CHAPTER 13
Gas-Liquid Systems

This chapter will focus on the systems where a dissolved gas contacts a liquid. In most cases the dissolved gas will undergo a reaction in the liquid phase. Systems where no reaction occurs are known as physical absorption while the systems where reaction occurs are referred to as chemical absorption. First we provide some examples of application areas where gas-liquid systems are encountered in the field of environmental reaction engineering. This is followed by a discussion of the types of equipment used for gas-liquid contacting. The theory of gas-liquid mass transfer and design of such systems then follow.

1. Application Areas

FCC Off Gas Cleaning

FCC (Fluid catalytic cracking) off gas from the catalyst regenerator is a major source of pollutants in oil refineries. These gases contain SO_2 (quantity depends on the sulfur content of feed), and NO_x. These gases are treated in a spray column. The spray is water with the other additives such as alkali, caustic soda or lime. A typical complete flowsheet for the process is shown in Figure 1. The design also removes particulate matter present in the off gas as well as NO_x. The pressure drop is low due to the open design.

FIGURE 1. FCC Off-Gas purification (to be added)

Wet Gas Scrubbing

This process is useful for highly diluted SO_2 from flue gases. The process uses a concentrated sodium sulfite solution. The absorbed SO_2 is thermally released from the bisulfite solution in an evaporator/crystallizer. SO_2 emerges as a concentrated gas containing 95% SO_2 by volume. This can be converted to elemental sulfur, liquid SO_2 or sulfuric acid. A typical flowsheet is shown in Figure 2.

FIGURE 2. SO_2 removal by wet gas scrubbing

H_2S removal and spent caustic oxidation

Caustic solutions are often used to treat H_2S and mercapten containing gas streams. This is an example of a gas-liquid reaction. The sulfides are formed as a result of reaction of caustic solution with the sulfur bearing compounds. This waste stream is highly toxic with high COD. Hence, the spent caustic solution has to be oxidized to form thiosulfate/sulfate. The treated stream is then suitable for wastewater treatment unit. A typical flow chart for spent caustic oxidation is shown in Figure 3. A series of mechanically agitated vessels are used as reactor here.

FIGURE 3. Spent caustic oxidation

Other applications include air/steam stripping of VOC from wastewater, ozone treatment, etc.

2. Common types of reactors.

Common reactors and their brief merits and demerits are discussed below:

1. Packed columns: Liquid and gas flow over a column filled with packings with the flow usually in a countercurrent manner. The design of such equipment need pressure drop calculations and establishment of flooding limits in addition to a mass transfer and reaction analysis. A column is said to flood when the gas flow is sufficient to blow the liquid against gravity and the column can not be operated at these flow rates. Gas pressure drop is usually low with packed columns. A variety of packings are available to suit the system requirements. The system is not suitable if the gas contains suspended solids.
2. Bubble columns: Mixing is caused by the gas flow alone and no additional energy input is necessary. The advantages are simplicity in construction, no moving parts and minimum energy requirements compared to other contactors. Disadvantage is the high pressure drop of the gas due to hydrostatic head of the liquid. Considerable backmixing of the liquid occurs here. Liquid hold up is high in these reactors and hence these are suitable for slow reactions in the liquid.
3. Mechanically agitated contactors: the liquid is agitated by an impeller. System has large liquid holdup and consumes considerable power. Very efficient mixing, moving parts require maintenance. Very suitable for small scale systems in pollution prevention and also for batch or near batch operations. As an example

we note that the black liquor oxidation (Na_2S solution) is carried out in a series of stirred vessels.

4. Spray columns: Liquid is dispersed in the form of fine spray into an empty column with gas upflow. The system is suitable for highly soluble gases, can also handle gas streams containing suspended impurities. Ultra low pressure drop for the gas phase. Some limitations are energy input to spray the liquid; low mass transfer efficiency due to bypassing and droplet coalescence and liquid entrainment in the gas phase. As an example, we note that the spray columns are used for SO_2 removal from FCCU flue gases because the gas stream contains suspended solids and also because SO_2 has a high solubility in the aqueous phase.

3. Fundamentals of gas-liquid mass transfer

The essential concepts to describe the gas-liquid mass transfer are:

(i) Henry's law

(ii) The two film model

Henry's law:

Henry's law describes the equilibrium concentration in the liquid exposed to a gas containing a solute. Various forms are used for Henry's law. These were summarized in Chapter 3 and students should recall them at this stage.

