\qquad W(R) = \frac{4}{3}\pi R^3 \rho\, N(R)$

$W = \frac{4}{3}\pi \overline{R^3} \rho N$

$$\dot{N}_{in}\, n_{in}(R)\,\Delta R - \dot{N}_{out}\, n_{out}(R)\,\Delta R + N n(R)\,\mathcal{R}\Big|_{R} - N n(R)\,\mathcal{R}\Big|_{R+\Delta R} = 0$$

$$\dot{N}_{in}\, n_{in}(R) - \dot{N}_{out}\, n_{out}(R) - N\frac{d}{dR}\left[n(R)\,\mathcal{R}\right] = 0$$

$$\boxed{n(R) = \frac{3W\,p(R)}{N\,4\pi R^3 \rho_s} \qquad n_{in} = \frac{3F_{in}\,p_{in}(R)}{\dot{N}_{in}\,4\pi R^3\rho_s}}$$

$$\cancel{\dot{N}_{in}}\frac{3F_{in}\,p_{in}(R)}{\cancel{\dot{N}_{in}}\,4\pi R^3\rho_s} - \cancel{\dot{N}_{out}}\frac{3F_{out}\,p_{out}(R)}{\cancel{\dot{N}_{out}}\,4\pi R^3\rho_s} - N\frac{3W}{N 4\pi\rho_s}\frac{d}{dR}\left[\frac{p(R)\,\mathcal{R}}{R^3}\right] = 0$$

CSTR $\longrightarrow$ CSTR $\quad \boxed{p_{out} = p}$

$$F_{in}\,p_{in}(R) - F_{out}\,p(R) - R^3 W\frac{d}{dR}\left[\frac{p(R)\,\mathcal{R}}{R^3}\right] = 0$$

$$F_{in}\,p_{in}(R) - F_{out}\,p(R) - W\frac{d}{dR}\left[p(R)\,\mathcal{R}\right] + \frac{3W}{R}p(R)\mathcal{R} = 0$$

$$\left(\frac{\text{mass generation}}{\text{in the size interval}}\right) = \rho_s \left(\begin{matrix}\text{number of particles}\\ \text{in the size interval}\end{matrix}\right)\left(\begin{matrix}\text{volume increase of}\\ \text{a particle in } \Delta t\end{matrix}\right)$$

$$= \rho_s\,\frac{3W\,p(R)}{4\pi\rho_s\,R^3}\,4\pi R^2\frac{dR}{dt}$$

$$= \frac{3W\,p\,\mathcal{R}}{R}$$

$$N(R) = \frac{W(R)}{\frac{4}{3}\pi R^3 \rho_s} = \frac{3\,W(R)}{4\pi R^3 \rho_s} = \frac{3W\,p(R)\,dR}{4\pi R^3 \rho_s}$$

$$n(R)\,dR = \frac{N(R)}{N} = \frac{3\,W\,p(R)\,dR}{N\,4\pi R^3 \rho_s}$$

For uniform solid density

$$N = \frac{W}{\frac{4}{3}\pi \bar{R}^3 \rho_s}$$

$$n(R) = \frac{3W\,p(R)}{N\,4\pi R^3 \rho_s}$$

Similarly (ρ_s = const)

$$\dot{n}_{in}(R) = \frac{3\,F_{in}\,p_{in}(R)}{\dot{N}_{in}\,4\pi R^3 \rho_s}$$

$$\dot{N}_{in}\,\frac{3\,F_{in}\,p_{in}(R)}{\dot{N}_{in}\,4\pi R^3 \rho_s} - \dot{N}_{out}\,\frac{3\,F_{out}\,p(R)}{\dot{N}_{out}\,4\pi R^3 \rho_s} - N\,\frac{d}{dR}\left[\frac{3W\,p(R)}{N\,4\pi R^3 \rho_s}\,\dot{R}\right] = 0$$

$$\frac{3}{4\pi R^3 \rho_s}\left[F_{in}\,p_{in}(R) - F_{out}\,p(R)\right] - \frac{3W}{4\pi R^3 \rho_s}\,\frac{d}{dR}\left[p\,\dot{R}\right]$$

$$+ \frac{9\,W\,p\,\dot{R}}{4\pi R^4 \rho_s} = 0$$

$$\boxed{F_{in}\,p_{in} - F_{out}\,p - W\,\frac{d}{dR}\left(p\,\dot{R}\right) + 3\,\frac{W}{R}\,p\,\dot{R} = 0}$$

basic balance is terms of weight (mass)

Fluidized Beds

Reaction of Solids Particles of Changing Size

Ass

- uniform gas composition
- uniform particle density
- CSTR of solids , steady state, isothermal

