Design Aspects – based on Shrinking core model

Processing:
- Batch – semi batch
- Solids in plug flow
- Solids in backmix flow

Analysis very much simplified if fluid assumed uniform throughout the reactor i.e. excess of fluid & recycling!

Residence time distribution of solids in reactor needed.

For particles of uniform size the derived equations apply directly

Particles of different sizes PFR of solids, uniform gas comp. (excess gas)

sizes R_i $R_{min} \le R_i \le R_{max}$

say R_i identified discretely by screening

Feed in the reactor

$$F = \sum_{R_i = R_{min}}^{R_i = R_{max}} F(R_i) \qquad (1)$$

~~From~~

Residence time in reactor for all solids $\bar{t}_p = \frac{V_R(1-\varepsilon)}{F}$

F – defined as volumetric feed rate of solids, when density does not change appreciably it is equivalent to the mass flow rate

Find $X_B(R_i)$ for any particle size from the derived equations using the knowledge of the rate controlling step.

The mean conversion is then given by:

$$\begin{pmatrix}\text{mean value}\\ \text{of fraction of}\\ \text{unconverted } B\end{pmatrix} = \sum \begin{pmatrix}\text{fraction of}\\ \text{reactant unconverted}\\ \text{in solids of size } R_i\end{pmatrix}\begin{pmatrix}\text{fraction of}\\ \text{feed of}\\ \text{size } R_i\end{pmatrix}$$

$$1-\bar{X}_B = \sum_{R_{min}}^{R_{max}} [1 - X_B(R_i)] \frac{F(R_i)}{F} \qquad 0 \le X_B \le 1$$

or in equivalent form

$$1-\bar{X}_B = \sum_{R(t_p=\tau)}^{R_{max}} [1 - X_B(R_i)] \frac{F(R_i)}{F}.$$

This model might be applied to moving beds

example lecture 5 p 22 ; Problem P2A

Particles of single size backmix flow of solids mixing par

Applicable to fluidized beds without carryover

The length of stay is now different for the particles: some come out early some late

$$\begin{pmatrix}\text{mean value}\\ \text{for fraction}\\ \text{of } B\\ \text{unconverted}\end{pmatrix} = \sum_{\text{all ages}} \begin{pmatrix}\text{fraction of } B\\ \text{unconverted for}\\ \text{particles staying}\\ \text{in the reactor}\\ \text{for time between } t \; t+dt\end{pmatrix}\begin{pmatrix}\text{fraction of}\\ \text{exit stream}\\ \text{which stayed}\\ \text{in reactor}\\ \text{between } t \;\&\; t+dt\end{pmatrix}$$

$$1-\bar{X}_B = \int_0^{\infty} [1 - X_B(t)] E(t)\,dt = \int_0^{\tau} (1 - X_B(t)) E(t)\,dt$$

find from derived equations using the knowledge of the controlling step.

$$E(t) = \frac{1}{\bar{t}} e^{-t/\bar{t}}$$

or

$$1-\bar{X}_B = \int_0^{\tau} [1 - X_B(t)] E(t)\,dt$$

Mixture of particles, CFSTR of solids, uniform gas

Applicable to fluidized beds. (solid density = const)

Since we have steady state: no change in particle size and the exit stream is representative of bed conditions the size distribution in the bed, feed & exit stream must be alike

$$\frac{F(R_i)}{F} = \frac{W(R_i)}{W} \text{ in reactor } = \frac{F_1(R_i)}{F_1}$$

The mean residence time for any size is equal to the mean residence time of solid

$$\bar{t} = \bar{t}(R_i) = \frac{W}{F}$$

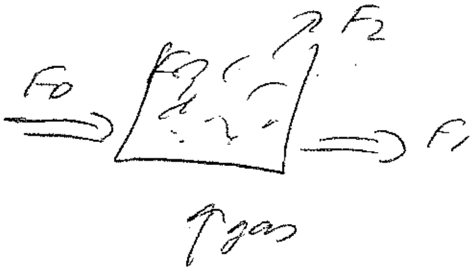
Then mean conversion for size R_i

$$1 - \bar{X}_B(R_i) = \int_0^{\tau(R_i)} [1 - X_B(R_i, t)]\, E(t)\, dt$$

The mean conversion for all solids:

$$1 - \bar{\bar{X}}_B = \sum_{R_{min}}^{R_{max}} [1 - \bar{X}_B(R_i)]\, \frac{F(R_i)}{F}$$