Two common definitions will be used here.

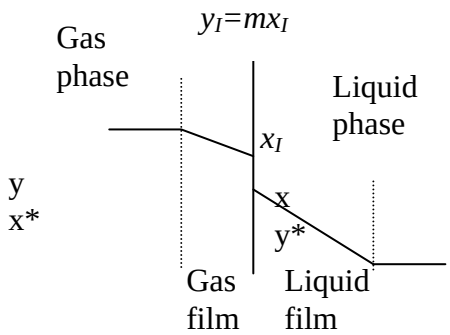
(1) $y = mx$ where m is the ratio of mole fraction in the gas to that liquid at equilibrium.

(2) $P = HC$ where H is the ratio of gas phase partial pressure to concentration in the liquid at equilibrium. The unit for H is then $Pa.m^3/mole$ or $atm.m^3/mole$.

Two film model:

The basic postulate is the existence of stagnant boundary layers on either side of the gas-liquid interface. Hence the terminology "two" film and these are referred to as gas film and liquid film. Concentration gradients are assumed to exist only in the films. Outside the film, the phases are assumed to be at uniform concentration and are referred to as gas bulk and liquid bulk. A mass transfer driving force is defined for transport in the gas film

liquid films individually. In the next section, we show how to combine this into an "overall" mass transfer coefficient.



where x* = hypothetical equilibrium value corresponding to y
= y / m

x_I, y_I are the interfacial mole fractions on the liquid side and gas side of the interface respectively. Interface is assumed to be in equilibrium. The mole fractions in the bulk gas are denoted as y and that in the bulk liquid as x.

$y_I = mx_I$ by Henry's law assuming interface to be in equilibrium

N_A = mass transfer rate per area = $k_y(y - y_I) = k_x(x_I - x)$

where k_x and k_y are mass transfer coefficients based on mole fraction driving force.

Overall coefficient is defined as $N_A = K_x(x^* - x)$

we find from the above equations:

$$\frac{1}{K_x} = \frac{1}{mk_y} + \frac{1}{k_x}$$

Alternative way of writing the transfer rate is:

$$N_A = K_y(y - y^*)$$

y^* = mx = hypothetical gas phase value corresponding to the bulk liquid.

$$\frac{1}{K_y} = \frac{1}{k_y} + \frac{m}{k_x}$$

Note that $N_A = K_x\left(x^* - x\right) = K_x\left(\frac{y}{m} - \frac{y^*}{m}\right) = \frac{K_x}{m}\left(y - y^*\right)$

Hence $K_y = \frac{K_x}{m}$

Mass transfer coefficients can also be based on concentration driving force:

$N_A = k_L\left(C_I^* - C_L\right)$

$k_x = k_L C_T$

C_T = total molar concentration in the liquid

Mass transfer coefficients for the gas phase is often based on gas phase partial pressure.

$N_A = k_g\left(p - p_I\right)$

$k_y = k_g P$

The overall coefficient K_L based on concentration driving force is then given as

$$\frac{1}{K_L} = \frac{1}{k_L} + \frac{1}{k_G H}$$

Similarly the coefficient K_G based on gas phase partial pressure as a driving force is given as

$$\frac{1}{K_G} = \frac{1}{k_g} + \frac{H}{k_L}$$

where H is in units of Pa m^3/mole.

Various definitions and inter-relationships should be noted.

Example 1:

At a point in a gas absorption column, we have air and water in contact. The solute is H_2S and the partial pressure of H_2S in the air is 5% and the concentration of H_2S dissolved in water is zero. Find the rate of mass transfer if the gas side and liquid side mass transfer coefficients are as follows:

$k_L = 2\text{x}10^{-4}$ m/s

$k_g = 6\text{x}10^{-6}$ mole/m^2 Pa s

Also find the overall transfer coefficient and express in terms of (a) an overall partial pressure driving force and (b) a liquid phase concentration driving force.

Solution:

Henry's law solubility coefficient of H_2S is needed and the value used here is 1950 Pa m^3/mol.

Overall transfer coefficient based on partial pressure driving force is K_G and is calculated as

$$\frac{1}{K_G} = \frac{1}{k_g} + \frac{H}{k_L}$$

K_G =1x10^{-7} mole/Pa m^2 s

Rate of absorption = $K_G P_G$ = 1 x 10^{-7} x 0.05 x 1 x 10^5

= 5.09x10-4 mole/m^2 s

Similarly

K_L =1.95 x10^{-4} m/s which is close to k_L.

