Fluidized Beds

Reaction of solids Particles of Changing Size

Ass

- uniform gas composition
- uniform particle density
- CSTR of solids, steady state, isothermal

Mass balance on reactor on size range R to $R+\Delta R$

$$\begin{pmatrix}\text{solids}\\ \text{in by}\\ \text{feed}\end{pmatrix} - \begin{pmatrix}\text{solids}\\ \text{out by}\\ \text{outflow}\end{pmatrix} - \begin{pmatrix}\text{solids}\\ \text{out by}\\ \text{carryover}\end{pmatrix} + \begin{pmatrix}\text{solids growing}\\ \text{into the interval}\\ \text{from a smaller}\\ \text{size}\end{pmatrix}$$

$$- \begin{pmatrix}\text{solids growing}\\ \text{out of the interval}\\ \text{to larger size}\end{pmatrix} + \begin{pmatrix}\text{solid mass}\\ \text{generation within}\\ \text{the interval by growth}\end{pmatrix} = 0$$

growth Rate $\qquad \mathcal{R} = \dfrac{dR}{dt}$

Particle size distribution in bed and outflow $p(R)$

Particle size distribution in ~~outflow~~ feed $p_0(R)$

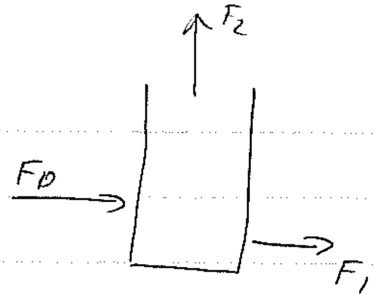
Particle size distribution in elutriated stream $p_2(R)$

$$\begin{pmatrix}\text{mass generation}\\ \text{within the size}\\ \text{interval}\end{pmatrix} = \rho_s \begin{pmatrix}\text{number of}\\ \text{particles in}\\ \text{the interval}\end{pmatrix}\begin{pmatrix}\text{volume increase}\\ \text{of a particle in}\\ \text{time } \Delta t\end{pmatrix}$$

$$= \rho_s \frac{W p(\bar{R})\Delta R}{\rho_s \frac{4}{3}\pi \bar{R}^3} \frac{dV}{dt} \times \Delta t$$

$V = \frac{4}{3}\pi \bar{R}^3$

$$= \frac{3 W p(\bar{R})\, \mathcal{R}(\bar{R})\, \Delta R}{\bar{R}}$$



$$R < \bar{R} < R + \Delta R$$

$$F_0\, p_0(\bar{R})\Delta R - F_1\, p(\bar{R})\Delta R - F_2\, p_2(\bar{R})\Delta R + W p(R)\,\mathcal{R}\Big|_{R} - W p(R)\mathcal{R}\Big|_{R+\Delta R}$$

$$+ \frac{3W\, p(\bar{R})\,\mathcal{R}(\bar{R})\,\Delta R}{\bar{R}} = 0$$

$$\lim_{\Delta R\to 0}\left(F_0\, p_0(\bar{R}) - F_1\, p(\bar{R}) - F_2\, p_2(\bar{R})\right) - W \lim_{\Delta R\to 0}\frac{(p\mathcal{R})_{R+\Delta R} - (p\mathcal{R})_R}{\Delta R}$$

$$+ \lim_{\Delta R\to 0}\frac{3W\, p(\bar{R})\,\mathcal{R}(\bar{R})}{\bar{R}}$$

$$F_0\, p_0(R) - F_1\, p(R) - F_2\, p_2(R) - W\frac{d(p\mathcal{R})}{dR} + \frac{3W p\mathcal{R}}{R} =$$

Now remember that in each a bed

$$\bar{t}(R_i) = \frac{W(R_i)}{F_0(R_i)} = \frac{W(R_i)}{F_1(R_i)+F_2(R_i)} = \frac{1}{\frac{F_1(R_i)}{W(R_i)} + \frac{F_2(R_i)}{W(R_i)}}$$

$$\frac{F_1(R_i)}{W(R_i)} = \frac{F_1}{W} \quad \text{and} \quad \frac{F_2(R_i)}{W(R_i)} = \kappa(R_i)$$

$$\bar{t} = \frac{1}{\frac{F_1}{W} + \kappa(R_i)} \quad \text{but} \quad \frac{F_2(R_i)}{W(R_i)} = \frac{p_2(R_i)\, F_2}{p(R_i)\, W}$$

$$p_2(R) = \frac{W}{F_2}\kappa(R)\, p(R) \qquad (A) \qquad (45)$$

$$\boxed{F_0\, p_0(R) - F_1\, p(R) - W\kappa(R)\,p(R) - W\frac{d}{dR}(p\mathcal{R}) + \frac{3W p\mathcal{R}}{R} = 0}$$

