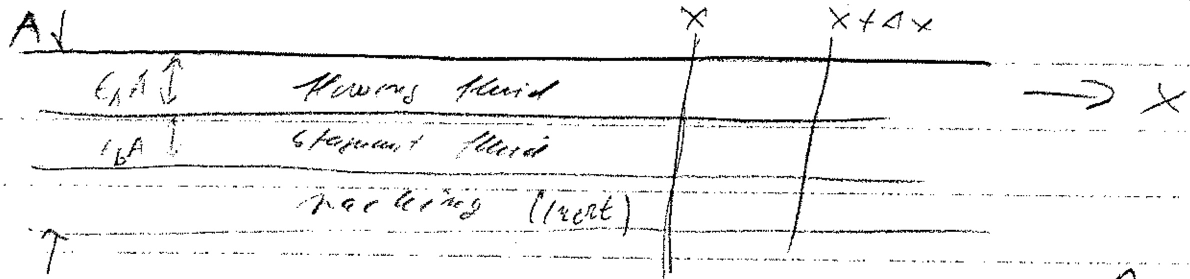
Consider Cross Flow Model for Packed Bed

$A$ = total cross sectional area

$\epsilon_a A$ = cross sectional area for flowing fluid

$\epsilon_b A$ = cross sectional area of stagnant fluid

$(\epsilon_a + \epsilon_b) A = \epsilon A$ = total fluid occupied cross sectional area




$u_a$ = mean actual velocity $= \frac{Q}{\epsilon_a A}$

$$\bar{t} = \frac{\epsilon A L}{Q} = \frac{\epsilon V}{Q} = \frac{\epsilon A L}{u_a \epsilon_a A} = \frac{\epsilon}{\epsilon_a}\frac{L}{u_a} = \epsilon \frac{L}{\bar{u}}$$

$\bar{u}$ = superficial velocity $= \frac{Q}{A} = \epsilon_a u_a$

$u_i$ = interstitial velocity $= \frac{Q}{\epsilon A} = \frac{\epsilon_a u_a}{\epsilon} = \frac{\bar{u}}{\epsilon}$

$$u_a \epsilon_a A C_1\Big|_x - u_a \epsilon_a A C_1\Big|_{x+\Delta x} - f A \Delta x (C_1 - kC_2) + R_1 \epsilon_a A \Delta x = \epsilon_a A \Delta x \frac{\partial C_1}{\partial t}$$

$f\left(\frac{m^3}{m^3 s}\right)$ = exchange flow rate per unit volume of total bed (space active + non active fluid)

$$R_2 \epsilon_b A \Delta x + f A \Delta x (C_1 - kC_2) = \epsilon_b A \frac{\partial C_2}{\partial t}$$

$R_1 \left(\frac{mol}{m^3 s}\right)$ — rate of formation of species 1 ($C_1$) per unit volume of active fluid (in active region)

$R_2 \left(\frac{mol}{m^3 s}\right)$ — rate of formation of species 2 ($C_2$) per unit volume of ~~stagnant~~ fluid (in inactive region)

Initially we assume $K \equiv 1 \left(\frac{\text{mol}/m^3 \text{ a. fluid}}{\text{mol}/m^3 \text{ s. fluid}}\right)$

If $\mathcal{R}\left(\frac{\text{mol}}{m^3 \text{ bed } s}\right)$ then $\frac{\mathcal{R}}{\epsilon}\left(\frac{\text{mol}}{m^3 \text{ fluid } s}\right)$

so that $R_1 = R_2 = \frac{R}{\epsilon}$

$$-u_a \epsilon_a \frac{\partial C_1}{\partial x} - f(C_1 - C_2) + \mathcal{R}\frac{\epsilon_a}{\epsilon} = \epsilon_a \frac{\partial C_1}{\partial t}$$

$$f(C_1 - C_2) + \mathcal{R}\frac{\epsilon_b}{\epsilon} = \epsilon_b \frac{\partial C_2}{\partial t}$$