Hence it is seen that the major resistance to transport lies in the liquid side of the film for this case.

<u>Gas-liquid Reaction : First order reactions.</u>

Model equations for transport in the film.

$$D_A \frac{d^2 A}{dx^2} = k_1 A \qquad (1)$$

where k_1 is the rate constant for the reaction. It is convenient to normalize the equation by defining the following dimensionless variable.

$$a = \frac{A}{A^*} \quad , \quad y = \frac{x}{\delta}$$

Dimensionless representation of Eq (1) is then

$$\frac{d^2 a}{dy^2} = Ha^2 \bullet a$$

where Ha^2 is $\delta^2 \frac{k_1}{D_A}$

Ha is an important dimensionless group for gas – liquid reactions and is known as the Hatta number. The physical significance of the Hatta number can be gauged by expressing it as a rate of two terms constants

$$Ha^2 = \frac{\delta^2 / D_A}{1/k_1} = \frac{t_D}{t_R} = \frac{diffusion\ \ time}{reaction\ \ time}$$

Thus a large value of Hatta number implies that diffusion time is much larger than reaction time and hence we would expect considerable reaction to take place in the film

itself. The concentration will drop significantly in the film for large values of Hatta number. The solution to Eq. (1) will refect these effects and is presented below:

We use the following boundary conditions for Eq(1).

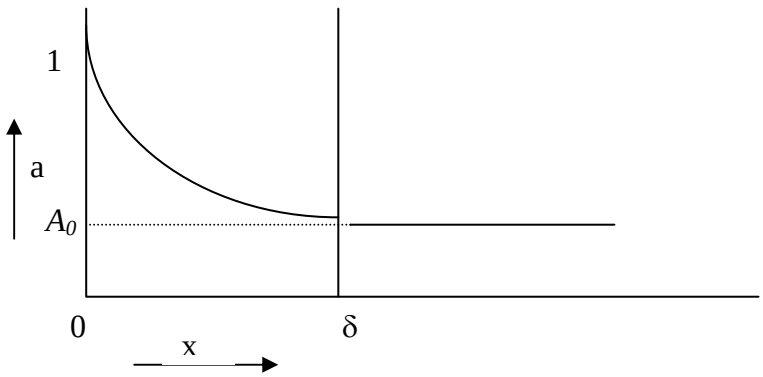
At $x = 0, A = A^*$ or $a = 1$

At $x = \delta, A = A_0$ or $a = a_0$

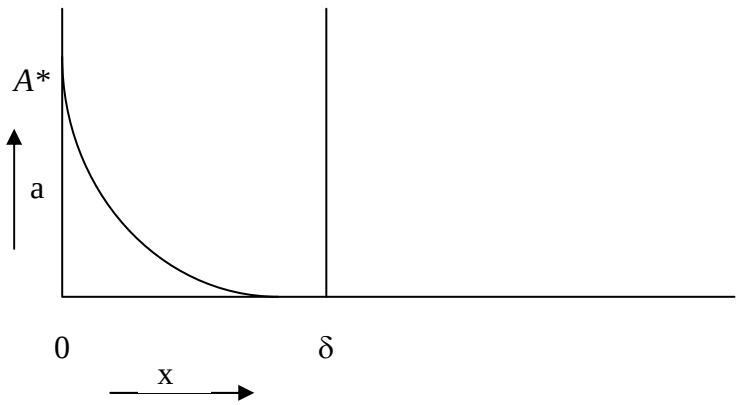
where A_0 is the concentration of dissolved A in the bulk liquid. The solution is then given as:

$$a = \frac{\sinh Ha(1-y)}{\sinh Ha} + a_0 \frac{\sinh Ha\, y}{\sinh Ha}$$

Typical concentration profiles are shown in Fig 4 as a plot of 'a' vs y for two values of Ha. For small values of Ha, some unreacted gas diffuse into the bulk. For large Ha all the dissolved gas reacts in the film.