Mixed gas flow & solids flow reactor with Elutriation



$$F_1 = \sum F_1(R_i)$$

$$F_0(R_i) = F_1(R_i) + F_2(R_i)$$

$$F_2(R_i) = \kappa(R_i)\, W(R_i)$$

$$\frac{F_1(R_i)}{W(R_i)} \equiv \frac{F_1}{W}$$

same for all particles
assumed well mixed bed

$$F_0(R_i) = F_1(R_i) + \kappa(R_i) \frac{W}{F_1} F_1(R_i)$$

$$F_0(R_i) = F_1(R_i)\left[1 + \kappa(R_i)\frac{W}{F_1}\right]$$

(*)
$$\boxed{F_1 = \sum F_1(R_i) = \sum \frac{F_0(R_i)}{1 + \kappa(R_i)\frac{W}{F_1}}}$$

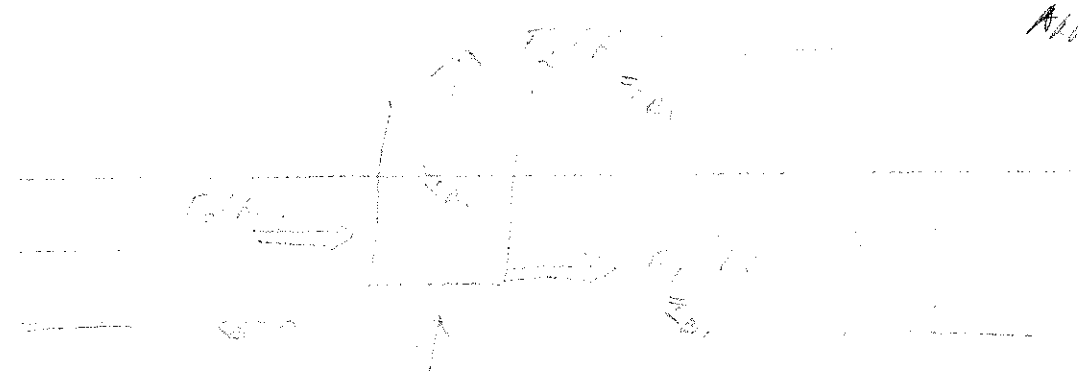
$$\bar{t}(R_i) = \frac{W(R_i)}{F_1(R_i) + F_2(R_i)} = \frac{1}{\frac{F_1}{W} + \kappa(R_i)}$$

$$F_2(R_i) = \kappa(R_i) \frac{F_1(R_i)}{F_1} W = \kappa(R_i)\, W(R_i)$$

$$1 - \bar{X}_B(R_i) = \int_0^{\tau} (1 - X_B(R_i))\, E(t)\, dt$$

$$1 - \bar{\bar{X}}_B = \sum (1 - \bar{X}_B(R_i)) \frac{F_0(R_i)}{F}$$

Approximate [illegible]



All solids have exponential RTD but their mean residence time depends on size

(3) ⇒ size distributions are the same

$\bar{t}(R_i) = \dfrac{1}{\frac{F_1}{W} + \kappa(R_i)}$

as $R_i \downarrow$ $\kappa(R_i)$ ... $\bar{t}(R_i)$

(5)

$= \dfrac{W(R_i)}{F_0(R_i)}$

(*) $F_1(R_i) \overset{\text{from (3)}}{=} \sum \frac{F_1}{W} W(R_i) \overset{\text{from (4)}}{=} \sum \frac{F_1}{W} \dfrac{F_0(R_i)}{\frac{F_1}{W} + \kappa(R_i)} = \sum \dfrac{F_0(R_i)}{1 + \frac{W}{F_1}\kappa(R_i)}$

$F_1(R_i) = \dfrac{F_0(R_i)}{1 + \frac{W}{F_1}\kappa(R_i)}$

(over)

Given: $F_0(R_i)$, W, $x(R_i)$

solve for F_1 by trial and error from (*)

i.e. guess F_1, get $F_1(R_i)$ and sum to get F_1.