$t = 0$; $C_1 = C_2 = 0$ ; $x = 0$ ; $C_1 = C_{10}$

Recall $u_a \epsilon_a = \bar{u}$

Let $y = \frac{x}{d_p}$ $\quad \gamma_1 = \frac{C_1}{C_{10}}$ ; $\gamma_2 = \frac{C_2}{C_{10}}$

$$-\frac{\bar{u} C_{10}}{d_p}\frac{\partial \gamma_1}{\partial y} - f C_{10}(\gamma_1 - \gamma_2) + \frac{\epsilon_a}{\epsilon}\mathcal{R} = \epsilon_a C_{10}\frac{\partial \gamma_1}{\partial t}$$

$$f C_{10}(\gamma_1 - \gamma_2) + \mathcal{R}\frac{\epsilon_b}{\epsilon} = \epsilon_b C_{10}\frac{\partial \gamma_2}{\partial t}$$

$$-\frac{\partial \gamma_1}{\partial y} - \frac{f d_p}{\bar{u}}(\gamma_1 - \gamma_2) + \frac{\epsilon_a}{\epsilon}\left(\frac{d_p}{\bar{u} C_{10}}\mathcal{R}\right) = \frac{d_p}{\bar{u}/\epsilon}\frac{\epsilon_a}{\epsilon}\frac{\partial \gamma_1}{\partial t}$$

Let $\tau = \frac{\bar{u}\, t}{d_p \epsilon}$ ; $S = \frac{\epsilon_b}{\epsilon}$ ; $\bar{\mathcal{R}} = \frac{d_p}{\bar{u} C_{10}}\mathcal{R}$ $\quad F = \frac{f d_p}{\bar{u}}$

$$\boxed{-\frac{\partial \gamma_1}{\partial y} - F(\gamma_1 - \gamma_2) + (1-S)\bar{\mathcal{R}} = (1-S)\frac{\partial \gamma_1}{\partial \tau}}$$

$$\frac{f d_p}{\bar{u}}(\gamma_1 - \gamma_2) + \frac{d_p}{\bar{u} C_{10}}\mathcal{R}\frac{\epsilon_b}{\epsilon} = \frac{\epsilon_b d_p \epsilon}{\bar{u}\,\epsilon}\frac{\partial \gamma_2}{\partial t}$$

$$\boxed{F(\gamma_1 - \gamma_2) + S\bar{\mathcal{R}} = S\frac{\partial \gamma_2}{\partial \tau}}$$

$\tau = 0$ $\quad \gamma_1 = \gamma_2 = 0$ $\qquad y = \frac{L}{d_p}$ $\quad \gamma_1 = \gamma_{1,\text{exit}}$

$y = 0$ $\quad \gamma_1 = 1$ (?)

Two cases are of interest.

Steady state reactor model: contains two parameters F and S.

Transient model yielding the E-curve.

Let us get F and S from known information about packed beds.

The Ergun equation for pressure drop is:

$$-\frac{dp}{dz} = \frac{150\,\mu\,(1-\epsilon)^2\,\bar{u}}{\epsilon^3\,d_p^2} + 1.75\left(\frac{1-\epsilon}{\epsilon^3}\right)\frac{\rho\,\bar{u}^2}{d_p}$$

$$d_p = \frac{6}{S_V}$$

We hypothesize (Hudgins et al) that the second term is due to turbulent wake phenomena caused by the momentum exchange between the flowing and stagnant zones.

Momentum balance then yields

$$-\epsilon_a\left(\frac{dp}{dz}\right)' = f\,\rho\,\frac{\bar{u}}{\epsilon_a}$$

$$-\left(\frac{dp}{dz}\right)' = f\,\rho\,\frac{\bar{u}}{\epsilon_a^2} = 1.75\left(\frac{1-\epsilon}{\epsilon^3}\right)\frac{\rho\,\bar{u}^2}{d_p}$$

$$\frac{f\,d_p}{\bar{u}} = 1.75\left(\frac{1-\epsilon}{\epsilon}\right)\left(\frac{\epsilon_a}{\epsilon}\right)^2$$

$$\boxed{F = 1.75\left(\frac{1-\epsilon}{\epsilon}\right)(1-S)^2}$$

We determine now the second parameter from the variance of the impulse response (note the spread of the curve around the mean is related to the stagnant volume size).