(a) Solution for small values of Ha



(b) Solution for large values of Ha

FIGURE 4. First-order reaction, film model (a) Solution of small Ha (b) Solution of large Ha

The flux of A at the interface is obtained by applying Fick's Law at the interface.

$$R_{Ai} = -D_A \left(\frac{dA}{dx} \right)_{x=0}$$

and is given by

$$R_{Ai} = k_L \left(A^* - \frac{A_0}{\cosh Ha} \right) \frac{Ha}{\tanh Ha} \qquad \text{(A)}$$

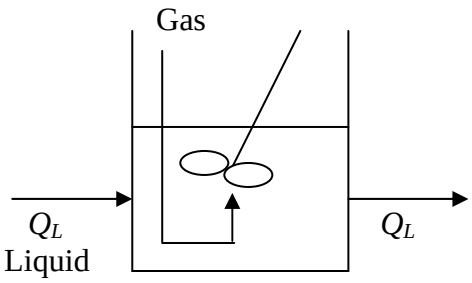
Similarly the flux into the bulk liquid is

$$R_{A\delta} = -D_A \left(\frac{dA}{dx} \right)_{x=\delta}$$

and the result is:

$$R_{A\delta} = \frac{k_L \, Ha}{\tanh Ha} \left(\frac{A^*}{\cosh Ha} - A_0 \right) \qquad \text{(B)}$$

The bulk liquid concentration has to be obtained by consideration of the processes in the bulk liquid and cannot be set arbitrarily. To illustrate how this is calculated, consider absorption in a completely backmixed liquid. The system can be schematically represented as shown in the following figure.



Assume that a pure gas is bubbled into the liquid so that there is no change in the gas phase partial pressure of reactant *A*. The interfacial concentration is therefore maintained at A^*. Liquid enters with a dissolved gas concentration A_{in} (usually zero) and leaves with a concentration A_{out}. The liquid volumetric flow rate is Q_L. The volume of the system is V and a_{gl} is the gas-liquid transfer area per unit volume of contactor.

Then writing a mass balance for bulk liquid yields:

In - Out = Reacted

$$R_{A\delta} a_{gl} V + Q_L A_{in} - Q_L A_{out} = \nu \varepsilon_l k_1 A_{out}$$

Equation can be made dimensionless by defining the following parameters:

$$\alpha_{gl} = k_L a_{gl} V / Q$$

$$\alpha_r = \varepsilon_l V k_1 / Q_L$$

Also $R_{A\delta}$ can be substituted from Eq(B) yielding

$$\frac{Ha}{\sinh Ha}\alpha_{gl} - \frac{Ha}{\tanh Ha}\alpha_{gl} a_0 - a_0 - \alpha_r a_0 = 0 \qquad \text{(C)}$$

where we set $A_{in} = 0$ for simplification.

Solution of Eq (C) gives the following equation for bulk liquid concentration of *A*.

$$a_0 = \frac{Ha\alpha_{gl}}{(\sinh Ha)(1+\alpha_r) + Ha\alpha_{gl}\cosh Ha}$$

For example, $Ha = 1$, $\alpha_{gl} = 1.5$ and $\alpha_r = 10$

We find $a_0 = 0.09841$

For large Ha, $a_0 \rightarrow 0$ indicating that film reactions dominate.

<u>Gas – Liquid Reactions, Second order reactions.</u>

For the reaction scheme A + ν B ➔ products, the differential equations in the film can be set up as follows

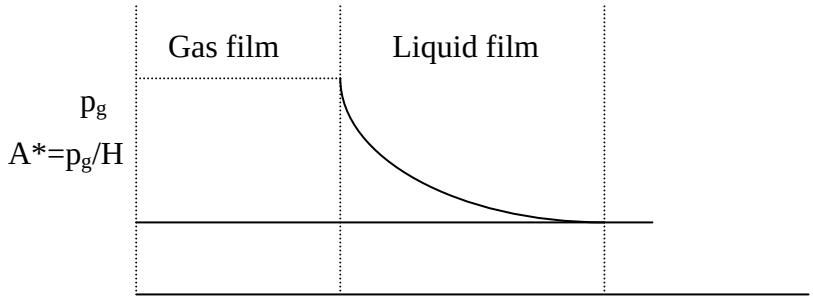
$$D_A \frac{d^2 A}{dx^2} = k_2 \text{ AB}$$