Mass balance on reactor in size range R to $R+\Delta R$

$$\begin{pmatrix}\text{solids}\\ \text{in by}\\ \text{feed}\end{pmatrix} - \begin{pmatrix}\text{solids}\\ \text{out by}\\ \text{outflow}\end{pmatrix} - \begin{pmatrix}\text{solids}\\ \text{out by}\\ \text{carryover}\end{pmatrix} + \begin{pmatrix}\text{solids growing}\\ \text{into the interval}\\ \text{from a smaller}\\ \text{size}\end{pmatrix}$$

$$- \begin{pmatrix}\text{solids growing}\\ \text{out of the interval}\\ \text{to larger size}\end{pmatrix} + \begin{pmatrix}\text{solid mass}\\ \text{generation within}\\ \text{the interval by growth}\end{pmatrix} = 0$$

Growth Rate $\quad \mathcal{R} = \dfrac{dR}{dt}$

Particle size distribution in bed and outflow $p(R)$

Particle size distribution in ~~outflow~~ feed $p_0(R)$

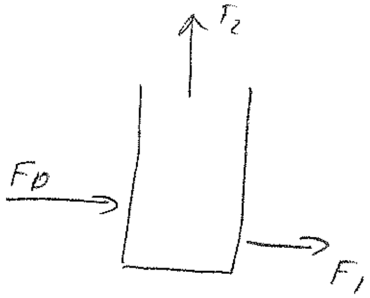
Particle size distribution in elutriated stream $p_2(R)$

$$\begin{pmatrix}\text{mass generation}\\ \text{within the size}\\ \text{interval}\end{pmatrix} = \rho_s \begin{pmatrix}\text{number of}\\ \text{particles in}\\ \text{the interval}\end{pmatrix}\begin{pmatrix}\text{volume increase}\\ \text{of a particle in}\\ \text{time } \Delta t\end{pmatrix}$$

$$= \rho_s \frac{W p(\bar{R}) \Delta R}{\rho_s \frac{4}{3}\pi \bar{R}^3} \frac{dV}{dt} \times \Delta t$$

$V = \frac{4}{3}\pi \bar{R}^3$

$$= \frac{3 W p(\bar{R})\, \mathcal{R}(\bar{R})\, \Delta R}{\bar{R}}$$



$$R < \bar{R} < R + \Delta R$$

$$F_0\, p_0(\bar{R})\Delta R - F_1\, p(\bar{R})\Delta R - F_2\, p_2(\bar{R})\Delta R + W p(R)\mathcal{R}\Big|_R - W p(R)\mathcal{R}\Big|_{R+\Delta R}$$

$$+ \frac{3W\, p(\bar{R})\,\mathcal{R}(\bar{R})\,\Delta R}{\bar{R}} = 0$$

$$\lim_{\Delta R \to 0} \left(F_0\, p_0(\bar{R}) - F_1\, p(\bar{R}) - F_2\, p_2(\bar{R})\right) - W \lim_{\Delta R \to 0} \frac{(p\mathcal{R})_{R+\Delta R} - (p\mathcal{R})_R}{\Delta R}$$

$$+ \lim_{\Delta R \to 0} \frac{3W\, p(\bar{R})\,\mathcal{R}(\bar{R})}{\bar{R}}$$

$$F_0\, p_0(R) - F_1\, p(R) - F_2\, p_2(R) - W \frac{d(p\mathcal{R})}{dR} + \frac{3W p \mathcal{R}}{R} = 0$$

Now remember that in mak - bal

$$\bar{t}(R_i) = \frac{W(R_i)}{F_0(R_i)} = \frac{W(R_i)}{F_1(R_i) + F_2(R_i)} = \frac{1}{\frac{F_1(R_i)}{W(R_i)} + \frac{F_2(R_i)}{W(R_i)}}$$

$$\frac{F_1(R_i)}{W(R_i)} = \frac{F_1}{W} \quad \text{and} \quad \frac{F_2(R_i)}{W(R_i)} = \kappa(R_i)$$

$$\bar{t} = \frac{1}{\frac{F_1}{W} + \kappa(R_i)} \quad \text{but} \quad \frac{F_2(R_i)}{W(R_i)} = \frac{p_2(R_i)\, F_2}{p(R_i)\, W}$$

$$p_2(R) = \frac{W}{F_2}\kappa(R)\, p(R) \qquad (A) \qquad (45)$$

$$\boxed{F_0\, p_0(R) - F_1\, p(R) - W\kappa(R)\, p(R) - W \frac{d}{dR}(p\mathcal{R}) + \frac{3W p\mathcal{R}}{R} = 0} \qquad (1)$$