(1)

$e^{a+b+c} = e^a e^b$

$$F_1 + F_2 - F_0 = \begin{pmatrix} \text{total solid} \\ \text{mass production} \\ \text{in the bed} \end{pmatrix} = \int_{R_{min}}^{R_{max}} \frac{3W p(R)\mathcal{R}(R)\,dR}{R} \quad (2)$$

$$F_1 + F_2 - F_0 > 0 \quad \text{particle growth} \quad (A4)$$
$$< 0 \quad \text{particle shrinkage}$$

Solution (Single Size Feed)

$$F_0\,\delta(R-R_i) - F_1 p - W\kappa p \underbrace{- W\frac{d}{dR}(p\mathcal{R})}_{-Wp\frac{d\mathcal{R}}{dR} - W\mathcal{R}\frac{dp}{dR}} + \frac{3Wp\mathcal{R}}{R} = 0$$

$$-W\mathcal{R}\frac{dp}{dR} - \left[W\frac{d\mathcal{R}}{dR} + F_1 + W\kappa - \frac{3W\mathcal{R}}{R}\right]p = -F_0\,\delta(R-R_i)$$

$$\frac{dp}{dR} + \left[\frac{1}{\mathcal{R}}\frac{d\mathcal{R}}{dR} + \frac{F_1}{W\mathcal{R}} + \frac{\kappa}{\mathcal{R}} - \frac{3}{R}\right]p = \frac{F_0\,\delta(R-R_i)}{W\mathcal{R}}$$

$$\frac{d}{dR}\left[e^{\int_{R_i}^{R}[\;]dR}\,p\right] = e^{\int_{R_i}^{R}[\;]dR}\,\frac{F_0}{W\mathcal{R}}\,\delta(R-R_i)$$

$$e^{\int_{R_i}^{R}[\;]dR}\,p(R) - 0 = \frac{F_0}{W|\mathcal{R}(R_i)|}\;\; e$$

p(R) clean't be negative

$$\int_{R_i}^{R}\left[\frac{d\ln|\mathcal{R}|}{dR} + \frac{F_1/W + \kappa}{\mathcal{R}} - \frac{3}{R}\right]dR =$$

$$= \ln\left|\frac{\mathcal{R}(R)}{\mathcal{R}(R_i)}\right| - 3\ln\frac{R}{R_i} + \int_{R_i}^{R}\frac{F_1/W + \kappa}{\mathcal{R}}\,dR$$

$$e^{\int_{R_i}^{R}[\;]dR} = \left|\frac{\mathcal{R}(R)}{\mathcal{R}(R_i)}\right|\left(\frac{R_i}{R}\right)^3 e^{\int_{R_i}^{R}\frac{F_1/W + \kappa(R)}{\mathcal{R}(R)}dR}$$

$$p(R) = \frac{F_0}{W\cancel{|\mathcal{R}(R_i)|}}\,\frac{\cancel{|\mathcal{R}(R_i)|}}{|\mathcal{R}(R)|}\left(\frac{R}{R_i}\right)^3 e^{-\int_{R_i}^{R}\frac{F_1/W + \kappa(R)}{\mathcal{R}(R)}dR}$$

$$p(R) = \frac{F_0}{w\,|\mathcal{R}(R)|}\left(\frac{R}{R_i}\right)^3 \underbrace{e^{-\int_{R_i}^{R} \frac{F_1/w + \kappa(R)}{\mathcal{R}(R)}\, dR}}_{I(R,\,R_i)\ \ (A2)} \qquad (3) \quad (A3)$$

Apply to eq (3)

$$\int_{R_i}^{R_\infty} p(R)\, dR = 1 \qquad R_\infty = R(t \to \infty)$$

to set

$$\frac{W}{F_0} = \int_{R_i}^{R_\infty} \frac{R^3}{R_i^3\,|\mathcal{R}(R)|}\, I(R, R_i)\, dR \qquad (4) \quad (A1)$$

1. Tabulate $I(R, R_i)$ for suitable increments of R or program it in.
2. get F_0, F_1 or w from (4). If in a bed of given w, F_1 and F_0 is to be found solution is direct. If F_1 or w is to be found solve (4) by trial and error
3. Calculate $p(R)$ by (3)
4. Calculate F_2 by (2)
5. Calculate $n_2(R)$ by (A)