We need to solve:

$$-\frac{\partial \gamma_1}{\partial \xi} - F(\gamma_1 - \gamma_2) = (1-S)\frac{\partial \gamma_1}{\partial \tau}$$

$$F(\gamma_1 - \gamma_2) = S\,\frac{\partial \gamma_2}{\partial \tau}$$

$$\tau = 0 \quad \gamma_1 = \gamma_2 = 0 \qquad ; \quad \xi = 0 \quad \gamma_1 = \delta(\tau)$$

Laplace transform of the above yields with $\bar{\gamma}_1 = \mathcal{L}\{\gamma_1\}$, $\bar{\gamma}_2 = \mathcal{L}\{\gamma_2\} = \int_0^{\infty} e^{-st}\gamma_1\,dt$

$$\frac{d\bar{\gamma}_1}{d\xi} = -F(\bar{\gamma}_1 - \bar{\gamma}_2) - (1-S)s\,\bar{\gamma}_1$$

$$F(\bar{\gamma}_1 - \bar{\gamma}_2) = S s\,\bar{\gamma}_2 \qquad \text{(call } \underline{s = p}\text{)}$$

Hence

$$\bar{\gamma}_2 = \frac{F\bar{\gamma}_1}{F + Ss}$$

$$\bar{\gamma}_1 - \bar{\gamma}_2 = \frac{Ss\,\bar{\gamma}_1}{F + Ss}$$

So that

$$\frac{d\bar{\gamma}_1}{d\xi} = -\left[\frac{FSs}{F + Ss} + (1-S)s\right]\bar{\gamma}_1$$

$$\xi = 0 \qquad \bar{\gamma}_1 = 1$$

$$\bar{\gamma}_1 = e^{-\left[\frac{FSs}{F+Ss} + (1-S)s\right]\xi} = e^{-\frac{FSs}{F+Ss} \ldots \xi}$$

we are interested in the Laplace transform of the exit concentration $\bar{\gamma}_1(\xi = \frac{L}{d_p})$ which is

$$\bar{\gamma}_{1exit} = e^{-\frac{L}{d_p}\left[\frac{FSs}{F+Ss} + (1-S)s\right]} \quad (*)$$

Since we want the variance we need to expand the above for small values of s.

$$\bar{\gamma}_{1exit} = e^{-\frac{L}{d_p}\left[F - \frac{F}{1+(S/F)s} + (1-S)s\right]}$$

$$= e^{-\frac{L}{d_p}\left[F - F\left(1 - \frac{S}{F}s + \frac{S^2}{F^2}s^2 - \frac{S^3}{F^3}s^3\right) + s - Ss\right]}$$

$$= e^{-\frac{L}{d_p}\left[s - \frac{S^2}{F}s^2 + \frac{S^3}{F^2}s^3 - \ldots\right]}$$

$$= e^{-\frac{L}{d_p}s\left[1 - \frac{S^2}{F}s + \frac{S^3}{F^2}s^2 \ldots\right]}$$

$$= 1 - \frac{L}{d_p}s\left[1 - \frac{S^2}{F}s + \frac{S^3}{F^2}s^2\right] + \frac{L^2}{2d_p^2}s^2\left[1 - \frac{S^2}{F}s \ldots\right]^2$$

$$= 1 - \frac{L}{d_p}s + \left[\frac{S^2}{F}\frac{L}{d_p} + \frac{1}{2}\left(\frac{L}{d_p}\right)^2\right]s^2 - O(s^3)$$

Comparison with

$$\bar{\gamma}_{1exit} = \mu_0 - \mu_1 s + \frac{\mu_2}{2}s^2 - \frac{\mu_3}{6}s^3 \ldots$$

yields

$$\mu_0 = 1$$

$$\mu_1 = \frac{L}{d_p}$$

$$\mu_2 = 2\left[\frac{S^2}{F}\frac{L}{d_p} + \frac{1}{2}\left(\frac{L}{d_p}\right)^2\right]$$