$e^{a+b\ldots}$

$$F_1 + F_2 - F_0 = \begin{pmatrix} \text{total solid} \\ \text{mass production} \\ \text{in the bed} \end{pmatrix} = \int_{R_{min}}^{R_{max}} \frac{3W\, p(R)\,\mathcal{R}(R)\, dR}{R} \qquad (2)$$

$$F_1 + F_2 - F_0 > 0 \qquad \text{particle growth} \qquad (A4)$$

$$< 0 \qquad \text{particle shrinkage}$$

Solution (Single size Feed)

$$F_0\,\delta(R-R_i) - F_1 p - W\kappa p \underbrace{- W\frac{d}{dR}(p\mathcal{R})}_{-Wp\frac{d\mathcal{R}}{dR} - W\mathcal{R}\frac{dp}{dR}} + \frac{3Wp\mathcal{R}}{R} = 0$$

$$-W\mathcal{R}\frac{dp}{dR} - \left[W\frac{d\mathcal{R}}{dR} + F_1 + W\kappa - \frac{3W\mathcal{R}}{R}\right] p = -F_0\,\delta(R-R_i)$$

$$\frac{dp}{dR} + \left[\frac{1}{\mathcal{R}}\frac{d\mathcal{R}}{dR} + \frac{F_1}{W\mathcal{R}} + \frac{\kappa}{\mathcal{R}} - \frac{3}{R}\right] p = \frac{F_0\,\delta(R-R_i)}{W\mathcal{R}}$$

$$\frac{d}{dR}\left[e^{\int_{R_i}^{R}[\;]dR}\, p\right] = e^{\int_{R_i}^{R}[\;]dR}\,\frac{F_0}{W\mathcal{R}}\,\delta(R-R_i)$$

$$e^{\int_{R_i}^{R}[\;]dR}\, p(R) - 0 = \frac{F_0}{W|\mathcal{R}(R_i)|}$$

$$\int_{R_i}^{R}\left[\frac{d\ln|\mathcal{R}|}{dR} + \frac{F_1/W + \kappa}{\mathcal{R}} - \frac{3}{R}\right]dR =$$

$$= \ln\left|\frac{\mathcal{R}(R)}{\mathcal{R}(R_i)}\right| - 3\ln\frac{R}{R_i} + \int_{R_i}^{R}\frac{F_1/W + \kappa}{\mathcal{R}}\,dR$$

$$e^{\int_{R_i}^{R}[\;]dR} = \left|\frac{\mathcal{R}(R)}{\mathcal{R}(R_i)}\right|\left(\frac{R_i}{R}\right)^3 e^{\int_{R_i}^{R}\frac{F_1/W + \kappa(R)}{\mathcal{R}(R)}dR}$$

$$p(R) = \frac{F_0}{W|\mathcal{R}(R_i)|}\,\frac{|\mathcal{R}(R_i)|}{|\mathcal{R}(R)|}\left(\frac{R}{R_i}\right)^3 e^{-\int_{R_i}^{R}\frac{F_1/W + \kappa(R)}{\mathcal{R}(R)}dR}$$

$$p(R) = \frac{F_0}{W\,|\mathcal{R}(R)|}\left(\frac{R}{R_i}\right)^3 \underbrace{e^{-\int_{R_i}^{R} \frac{F_1/W + K(R)}{\mathcal{R}(R)}\,dR}}_{I(R,R_i)\ (A2)} \qquad (3)\ (A3)$$

Apply to eq (2)

$$\int_{R_i}^{R_\infty} p(R)\,dR = 1 \qquad R_\infty = R(t \to \infty)$$

to get

$$\frac{W}{F_0} = \int_{R_i}^{R_\infty} \frac{R^3}{R_i^3\,|\mathcal{R}(R)|}\,I(R,R_i)\,dR \qquad (4)\ (A1)$$

1. Tabulate $I(R, R_i)$ for suitable increments of R or program it in.
2. get F_0, F_1 or W from (4).
   If in a bed of given W, F_1 and F_0 is to be found solution is direct.
   If F_1 or W is to be found solve (4) by trial and error
3. Calculate $p(R)$ by (3)
4. Calculate F_2 by (2)
5. Calculate $n_2(R)$ by (A)