$$D_B \frac{d^2 B}{dx^2} = \nu k_2 \text{ AB}$$

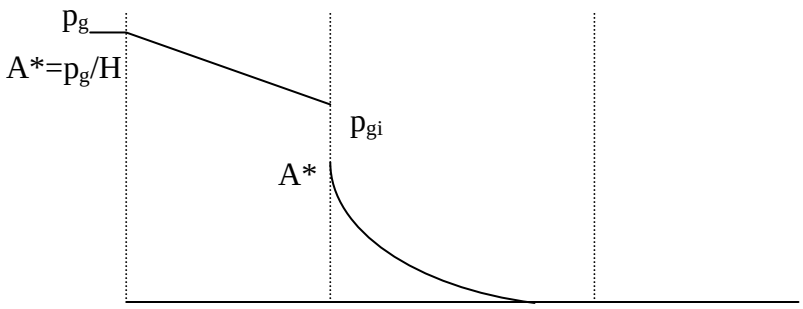
We introduce the following dimensionless variables

$$a = \frac{A}{A^*} \quad , \quad y = \frac{x}{\delta} \quad , \quad b = \frac{B}{B_0}$$

The value of A* depends on the magnitude of the gas film resistance. The two cases are shown in the following figure



Case 1: No gas side resistance



Case 2: With gas film resistance

FIGURE x: concentration drop in the gas film showing the effect of gas film resistance.

For case (1), A^* is the interfacial concentration while for case (2) A^* is equivalent to bulk gas concentration. $A^* = p_g / H$ where p_g is the partial pressure of A in the bulk gas.

Enhancement Factor is defined as

$$\phi = \frac{R_{Ai}}{k_L A^*} = \frac{Actual\ rate\ of\ absorption}{Maximum\ rate\ of\ physical\ absorption}$$

and it is a measure of rate of absorption of A into the liquid.

<u>Interfacial Concentration of B</u>

Equations for diffusion and reaction for transport of A and B can then be reduced to dimensionless form as;

$$\frac{d^2a}{dy^2} = Ha^2ab \qquad (1) \qquad y = x/\delta$$

$$\frac{d^2b}{dy^2} = \frac{Ha^2ab}{q} \qquad (2) \qquad q = \frac{D_B B_0}{zD_A A^*}$$

The parameter Ha^2 is now defined as:

$$Ha^2 = \delta^2 k_2 B_0 / D_A$$

The additional parameter is now 'q' which is an effective concentration ratio of reactant B to A. The relative magnitude of Ha and q determines the concentration profiles in the film. Complete solution can be done only numerically and here we present approximate analytical solutions.

First, we note that if the interfacial concentration of B denoted as B_i is nearly the same as B_o, the bulk concentration, then the problem reduces to a first order reaction. The enhancement factor is then given as

$$\phi = \frac{Ha}{\tanh Ha}$$

This will be referred to as a "no depletion" case or the fast pseudo-first order case.

Combining Eqs(1) and (2) we find that 'a' and 'b' are related by:

$$\frac{d^2 a}{dy^2} = q \frac{d^2 b}{dy^2} \qquad (3)$$

Also $R_{Ai} = -D_A \left(\frac{dA}{dx} \right)_{x=0}$

or

$$\frac{R_{Ai}}{\frac{D_A A^*}{\delta}} = -\left(\frac{da}{dy} \right)_{y=0}$$

Hence the enhancement factor is

$$\phi = -\left(\frac{da}{dy} \right)_{y=0}$$

Integrating Eq (3) once we have

$$\left(\frac{da}{dy} \right) = q \frac{db}{dy} + c_1$$

Using the boundary values at y = 0, we obtain

$- \phi = 0 + c_1$

Hence $c_1 = - \phi$

Integrating a second time

$a = qb - \phi y + c_2$

At y = 1

$0 = q - \phi + c_2$ or $c_2 = \phi - q$

Here we assume that the reaction in the bulk is rapid so as to keep a zero concentration of dissolved a.