Then

$$\begin{pmatrix}\text{fraction of} \\ \text{all solids unconverted}\end{pmatrix} = \sum_{\text{all sizes}} \begin{pmatrix}\text{fraction} \\ \text{of size } R_i \\ \text{unconverted}\end{pmatrix}\begin{pmatrix}\text{fraction of} \\ \text{that size} \\ \text{in the feed}\end{pmatrix}$$

$$1-\bar{\bar{X}}_B = \sum_{R_i=R_{min}}^{R_{max}} [1-\bar{X}_B(R_i)] \frac{F_0(R_i)}{F_0}$$

This means that conversion is defined on total solids:

$$\bar{\bar{X}}_B = \frac{F_{B0} - (F_{1B} + F_{2B})}{F_{B0}}$$

This can be written as:

$$1-\bar{\bar{X}}_B = \frac{F_1(1-\bar{\bar{X}}_{Bov}) + F_2(1-\bar{\bar{X}}_{Bel})}{F_0}$$

where

$$1-\bar{\bar{X}}_{Bov} = \sum_{R_i=R_{min}}^{R_{max}} [1-\bar{X}_B(R_i)] \frac{F_1(R_i)}{F_1}$$

$$1-\bar{\bar{X}}_{Bel} = \sum_{R_i=R_{min}}^{R_{max}} [1-\bar{X}_B(R_i)] \frac{F_2(R_i)}{F_2}$$

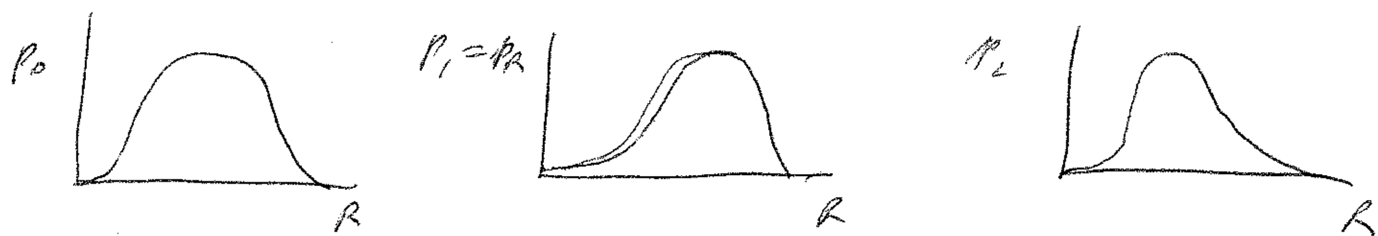
$$\bar{\bar{X}}_{Bov} = \frac{F_1 - F_{1B}}{F_1} \qquad F_{1B} = F_1(1-\bar{\bar{X}}_{Bov})$$

Same problem can be approached from the point of view of cavitation size distribution

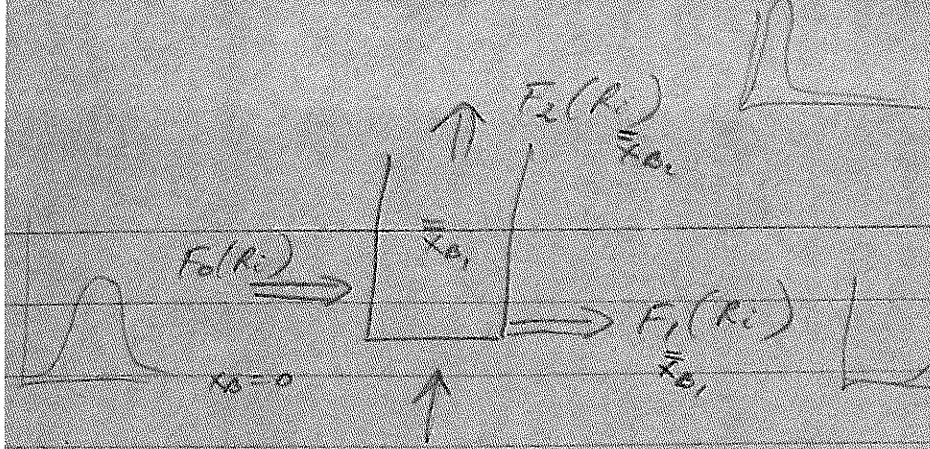
$$P_0(R) \neq P_1(R) = P_R(R) \neq P_2(R)$$

