

Fluidized Beds

Reaction of Solids Particles of Changing Size

Ass

- uniform gas composition
- uniform particle density
- CSTR of solids, steady state, isothermal

Mass balance on reactor on size range R to $R + \Delta R$

$$\begin{aligned} & \left(\begin{array}{l} \text{solids} \\ \text{in by} \\ \text{feed} \end{array} \right) - \left(\begin{array}{l} \text{solids} \\ \text{out by} \\ \text{outflow} \end{array} \right) - \left(\begin{array}{l} \text{solids} \\ \text{out by} \\ \text{carriover} \end{array} \right) + \left(\begin{array}{l} \text{solids growing} \\ \text{into the interval} \\ \text{from a smaller} \\ \text{size} \end{array} \right) \\ & - \left(\begin{array}{l} \text{solids growing} \\ \text{out of the interval} \\ \text{to larger size} \end{array} \right) + \left(\begin{array}{l} \text{solid mass} \\ \text{generation within} \\ \text{the interval by growth} \end{array} \right) = 0 \end{aligned}$$

Growth Rate

$$R = \frac{dR}{dt}$$

Particle size distribution in bed and outflow $p(R)$

Particle size distribution in ~~the~~ feed $p_0(R)$

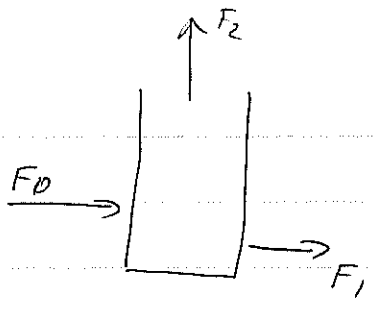
Particle size distribution in elutriated stream $p_2(R)$

$$\left(\begin{array}{l} \text{mass generation} \\ \text{within the size} \\ \text{interval} \end{array} \right) = \rho_s \left(\begin{array}{l} \text{number of} \\ \text{particles in} \\ \text{the interval} \end{array} \right) \left(\begin{array}{l} \text{volume increase} \\ \text{of a particle in} \\ \text{time } \Delta t \end{array} \right)$$

$$= \rho_s \frac{W p(\bar{R}) \Delta R}{\rho_s \frac{4}{3} \pi \bar{R}^3} \frac{dV}{dt} \times \Delta t$$

$$V = \frac{4}{3} \pi \bar{R}^3$$

$$= \frac{3 W p(\bar{R}) R(\bar{R}) \Delta R}{\bar{R}}$$



$$R < \bar{R} < R + \Delta R$$

$$F_0 p_0(\bar{R}) \Delta R - F_1 p(\bar{R}) \Delta R - F_2 p_2(\bar{R}) \Delta R + W p(R) R \Big|_R - W p(R) \bar{R} + \frac{3W p(\bar{R}) R(\bar{R}) \Delta R}{\bar{R}} = 0$$

$$\lim_{\Delta R \rightarrow 0} (F_0 p_0(\bar{R}) - F_1 p(\bar{R}) - F_2 p_2(\bar{R})) - W \lim_{\Delta R \rightarrow 0} \frac{(pR)_{R+\Delta R} - (pR)_R}{\Delta R} + \lim_{\Delta R \rightarrow 0} \frac{3W p(\bar{R}) R(\bar{R}) \Delta R}{\bar{R}}$$

$$F_0 p_0(R) - F_1 p(R) - F_2 p_2(R) - W \frac{d(pR)}{dR} + \frac{3W pR}{R} = 0$$

Now remember that in such a bed

$$\bar{z}(R_i) = \frac{W(R_i)}{F_0(R_i)} = \frac{W(R_i)}{F_1(R_i) + F_2(R_i)} = \frac{1}{\frac{F_1(R_i)}{W(R_i)} + \frac{F_2(R_i)}{W(R_i)}}$$

$$\frac{F_1(R_i)}{W(R_i)} = \frac{F_1}{W} \quad \text{and} \quad \frac{F_2(R_i)}{W(R_i)} = \kappa(R_i)$$

$$\bar{z} = \frac{1}{\frac{F_1}{W} + \kappa(R_i)} \quad \text{but} \quad \frac{F_2(R_i)}{W(R_i)} = \frac{p_2(R_i) F_2}{p(R_i) W}$$