Therefore

$a = qb - \phi y + \phi - q$

The interfacial concentration of B is obtained for case (1) (No gas film resistance) directly by substituting a = 1, y = 0 and rearranging the above equation. The result is

$$b_i = 1 + \frac{1}{q} - \frac{\phi}{q} \qquad (4)$$

or in terms of dimensional variables

$$B_i = B_0 + \frac{vD_A}{D_B} A^* - \frac{vD_A}{D_B} \frac{R_{Ai}}{k_L}$$

Hoftizer-van Krevelen, Hikita and Asai suggest that the pseudo first order equation can still be used in presence of depletion provided the Ha parameter is now based on B_i and not B_0

$$\frac{D k_2 B_i}{k_L^{\ 2}} = \frac{D k_2 B_0}{k_L^{\ 2}} \frac{B_i}{B_0}$$

Ha^2 (depletion) = $Ha.b_i$

Therefore the enhancement factor in presence of depletion can be calculated as

$$\phi = \frac{Ha\sqrt{b_i}}{\tanh(Ha\sqrt{b_i})} \qquad (5)$$

Simultaneous solution of (4) and (5) is necessary for this case. This procedure is again quite general except for the assumption of fast reaction in the bulk and some error associated with the Hikita-Asai approximation for the rate.

Eq. (4) and (5) can be combined to give an expression for b_i.

$$b_i = \frac{1}{q}\left(q + 1 - Ha\sqrt{b_i} / \tanh\left(Ha\sqrt{b_i}\right)\right) \qquad (6)$$

Calculation procedure is illustrated by the following problem.

Problem 2

Lo-Cat is a gas treating process that removes hydrogen sulfide out of gas streams and converts to solid sulfur. In this process, chelated iron Fe(3+) EDTA solution is used as a reactant. A simple representation of the reaction is

$$H_2S_{(g-l)} + 2Fe^{+++} \rightarrow 2H^+ + S^0 + 2Fe^{++}$$

Find the rate of absorption of H_2S for the following process conditions. Partical pressure of H_2S in the gas phase = 0.05 atm, concentration of Fe^{+++} in the liquid = 60 mol/m^3

The kinetic parameters and physico – chemical properties are as follows:

$D_A = 1.44 \times 10^{-9}$ m^2/s $D_B = 0.54 \times 10^{-9}$ m^2/s

Henry's law constant = 1950 Pa m^3/mol

Reaction rate constant = 9 m^3/mol s

The mass tranfer coefficient $k_L = 2 \times 10^{-4}$ m/s for liquid film

Ignore the effect of gas side resistance as an approximation.

Solution

$$A^* = \frac{p_g}{H} = \frac{0.05\ atm \cdot \frac{10^5\ Pa}{1\ atm}}{1950\ Pa\ m^3/mol} = 2.56\ \text{mol/m}^3$$

$$\text{Ha} = \frac{\sqrt{D_A k_2 B_0}}{k_L} = 4.4$$

$$q = \frac{D_B B_0}{\nu D_A A^*} = 4.3945$$

(note $\nu = 2$ here) All units are in S.I. here and therefore consistent. Solving Eq. (6) for b_i, iteratively one finds $b_i = 0.5085$. Hence the concentration of Fe^{+++} at the interface is only 50% of that in the bulk liquid.

Hence the enhancement factor is

$$\phi = \frac{(Ha\sqrt{b_i})}{\tanh(Ha\sqrt{b_i})} = 3.12$$

Rate of absorption of H_2S $= \phi\, k_L\, A^*$

$= 3.12 \times 2 \times 10^{-4} \times 2.56 \quad = 0.0016\ \text{mol/m}^2\,\text{s}$

Regimes of absoprtion

Consider eq.(6) which can be expressed as

$$b_i = \frac{1}{q}\left(q + 1 - Ha\sqrt{b_i}\right) \text{ as an approximation if } Ha\sqrt{b_i} > 3.$$

From equation (2) we find that as Ha increases in comparison with 1 + q, the depletion increases significantly. Based on this the following two criteria are suggested:

$Ha \;>\; 10\,(1+q)$ complete depletion ($b_i = 0$)

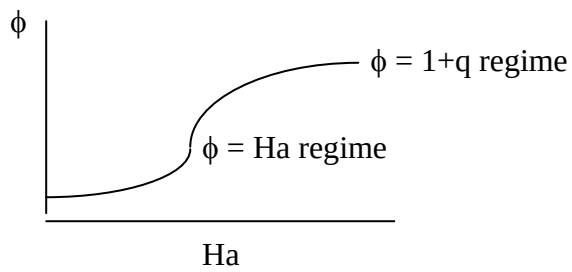
Here ϕ tends to an asymptotic value of 1+q.

$Ha \;<\; \dfrac{(1+q)}{2}$ no deplection ($b_i \cong 1$)

(pseudo first order reactions)

Here ϕ tends to Ha.