$$F_1(R) = F_1 \, P_1(R) \, dR$$

$$W_1(R) = W \cdot P_R(R) \, dR$$



Approximate $\bar{t}_s \approx$ const



All solids have exponential RTD but their mean residence time depends on size

(1) $F_0 = F_1 + F_2$

(2) $F_0(R_i) = F_1(R_i) + F_2(R_i)$

perfect mixing assumption: requires

(3) $\boxed{\frac{F_1(R_i)}{F_1} = \frac{W(R_i)}{W}}$ ⇒ size distributions are the same

Different $\bar{t}(R_i)$ for various R_i

(4) $\bar{t}(R_i) = \frac{W(R_i)}{F_0(R_i)} = \frac{W(R_i)}{F_1(R_i)+F_2(R_i)} = \frac{1}{\frac{F_1}{W} + \frac{F_2(R_i)}{W(R_i)}}$

(5) $1-\bar{x}_B(R_i) = \int_0^{\tau(R_i)} [1 - x_B(R_i)] \frac{e^{-t/\bar{t}(R_i)}}{\bar{t}(R_i)} \, dt$

$\bar{t}(R_i) = \frac{1}{\frac{F_1}{W} + \kappa(R_i)}$

$1-\bar{\bar{x}}_B = \sum^{R_m} [1-\bar{x}_B(R_i)] \frac{F_0(R_i)}{F_0}$

as $R_i \downarrow$ $\kappa(R_i)$ $\uparrow$

$t(R_i)$ $\downarrow$

$-\frac{d(\text{No of tracer particles})}{dt} = \kappa(\text{number of tracer particles})$

$\kappa \sim \frac{(\text{gas velocity})^4}{(\text{bed height})(\text{particle size})^{2 \text{ to } 3}}$

$\kappa(R_i) = \frac{F_2(R_i)}{W(R_i)}$ $\quad \bar{t}(R_i) = \frac{1}{\frac{F_1}{W} + \kappa(R_i)} = \frac{W(R_i)}{F_0(R_i)}$

(*) $F_1 = \sum^{R_m} F_1(R_i) \overset{\text{from (3)}}{=} \sum \frac{F_1}{W} W(R_i) \overset{\text{from (4)}}{=} \sum \frac{F_1}{W} \frac{F_0(R_i)}{\frac{F_1}{W} + \kappa(R_i)} = \sum \frac{F_0(R_i)}{1 + \frac{W}{F_1}\kappa(R_i)}$

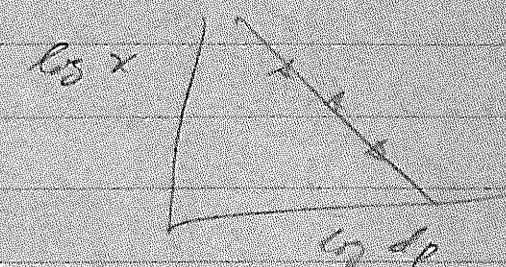
$F_1 = \sum^{R_m} \frac{F_0(R_i)}{1 + \frac{W}{F_1}\kappa(R_i)}$ $\quad$ Trial & error $\quad F_1(R_i) = \frac{F_0(R_i)}{1+\frac{W}{F_1}\kappa(R_i)}$ (over)

However

(12-25) Elutriation experiment

$$-\frac{dP(R_i)}{dt} = \kappa P(R_i)$$

$$-\ln \frac{P(R_i)_{t=t}}{P(R_i)_{t=0}} = \kappa(R_i)\, t$$



$$\kappa(70) = \frac{1}{2} \ln \frac{450}{36} = 1.263$$

$$\kappa(98) = \frac{1}{2} \ln \frac{450}{159} = 0.52$$

$$\kappa(146) = \frac{1}{2} \ln \frac{450}{302} = 0.196$$

slope $= -2.6$

$$\boxed{\kappa(R) = 1.3 \times 10^{4} \left(\mu^{2.6}\,\text{min}^{-1}\right) \underbrace{R}_{\text{microns}}^{-2.6}}$$