We recognize that $\bar{E}_1(\tau)$ is not the $E(t)$ or $\bar{E}_\theta(t)$ since $\tau = \frac{\bar{u}t}{\epsilon d_p}$ while

$$\theta = \frac{\bar{u}t}{\epsilon L} = \tau\left(\frac{d_p}{L}\right)$$

$$\bar{\theta} = \int_0^\infty \theta E_\theta(\theta)\,d\theta = \left(\frac{d_p}{L}\right)\int_0^\infty \tau\, E_\tau(\tau)\,d\tau = \left(\frac{d_p}{L}\right)\times\left(\frac{L}{d_p}\right) = 1 \quad Q.E.D.$$

$$\sigma_\theta^2 = \int_0^\infty (\theta-1)^2 E_\theta(\theta)\,d\theta = \left(\frac{d_p}{L}\right)^2 \int_0^\infty \left(\tau - \frac{L}{d_p}\right)^2 E_\tau(\tau)\,d\tau$$

$$= \frac{\int_0^\infty \left(\tau - \frac{L}{d_p}\right)^2 E_\tau(\tau)\,d\tau}{(L/d_p)^2} = \overline{\sigma_\tau^2} = \frac{\mu_2 - \mu_1^2}{\mu_1^2}$$

Hence

$$\sigma_D^2 = \frac{2\,\frac{S^2}{F}\frac{L}{d_p} + \left(\frac{L}{d_p}\right)^2 - \left(\frac{L}{d_p}\right)^2}{\left(\frac{L}{d_p}\right)^2} = \frac{2\,S^2/F}{(L/d_p)}$$

$$\boxed{\sigma_D^2 = 2\,\frac{S^2}{F}\left(\frac{d_p}{L}\right)}$$

Now we know from before (lecture)

$$\sigma_D^2 = \frac{2}{Pe_z} = \frac{2\,E_z}{\bar{u}L} = \frac{2\,E_z}{\bar{u}\,d_p}\times\left(\frac{d_p}{L}\right)$$

Let $\frac{\bar{u}\,d_p}{E_z} = Bo = Pe_{p\,ax}$

$Bo = f(Re_p)$ $\quad Re_p > 100 \quad \underline{\underline{Bo = 2}}$

Equating the variances we get

$$\frac{S^2}{F}\left(\frac{d_p}{L}\right) = \frac{1}{Bo}\left(\frac{d_p}{L}\right) \qquad \boxed{\frac{S^2}{F} = \frac{1}{Bo}}$$

or $\boxed{F = S^2\,Bo}$

$$S^2 Bo = 1.75\left(\frac{1-\epsilon}{\epsilon}\right)(1-S)^2$$

$$\frac{S}{1-S} = \sqrt{\frac{1.75}{Bo}\left(\frac{1-\epsilon}{\epsilon}\right)}$$

$$\boxed{S = \frac{\sqrt{\frac{1.75}{Bo}\left(\frac{1-\epsilon}{\epsilon}\right)}}{1+\sqrt{\frac{1.75}{Bo}\left(\frac{1-\epsilon}{\epsilon}\right)}}}$$

$$\boxed{F = \frac{1.75\left(\frac{1-\epsilon}{\epsilon}\right)}{\left[1+\sqrt{\frac{1.75}{Bo}\left(\frac{1-\epsilon}{\epsilon}\right)}\right]^2}}$$

Typically
$Bo = 2$
$\epsilon \simeq 0.5$

$S \approx 0.342$

$F \approx 0.467$

The steady state model to be solved is

$$\frac{d\gamma_1}{dy} = -F(\gamma_1-\gamma_2) + (1-S)\bar{R}\big|_{\gamma_1} \quad (1)$$

$$F(\gamma_1-\gamma_2) + S\bar{R}\big|_{\gamma_2} = 0 \quad (2)$$

$$y=0 \qquad \gamma_1 = 1$$

$\gamma_{1exit} = \gamma_1(y = \frac{L}{d_p})$

where $\bar{R}$ is the dimensionless rate of formation

for an n-th order reaction

$R = kC^n$ $\left(\frac{mol}{m^3 bed}\right)$

$$\bar{R} = \frac{d_p}{u C_{10}} k C^n = \frac{d_p k C_{10}^{n-1}}{u} \gamma_1^n$$

Let $Da_p = \frac{d_p}{u} k C_{10}^{n-1}$

The rate will (for a monocatalytic reaction) normally be scaled by:

$R_f = k_f C^n$ $\left(\frac{mol}{m^3 fluid}\right)$

$\boxed{k_f \epsilon = k}$

Consider 1st order reaction

$$Da_p = \frac{d_p}{u} k$$

The solution is given by (Weisz ?)

$$\gamma_{exit} = e^{-\left[\frac{F S Da_p}{F + S Da_p} + (1-S) Da_p\right] \frac{L}{d_p}}$$

In the limit if there is no exchange

$$F \equiv 0$$

$$\gamma_{exit} = e^{-(1-S) Da_p \frac{L}{d_p}} = e^{-(1-S) k \frac{L}{u}}$$

$$\cancel{= e^{-(1-\frac{\epsilon_b}{\epsilon}) k_r \frac{L}{u/\epsilon}}}$$

QED

---

For nonlinear rate forms we would have to solve the nonlinear eq (2) for each step in $y$. Instead solve (2) at $y = 0$ to get $\gamma_{20}$ from the solution of

$$F(1 - \gamma_{20}) + S \bar{R}|_{\gamma_{20}} = 0$$

Then differentiate eq (2)

$$F \frac{d\gamma_1}{dy} - F \frac{d\gamma_2}{dy} + S \frac{d\bar{R}}{d\gamma_2} \frac{d\gamma_2}{dy} = 0$$

$$\left(F - S \frac{d\bar{R}}{d\gamma_2}\right) \frac{d\gamma_2}{dy} = F \frac{d\gamma_1}{dy} = -F^2(\gamma_1 - \gamma_2) + F(1-S) \bar{R}$$

Integrate simultaneously for $y = \frac{L}{d_p}$

Let us find the impulse response

$$\bar{\gamma}_{1exit} = e^{-\left[\frac{FSs}{F+Ss} + (1-S)s\right]\frac{L}{d_p}}$$

$$= e^{-\frac{L}{d_p}\left[F - \frac{F^2/S}{s + (F/S)} + (1-S)s\right]}$$

$$= \underbrace{e^{-\frac{L}{d_p}F}}_{\text{constant}} \underbrace{e^{-\frac{L}{d_p}(1-S)s}}_{\text{time lag}} e^{\frac{L}{d_p}(F^2/S)/(s+F/S)}$$

---

Let $\bar{f}(s) = e^{\frac{L}{d_p}(F^2/S)/(s+F/S)}$

$$\bar{\gamma}_{1exit} = e^{-\frac{L}{d_p}F} e^{-\frac{L}{d_p}(1-S)s} \cdot \bar{f}(s)$$

$$\gamma_{1exit} = e^{-\frac{L}{d_p}F} \int_0^{\tau} f(\tau)\, \delta\!\left(\tau - \tfrac{L}{d_p}(1-S)\right) d\tau$$

$$= e^{-\frac{L}{d_p}F} f\!\left(\tau - \tfrac{L}{d_p}(1-S)\right) H\!\left(\tau - \tfrac{L}{d_p}(1-S)\right)$$

---

Consider now

$$\bar{f}(s) = e^{\frac{L}{d_p}(F^2/S)/(s+F/S)} = \bar{g}\left(s + \tfrac{F}{S}\right)$$

<u>Rule 1</u> $\mathcal{L}^{-1}\{\bar{g}(s+a)\} = e^{-a\tau}\, \mathcal{L}^{-1}\{\bar{g}(s)\}$

where $\bar{g}(s) = e^{\frac{L}{d_p}(F^2/S)/s}$

<u>Rule 2</u>: $\mathcal{L}^{-1}\{e^{\alpha/s} - 1\} = \sqrt{\frac{\alpha}{\tau}}\, I_1(2\sqrt{\alpha\tau})$