A typical Ha vs ϕ for a given q then looks like this:



Effect of gas film resistance.

The boundary condition at x = 0 is now represented as

$$k_g\,(p_g - p_{gi}) \;=\; -D_A\left(\frac{dA}{dx}\right)_{x=0} \qquad (6)$$

We have $p_{gi} = H(A)_{x=0}$

Let $A^* = \dfrac{p_g}{H}$ = equilibrium concentration of A corresponding to bulk gas partial pressure.

Eq (6) can be expressed in dimensionless form as

$$Bi_g\,(1-a_{y=0}) \quad = \quad -\left(\frac{da}{dy}\right)_{y=0}$$

where

$$Bi_g \quad = \quad \frac{k_g H}{k_L}$$

The parameter q is now defined as $\dfrac{D_B B_o}{z D_A A^*}$

The enhancement factor is now $\phi = \dfrac{R_A}{k_L A^*}$

Then the following set of equations van be derived for calculating b_i and ϕ

$$b_i = 1 + \frac{1}{q} - \frac{\phi}{q}\,[1 + \frac{1}{Bi_g}]$$

and

$$\phi \quad = \quad \frac{Ha\sqrt{b_i}\,\coth Ha\sqrt{b_i}}{1 + \dfrac{Ha\sqrt{b_i}\,\coth Ha\sqrt{b_i}}{Bi_g}}$$

Problem 3

For the H_2S removal case shown in Problem 1, indicate the magnitude if gas side resistance of the value of k_g is $6\text{x}10^{-6}$ mole/m^2 Pa s.

Solution:

Rearranging Eq.(6) we find

$$\Delta p_g = \frac{R_A}{k_g}$$

where Δp_g is partial pressure drop in the gas film. Using the same value of R_A computed from Problem 1 as an approximation, we find

Δp_g = 0.0016 / $6\text{x}10^{-6}$ = 266 Pa.

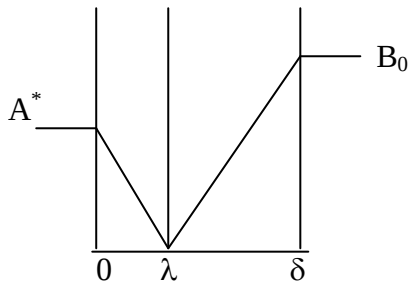
The bulk gas partial pressure of H_2S is 5000 Pa and the interfacial partial pressure is expected to be 4734 (5000-266). Hence the gas side resistance can be neglected for this problem.

<u>Instantaneous Reactions</u>

Instantaneous reactions can be analyzed directly as shown in this section.

For the case of complete depletion ϕ approaches an asymptotic value of (1+q) which can be proved as follows.

Assume now that the reaction takes place a reaction plane that separates A and B as shown in the following figure.



Equating the fluxes at the reaction plane

$$\frac{1}{z}\frac{D_B B_0}{\delta - \lambda} = \frac{D_A A^*}{\lambda} = R_{Ai}$$

Eliminating

λ we have $\lambda = \delta \dfrac{1}{1+q}$

$$\phi = \frac{R_{Ai}}{D_A A^*/\delta} = \frac{\delta}{\lambda} = 1 + q$$

Problem 4:

Reaction of H_2S with amines can be treated as instantaneous as it is a proton transfer process.

$H_2S + R_2NH = HS^- + R_2NH_2 +$

secondary

amine

Calculate the rate of absorption H_2S if the gas is at 20 atm, contains 1% H_2S while the amine solution contains 0.25 M amine. Diffusivity of H_2S is 1.5 times the diffusivity of the amine, $k_L a_B = 0.030\ s^{-1}$.

$H_A = 10^{-4}$ mol/cm^3 atm

Solution

$A^* = (10^{-4})\ (20)\ (0.01) = 2 \times 10^{-5}$ mol/cm^3

$$q = \left(\frac{1}{1.5}\right)\frac{2.5\times 10^{-4}}{2\times 10^{-5}} = 8.33 \quad or \quad \phi = 9.33$$

$R'_A = 9.33\ (0.030)\ (2 \times 10^{-5})$

$$= 5.598 \times 10^{-6}\ \frac{mol}{(cm^3\ volume)\ (s)}$$

Note that the gas side mass transport resistance is neglected here which is not a realistic assumption for this case.