$$p_2(R) = \frac{W}{F_2} \kappa(R) p(R) \quad (A) \quad (45)$$

$$F_0 p_0(R) - F_1 p(R) - W \kappa(R) p(R) - W \frac{d(pR)}{dR} + \frac{3W pR}{R} = 0$$

$$e^{a+bc} = e^a e^{bc}$$

$$F_1 + F_2 - F_0 = \left(\begin{array}{l} \text{total solid} \\ \text{mass production} \\ \text{in the bed} \end{array} \right) = \int_{R_{min}}^{R_{max}} \frac{3Wp(R)R(R)dR}{R} \quad (2)$$

$$F_1 + F_2 - F_0 > 0 \quad \text{particle growth} \quad (A4)$$

$$LO \quad \text{particle shrinkage}$$

Solution (Single Size Feed)

$$F_0 \delta(R-R_i) - F_1 p - Wk p - \underbrace{W \frac{d}{dR}(pR)} + \frac{3WpR}{R} = 0$$

$$- Wp \frac{dR}{dR} - WR \frac{dp}{dR}$$

$$- WR \frac{dp}{dR} - \left[W \frac{dR}{dR} + F_1 + Wk - \frac{3WR}{R} \right] p = - F_0 \delta(R-R_i)$$

$$\frac{dp}{dR} + \left[\frac{1}{R} \frac{dR}{dR} + \frac{F_1}{WR} + \frac{k}{R} - \frac{3}{R} \right] p = \frac{F_0 \delta(R-R_i)}{WR}$$

$$\frac{d}{dR} \left[e^{\int_{R_i}^R \left[\frac{1}{R} + \frac{F_1}{WR} + \frac{k}{R} - \frac{3}{R} \right] dR} p \right] = e^{\int_{R_i}^R \left[\frac{1}{R} + \frac{F_1}{WR} + \frac{k}{R} - \frac{3}{R} \right] dR} \frac{F_0 \delta(R-R_i)}{WR}$$

$$e^{\int_{R_i}^R \left[\frac{1}{R} + \frac{F_1}{WR} + \frac{k}{R} - \frac{3}{R} \right] dR} p(R) - 0 = \frac{F_0}{W|R(R_i)|} e^{\int_{R_i}^R \left[\frac{1}{R} + \frac{F_1}{WR} + \frac{k}{R} - \frac{3}{R} \right] dR}$$

$$\int_{R_i}^R \left[\frac{d \ln |R|}{dR} + \frac{F_1/W + k}{R} - \frac{3}{R} \right] dR =$$

$$= \ln \left| \frac{R(R)}{R(R_i)} \right| - 3 \ln \frac{R}{R_i} + \int_{R_i}^R \frac{F_1/W + k}{R} dR$$

$$e^{\int_{R_i}^R \left[\frac{1}{R} + \frac{F_1}{WR} + \frac{k}{R} - \frac{3}{R} \right] dR} = \frac{|R(R)|}{|R(R_i)|} \left(\frac{R_i}{R} \right)^3 e^{\int_{R_i}^R \frac{F_1/W + k(R)}{R(R)} dR}$$

$$p(R) = \frac{F_0}{W|R(R_i)|} \frac{|R(R_i)|}{|R(R)|} \left(\frac{R}{R_i} \right)^3 e^{-\int_{R_i}^R \frac{F_1/W + k(R)}{R(R)} dR}$$

$$p(R) = \frac{F_0}{W |R(R)|} \left(\frac{R}{R_i}\right)^3 e^{-\int_{R_i}^R \frac{F_1/W + x(R)}{R(R)} dR} \quad (3)$$

4

(A2) (A3)

Apply to eq (3)

$$\int_{R_i}^{R_\infty} p(R) dR = 1 \quad \text{to get} \quad R_\infty = R(t \rightarrow \infty)$$

$$\frac{W}{F_0} = \int_{R_i}^{R_\infty} \frac{R^3}{R_i^3 |R(R)|} I(R, R_i) dR \quad (4)$$

(A1)

1. Tabulate $I(R, R_i)$ for suitable increments of R or program it in.
2. get F_0, F_1 or W from (4).
If in a bed of given W, F_1 and F_0 is to be found solution is direct.
If F_1 or W is to be found solve (4) by trial and error.
3. Calculate $p(R)$ by (3)
4. Calculate F_2 by (2)
5. Calculate $\eta_2(R)$ by (A)