Hence

$$\mathcal{L}^{-1}\{e^{\alpha/s}\} = \delta(\tau) + \sqrt{\frac{\alpha}{\tau}}\, I_1(2\sqrt{\alpha\tau})$$

$$\alpha = \frac{F^2}{S}\frac{L}{d_p}$$

Therefore

$$\mathcal{L}^{-1}\left\{ e^{\frac{L}{d_p}(F^2/s)/p} \right\} = \delta(\tau) + \sqrt{\frac{F^2 L}{s\,\tau d_p}}\, I_1\left(2\sqrt{\frac{F^2}{s}\tau\frac{L}{d_p}}\right) = g(t)$$

$$\mathcal{L}^{-1}\left\{ e^{\frac{L}{d_p}(F^2/s)/(p+F/s)} \right\} = e^{-\frac{F}{s}\tau}\left\{ \delta(\tau) + \frac{F}{\sqrt{s\,\tau\frac{d_p}{L}}} I_1\left(2F\sqrt{\frac{\tau L}{s\, d_p}}\right)\right\}$$

Finally

$$\gamma_{1exit}(\tau) = e^{-\frac{L}{d_p}F}\, e^{-\frac{F}{s}(\tau - \frac{L}{d_p}(1-s))}\left\{ \delta(\tau - \tfrac{L}{d_p}(1-s)) \right.$$

$$\left. + \frac{F}{\sqrt{s(\tau - \frac{L}{d_p}(1-s))\frac{d_p}{L}}}\, I_1\left(2F\sqrt{\frac{L(\tau - \frac{L}{d_p}(1-s))}{d_p\, s}}\right)\right\} H(\tau - \tfrac{L}{d_p}(1-s))$$

$$\gamma_{1exit}(\tau)\, d\tau = E_\theta(\theta)\, d\theta = E(t)\, dt$$

$$\theta = \frac{\bar{u}}{\epsilon L} t = \left(\frac{d_p}{L}\right)\tau \qquad \tau = \frac{L}{d_p}\theta$$

$$E_\theta(\theta) = \left(\frac{L}{d_p}\right)\gamma_{1exit} \qquad E(t) = \frac{1}{t} E_\theta(\theta)$$

$$E_\theta(\theta) = e^{-\frac{LF}{d_p}}\, e^{-\frac{FL}{d_p s}(\theta - (1-s))}\left\{ \frac{L}{d_p}\delta(\theta - (1-s)) + \right.$$

$$\left. + \frac{F\frac{L}{d_p}}{\sqrt{s(\theta - (1-s))}}\, I_1\left(2F\frac{L}{d_p}\sqrt{\frac{\theta - (1-s)}{s}}\right)\right\} H(\theta - (1-s))$$

$$E(t) = e^{-\frac{LF}{d_p}}\, e^{-\frac{FL}{d_p s}\left(\frac{\bar{u}}{\epsilon L}t - (1-s)\right)}\left\{ \frac{L}{d_p}\delta\left(\frac{\bar{u}t}{\epsilon L} - (1-s)\right) + \right.$$

$$\left. + \frac{F\frac{L}{d_p}\frac{1}{t}}{\sqrt{s(\frac{\bar{u}t}{\epsilon L} - (1-s))}}\, I_1\left(2F\frac{L}{d_p}\sqrt{\frac{\frac{\bar{u}t}{\epsilon L} - (1-s)}{s}}\right)\right\} H\left(t - (1-s)\frac{\epsilon L}{\bar{u}}\right)$$

Krall $\quad S = \frac{\epsilon_b}{\epsilon} \quad\quad 1-S = \frac{\epsilon_a}{\epsilon}$

$$F = \frac{f\, d_p}{\bar{u}}$$

$$E(t) = e^{-\frac{fL}{\bar{u}}}\, e^{-\frac{fL\epsilon}{\bar{u}\epsilon_b}\left(\frac{\bar{u}}{\epsilon L}t - \frac{\epsilon_a}{\epsilon}\right)} \Big\{ \delta\left(t - \frac{\epsilon_a L}{\bar{u}}\right)$$

$$+ \frac{\frac{f d_p}{t\,\bar{u}}\,\frac{L}{d_p}}{\sqrt{\frac{\epsilon_b}{\epsilon}\left(\frac{\bar{u}t}{\epsilon L} - \frac{\epsilon_a}{\epsilon}\right)}}\, I_1\left(2\frac{f d_p}{\bar{u}}\frac{L}{d_p}\sqrt{\frac{\frac{\bar{u}t}{\epsilon L} - \frac{\epsilon_a}{\epsilon}}{\frac{\epsilon_b}{\epsilon}}}\right)\Big\}$$