Mixing cell model for gas-liquid reactions.

A. Co-current flow case

Gas phase balance

$$Q_G\,(C_{A,k-1} - C_{A,k}) = a_{gl}\ V_{cell}\ (N_A)_{z=0} \qquad (1)$$

where N_A is the flux of gas A into the liquid at the gas-liquid interface (z=0)

Liquid phase balance

$$Q_L\,(C_{B,k-1} - C_{B,k}) = -a_{gl}\ V_{cell}\ (N_B)_{z=\delta} \qquad (2)$$

Where $(N_B)_{z=\delta}$ is the flux of B at the end of the film, $z=\delta$

In Eq (2), we neglect reaction in the bulk liquid and assume that all the reaction is complete in the film itself.

By stoichiometry

$$-(N_B)_{z=\delta} = \upsilon (N_A)_{z=0} \qquad (3)$$

since the amount of B diffusing into δ should balance the amount of A reacted in the film. Again note that there is no A diffusing into the bulk liquid for fast reactions.

It is convenient to represent the model in terms of some basic dimensionless parameters.

Let $a_g = \frac{C_{A,g}}{C_{Ag,0}}$ and $b_\ell = \frac{C_{B,L}}{C_{B\ell,0}}$ for any cell represent the dimensionless concentration in the exit of that cell.

$(N_A)_{z=0}$ for any cell can be expressed in terms of the enhancement factor, E_k for that cell

$N_A = E_k\, k_L\, A_g^*$

where A_g^* is the equilibrium gas concentration for the cell

$$A_g^* = \frac{pg}{H_A} = \frac{C_{Ag} RT}{H_A}$$

Let $\frac{H_A}{RT} = H_{AC}$ concentration based Henry's law constants

Also define the following dimensionless group.

$$\alpha_{g\ell} = \frac{k_L a_{gl} L}{u_l} \quad and \quad \gamma = \frac{u_l}{u_g H_{AC}}$$

Eq (1) then reduces to

$$a_{g,k-1} - a_{g,k} = \frac{\alpha_{gl}}{N} \gamma\, E(k)\, a_{g,k}$$

Combining Eq (2) and (1) using Eq (3) we obtain

$Q_L\,(C_{B,k-1} - C_{B,k}) = \upsilon\, Q_G\,(C_{A,k-1} - C_{A,k})$

which represents an overall mass balances for a given cell. Rearranging

$$b_{l,k} = b_{l,k-1} - \frac{\upsilon Q_c}{Q_L} \frac{A_{g,m}}{B_{l,m}} \left(C_{A,k-1} - C_{A,k}\right)$$

Let $Q_{BA} = \frac{B_{l,m}}{\upsilon A_{g,m}^*}$

Then the above equation can be represented as

$$b_{l,k} = b_{l,k-1} - \frac{1}{Q_{BA}\,\gamma} (a_{g,k-1} - a_{g,k})$$

Finally the local enhancement factor E(k) needs to be calculated based on local values of the dimensionless groups needed in the film model

E(k) = E (Ha(k), q(k), B_{iG})

where

$$Ha(k) = Ha_0 \sqrt{b_k}$$

$$q = Q \frac{b_k}{a_{g\,k}} \cdot \frac{D_B}{D_A}$$

where Ha_0 is based on inlet liquid concentration

B.Counter-current flow

The schematic of the k the cell is shown in the figure. The concentration entering a cell are denoted by the subscript, k-1 while those leaving the cell by subscript k

The mass balance for each cell follows the similar line as for the co-current flow.

$$a_{g,k-1} - a_{g,k} = \frac{\alpha_{gl}}{N} \gamma E\, a_{g,k}$$

$$b_{\lg,k-1} - b_{l,k} = \frac{\alpha_{gl}}{N} \frac{1}{Q_{BA}} E\, a_{g,k}$$

where E is now based on $a_{g,k}$ and $b_{l,k}$ values.

It is easier to eliminate the commmon term $\left(\frac{\alpha_{gl}}{N}\right) E\, a_{g,k}$ to get on expression for $b_{l,k}$ for each cell.

$$b_{l,k} = b_{l,k-1} - \frac{1}{Q_{BA}\,\gamma}(a_{g,k-1} - a_{g,k})$$

Calculation procedure requires an iterative scheme since all the incoming concentrations are not known in contrast to the co-current case.