Single Size Feed

$$(*) \quad p(R) = \frac{F_0}{W |R(R)|} \left(\frac{R}{R_i}\right)^3 e^{-\int_{R_i}^R \frac{F/W + X(R)}{R(R)} dR}$$

$$\frac{W}{F_0} = \int_{R_i}^{R_{\infty}} \frac{R^3}{R_i |R(R)|} I(R, R_i)$$

$$F_1 + F_2 - F_0 = \int_{R_i}^{R_{\infty}} \frac{3W p(R) R(R) dR}{R}$$

$R_{\infty} \rightarrow \infty$ growing par holes

$R_{\infty} \rightarrow 0$ shrinking par holes

Feed of various par hole sizes growing par holes

$$\left(\begin{array}{l} \text{fraction of} \\ \text{exit stream of} \\ \text{size } R \end{array} \right) = \sum_{\substack{\text{all feed} \\ \text{sizes not} \\ \text{larger than } R}} \left(\begin{array}{l} \text{fraction of seeds} \\ \text{leaving of size } R \\ \text{and original feed} \\ \text{size } R_i \end{array} \right) \left(\begin{array}{l} \text{fraction of} \\ \text{seeds of} \\ \text{size } R_i \end{array} \right)$$

$$p(R) = \int_{R_{\min}}^R p_1(R, R_i) p_0(R_i) dR_i$$

given above (*)

$$p_1(R, R_i) = \int_{R_i}^R \left(\frac{R}{R_i}\right)^3 \frac{F_0}{W |R(R)|} I(R, R_i) p_0(R_i) dR_i$$

$$p(R) = \frac{F_0}{W} \int_{R_{\min}}^R \left(\frac{R}{R_i}\right)^3 \frac{1}{|R(R)|} I(R, R_i) p_0(R_i) dR_i$$

$$p(R) = \frac{F_0 R^3}{W |R(R)|} \int_{R_{\min}}^R \frac{I(R, R_i) p_0(R_i)}{R_i^3} dR_i$$

Flow over

$$I(R, R_i) = e^{-\int_{R_i}^R \epsilon \, dR} = e^{-\left[\int_{R_{min}}^R \epsilon \, dR - \int_{R_{min}}^{R_i} \epsilon \, dR \right]}$$

$$= I(R, R_{min}) I(R_{min}, R_i) = \frac{I(R, R_{min})}{I(R_i, R_{min})}$$

$$p(R) = \frac{F_0 R^3}{W |R(R)|} I(R, R_{min}) \int_{R_{min}}^R \frac{p_0(R_i) dR_i}{R_i^3 I(R_i, R_{min})}$$

$$\frac{W}{F_0} = \int_{R_{min}}^{R \rightarrow \infty} \frac{R^3}{|R(R)|} I(R, R_{min}) \int_{R_{min}}^R \frac{p_0(R_i) dR_i}{R_i^3 I(R_i, R_{min})}$$

For shear layer jet holes

$\left(\begin{array}{l} \text{fraction of} \\ \text{exit stream of} \\ \text{hole } R \end{array} \right) = \sum_{\substack{\text{all feed} \\ \text{holes that} \\ \text{smaller than } R}} \left(\begin{array}{l} \text{fraction of} \\ \text{winds leaving} \\ \text{of hole } R_i \text{ and} \\ \text{original feed hole } R_i \end{array} \right) \left(\begin{array}{l} \text{fraction of} \\ \text{feed} \\ \text{of hole } R_i \end{array} \right)$

$$p(R) = \int_R^{R_{max}} p_1(R, R_i) p_0(R_i) dR_i$$

$$p(R) = \frac{F_0 R^3}{W |R(R)|} \int_R^{R_{max}} \frac{I(R, R_i) p_0(R_i)}{R_i^3} dR_i$$

$$I(R, R_i) = \frac{I(R_{max}, R_i)}{I(R_{max}, R)} = \frac{I(R, R_{max})}{I(R_i, R_{max})}$$

$$p(R) = \frac{F_0 R^3}{W |R(R)|} I(R, R_{max}) \int_R^{R_{max}} \frac{p_0(R_i) dR_i}{R_i^3 I(R_i, R_{max})}$$

$$\frac{W}{F_0} = \int_R^{R_{max}} \frac{R^3}{|R(R)|} I(R, R_{max}) \int_R^{R_{max}} \frac{p_0(R_i) dR_i}{R_i^3 I(R_i, R_{max})}$$