$$\times H\left(t - \frac{\epsilon_a L}{\bar{u}}\right)$$

$$\bar{t} = \frac{\epsilon L}{\bar{u}} \qquad \frac{L}{\bar{u}} = \frac{\bar{t}}{\epsilon}$$

$$E(t) = e^{-\frac{f\bar{t}}{\epsilon}}\, e^{-\frac{f\bar{t}}{\epsilon_b}\left(\frac{t}{\bar{t}} - \frac{\epsilon_a}{\epsilon}\right)} \Big[\frac{L}{d_p}\delta\left(t - \frac{\epsilon_a}{\epsilon}\bar{t}\right)$$

$$+ \frac{f\frac{\bar{t}}{t\epsilon}\frac{d_p}{L}\times\frac{L}{d_p}}{\sqrt{\frac{\epsilon_b}{\epsilon}\left(\frac{t}{\bar{t}} - \frac{\epsilon_a}{\epsilon}\right)}}\, I_1\left(2 f\frac{\bar{t}}{\epsilon}\sqrt{\frac{\frac{t}{\bar{t}} - \frac{\epsilon_a}{\epsilon}}{\frac{\epsilon_b}{\epsilon}}}\right)\Big] H\left(t - \frac{\epsilon_a}{\epsilon}\bar{t}\right)$$

$$E(t) = e^{-\frac{f\bar{t}}{\epsilon}}\, e^{-\frac{f}{\epsilon_b}\left(t - \frac{\epsilon_a}{\epsilon}\bar{t}\right)} \Big\{ \delta\left(t - \frac{\epsilon_a}{\epsilon}\bar{t}\right)$$

$$+ \frac{f}{t\sqrt{\frac{\epsilon_b}{\epsilon}\left(\frac{t}{\bar{t}} - \frac{\epsilon_a}{\epsilon}\right)}}\, I_1\left(2 f\frac{\bar{t}}{\epsilon}\sqrt{\frac{t - \frac{\epsilon_a}{\epsilon}\bar{t}}{\frac{\epsilon_b}{\epsilon}\bar{t}}}\right)\Big\}$$

$$H\left(t - \frac{\epsilon_a}{\epsilon}\bar{t}\right)$$

Old Soln A-12

$$E(t) = e^{-\alpha}\, e^{-\frac{\alpha}{(1-\beta)\bar{t}}(t-\beta\bar{t})} \Big\{ \delta(t-\beta\bar{t})$$

$$+ \frac{\alpha}{\sqrt{(1-\beta)\,\bar{t}\,(t-\beta\bar{t})}}\, I_1\left(2\alpha\sqrt{\frac{t-\beta\bar{t}}{(1-\beta)\bar{t}}}\right)\Big\}\, H(t-\beta\bar{t})$$

$$\frac{\epsilon_a}{\epsilon} = \beta = 1-S \qquad \alpha = \frac{k\bar{t}}{\epsilon}$$

$$1-\beta = \frac{\epsilon_b}{\epsilon} = S$$

$$E(t) = e^{-\frac{k\bar{t}}{\epsilon}}\, e^{-\frac{k}{\epsilon_b}\left(t-\frac{\epsilon_a}{\epsilon}\bar{t}\right)} \Big\{ \delta\left(t - \frac{\epsilon_a}{\epsilon}\bar{t}\right)$$

$$+ \frac{k\bar{t}}{\epsilon\sqrt{\frac{\epsilon_b}{\epsilon}\bar{t}\left(t-\frac{\epsilon_a}{\epsilon}\bar{t}\right)}}\, I_1\left(2\frac{k\bar{t}}{\epsilon}\sqrt{\frac{t-\frac{\epsilon_a}{\epsilon}\bar{t}}{\frac{\epsilon_b}{\epsilon}\bar{t}}}\right)\Big\}\, H\left(t-\frac{\epsilon_a}{\epsilon}\bar{t}\right)$$

$$+ \frac{k}{\epsilon\sqrt{\frac{\epsilon_b}{\epsilon}\left(\frac{t}{\bar{t}} - \frac{\epsilon_a}{\epsilon}\right)}}$$

& to D