Single Size Feed

(*) $$n(R) = \frac{F_0}{W|\mathcal{R}(R)|}\left(\frac{R}{R_i}\right)^3 \underbrace{e^{-\int_{R_i}^{R} \frac{F_1/W + \kappa(R)}{\mathcal{R}(R)}\, dR}}$$

$$\frac{W}{F_0} = \int_{R_i}^{R_\infty} \frac{R^3}{R_i^3 |\mathcal{R}(R)|}\, I(R, R_i)$$

$$F_1 + F_2 - F_0 = \int_{R_i}^{\infty} \frac{3W\, n(R)\, \mathcal{R}(R)}{R}\, dR$$

$R_f \to \infty$ growing particles

$R_f \to 0$ shrinking particles

Feed of various particle sizes

growing particles

$$\left(\begin{array}{c}\text{fraction of} \\ \text{exit stream of} \\ \text{size } R\end{array}\right) = \sum_{\substack{\text{all feed} \\ \text{solids not} \\ \text{larger than } R}} \left(\begin{array}{c}\text{fraction of solids} \\ \text{of size } R \\ \text{and original feed} \\ \text{size } R_i\end{array}\right)\left(\begin{array}{c}\text{fraction of} \\ \text{feed of} \\ \text{size } R_i\end{array}\right)$$

$$p(R) = \int_{R_{min}}^{R} \underbrace{n_1(R, R_i)}_{\text{given above } (*)}\, n_0(R_i)\, dR_i$$

$$n_1(R, R_i) = \int_{R_i}^{R_\infty} \left(\frac{R}{R_i}\right)^3 \frac{F_0}{W|\mathcal{R}(R)|}\, I(R, R_i)\, n_0(R_i)\, dR_i$$

$$n(R) = \frac{F_0}{W}\int_{R_{min}}^{R} \left(\frac{R}{R_i}\right)^3 \frac{1}{|\mathcal{R}(R)|}\, I(R, R_i)\, n_0(R_i)\, dR_i$$

$$n(R) = \frac{F_0 R^3}{W|\mathcal{R}(R)|}\int^{R} I(R, R_i)\, n_0(R_i) \ldots$$

However

$$I(R,R_i) = e^{-\int_{R_i}^{R}[\ ]\,dR} = e^{-\left[\int_{R_{min}}^{R}[\ ]\,dR - \int_{R_{min}}^{R_i}[\ ]\,dR\right]}$$

$$= I(R,R_{min})\, I(R_{min},R_i) = \frac{I(R,R_{min})}{I(R_i,R_{min})}$$

$$p(R) = \frac{F_0 R^3}{w\,|\mathcal{R}(R)|}\, I(R,R_{min}) \int_{R_{min}}^{R} \frac{p_0(R_i)\,dR_i}{R_i^3\, I(R_i,R_{min})}$$

$$\frac{W}{F_0} = \int_{R_{min}}^{R_\infty \to \infty} \frac{R^3}{|\mathcal{R}(R)|}\, I(R,R_{min}) \int_{R_{min}}^{R} \frac{p_0(R_i)\,dR_i}{R_i^3\, I(R_i,R_{min})}$$

For shrinking particles

$$\begin{pmatrix}\text{fraction of}\\ \text{exit stream of}\\ \text{size } R\end{pmatrix} = \sum_{\substack{\text{all feed}\\ \text{sizes not}\\ \text{smaller than } R}} \left(\begin{matrix}\text{fraction of}\\ \text{solids leaving}\\ \text{at size } R \text{ and}\\ \text{original feed size } R_i\end{matrix}\right) \Bigg/ \left(\begin{matrix}\text{fraction of}\\ \text{feed}\\ \text{of size } R_i\end{matrix}\right)$$

$$p(R) = \int_{R}^{R_{max}} p_1(R,R_i)\, p_0(R_i)\,dR_i$$

$$p(R) = \frac{F_0 R^3}{w\,|\mathcal{R}(R)|} \int_{R}^{R_{max}} \frac{I(R,R_i)\, p_0(R_i)}{R_i^3}\,dR_i$$

$$I(R,R_i) = \frac{I(R_{max},R_i)}{I(R_{max},R)} = \frac{I(R,R_{max})}{I(R_i,R_{max})}$$

$$p(R) = \frac{F_0 R^3}{w\,|\mathcal{R}(R)|}\, I(R,R_{max}) \int_{R}^{R_{max}} \frac{p_0(R_i)\,dR_i}{R_i^3\, I(R_i,R_{max})}$$

$$\frac{W}{F_0} = \int_{R\to 0}^{R_{max}} \frac{R^3}{|\mathcal{R}(R)|}\, I(R,R_{max}) \int_{R}^{R_{max}} \frac{p_0(R_i)\,dR_i}{R_i^3\, I(R_i,R_{max})}$$