Single Size Feed

$$(*)\quad p(R) = \frac{F_0}{W|\mathcal{R}(R)|}\left(\frac{R}{R_i}\right)^3 \underbrace{e^{-\int_{R_i}^{R}\frac{F_1/W + K(R)}{\mathcal{R}(R)}dR}}$$

$$\frac{W}{F_0} = \int_{R_i}^{R_\infty} \frac{R^3}{R_i|\mathcal{R}(R)|}\, I(R, R_i)$$

$$F_1 + F_2 - F_0 = \int_{R_i}^{R_\infty} \frac{3W\, p(R)\, \mathcal{R}(R)\, dR}{R}$$

$R_\infty \to \infty$ growing particles

$R_\infty \to 0$ shrinking particles

Feed of various particle sizes

growing particles

(fraction of exit stream of size R) = $\sum_{\text{all feed solids not larger than } R}$ (fraction of solids leaving of size R and original feed size R_i)(fraction of feed of size R_i)

$$p(R) = \int_{R_{min}}^{R} \underbrace{p_1(R, R_i)}_{\text{given above } (*)} p_0(R_i)\, dR_i$$

$$p_1(R, R_i) = \int_{R_i}^{R_\infty} \left(\frac{R}{R_i}\right)^3 \frac{F_0}{W|\mathcal{R}(R)|}\, I(R, R_i)\, p_0(R_i)\, dR_i$$

$$p(R) = \frac{F_0}{W}\int_{R_{min}}^{R} \left(\frac{R}{R_i}\right)^3 \frac{1}{|\mathcal{R}(R)|}\, I(R, R_i)\, p_0(R_i)\, dR_i$$

$$p(R) = \frac{F_0 R^3}{W|\mathcal{R}(R)|}\int_{R_{min}}^{R} \frac{I(R, R_i)\, p_0(R_i)}{R_i^3}\, dR_i$$

However

$$I(R,R_i) = e^{-\int_{R_i}^{R} [\ ]\,dR} = e^{-\left[\int_{R_{min}}^{R}[\ ]\,dR - \int_{R_{min}}^{R_i}[\ ]\,dR\right]}$$

$$= I(R,R_{min})\, I(R_{min},R_i) = \frac{I(R,R_{min})}{I(R_i,R_{min})}$$

$$p(R) = \frac{F_0 R^3}{w\,|\mathcal{R}(R)|}\, I(R,R_{min}) \int_{R_{min}}^{R} \frac{p_0(R_i)\,dR_i}{R_i^3\, I(R_i,R_{min})}$$

$$\frac{W}{F_0} = \int_{R_{min}}^{R_\infty \to \infty} \frac{R^3}{|\mathcal{R}(R)|}\, I(R,R_{min}) \int_{R_{min}}^{R} \frac{p_0(R_i)\,dR_i}{R_i^3\, I(R_i,R_{min})}$$

For shrinking particles

$$\begin{pmatrix}\text{fraction of}\\ \text{exit stream of}\\ \text{size } R\end{pmatrix} = \sum_{\substack{\text{all feed}\\ \text{sizes not}\\ \text{smaller than } R}} \begin{pmatrix}\text{fraction of}\\ \text{solids leaving}\\ \text{of size } R \text{ and}\\ \text{original feed size } R_i\end{pmatrix}\begin{pmatrix}\text{fraction of}\\ \text{feed}\\ \text{of size } R_i\end{pmatrix}$$

$$p(R) = \int_{R}^{R_{max}} p_1(R,R_i)\, p_0(R_i)\,dR_i$$

$$p(R) = \frac{F_0 R^3}{w\,|\mathcal{R}(R)|} \int_{R}^{R_{max}} \frac{I(R,R_i)\, p_0(R_i)}{R_i^3}\,dR_i$$

$$I(R,R_i) = \frac{I(R_{max},R_i)}{I(R_{max},R)} = \frac{I(R,R_{max})}{I(R_i,R_{max})}$$

$$p(R) = \frac{F_0 R^3}{w\,|\mathcal{R}(R)|}\, I(R,R_{max}) \int_{R}^{R_{max}} \frac{p_0(R_i)\,dR_i}{R_i^3\, I(R_i,R_{max})}$$

$$\frac{W}{F_0} = \int_{R\to 0}^{R_{max}} \frac{R^3}{|\mathcal{R}(R)|}\, I(R,R_{max}) \int_{R}^{R_{max}} \frac{p_0(R_i)\,dR_i}{R_i^3\, I(R_i,R_{max})}$$