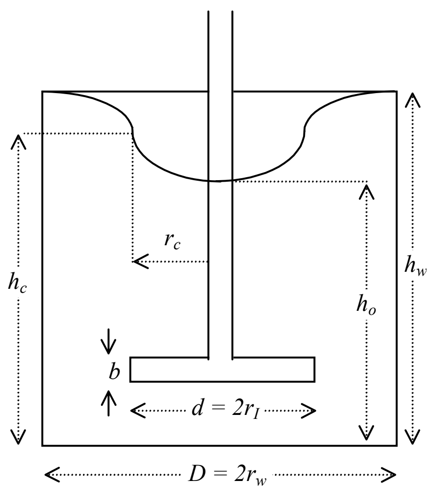
# I. Stirred Tank Reactor

Ideal CSTR has an exponential RTD and is perfectly mixed on a molecular level <u>i.e</u>, is in the state of maximum mixedness.

Can the exponential RTD and perfect molecular mixing be approached in practice? It depends on the design and operation of the stirred tank and on the characteristic reaction time.

Consider briefly mixing in a tank via a paddle or turbine.

**FIGURE 1:** Tank of diameter *D* mixed with a paddle of diameter *d* and height *b*.




In the inner region $r < r_c$ liquid is moved by the impeller and forms a <u>forced vortex</u> region with tangential velocity $u_t = \omega r$. In the outer region $r > r_c$ liquid behaves more as in a <u>free vortex</u> with tangential velocity $u_t r$ = const.

$$u_t = \begin{cases} r\omega & r \le r_c \quad \textit{forced vortex} \\ \dfrac{\omega r_c^2}{r} & r \ge r_c \quad \textit{free vortex} \end{cases}$$

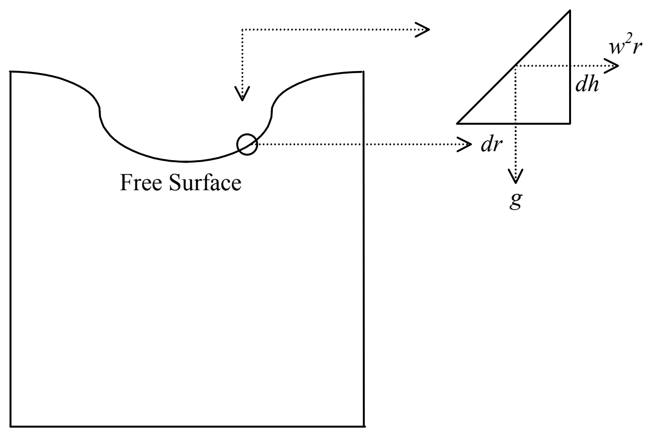
Naturally in the very vicinity of the wall $u_t = c\,(r_w - r)$

The <u>relative</u> velocity (tangential) between the impeller and liquid is

$$u_t = \omega\left(r - \frac{r_c^2}{r}\right) \qquad r_c \le r \le r_I$$

Since the liquid surface is <u>isobaric</u> its shape is determined by the balance of centrifugal and gravitational forces

**FIGURE 2:** Balance on the free liquid surface




$$\frac{dh}{dr} = \frac{\omega^2 r}{g}$$

$$h = h_o + \frac{\omega^2 r^2}{2g} = h_o + \frac{u^2}{2g}$$

Outside this region by Bernoulli's balance

$$h + \frac{u^2}{2g} = const \quad u = \frac{\omega r_c^2}{r}$$

$$h + \frac{\omega^2 r_c^4}{2gr^2} = h_c + \frac{\omega^2 r_c^4}{2gr_c^2} = h_o + \frac{\omega^2 r_c^2}{2g} + \frac{\omega^2 r_c^2}{2g} = h_o + \frac{\omega^2 r_c^2}{g}$$

$$h = h_o + \frac{\omega^2 r_c^2}{2g}\left[2 - \frac{r_c^2}{r^2}\right]$$

$$h_w = h_o + \frac{\omega^2 r_c^2}{2g}\left[2 - \frac{r_c^2}{r_w^2}\right]$$

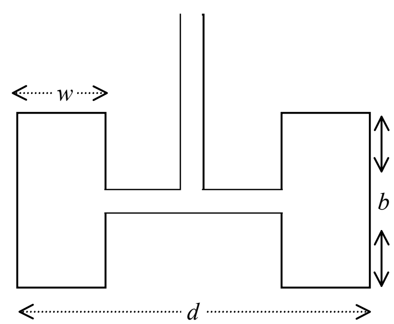
Solving for $r_c$ we get:

$$\left(r_c^2\right)^2 - 2r_\omega^2 r_c^2 + \frac{2g}{\omega^2} r_w^2 \left(h_w - h_o\right) = 0$$

$$r_c^2 = r_w^2 \left[ 1 - \sqrt{\frac{2g\left(h_w - h_o\right)}{\omega^2 r_w^2}} \right]$$

$$\omega = \frac{2N\pi}{60} = \frac{N\pi}{30} \quad N(RPM) \qquad \omega = \frac{2N\pi}{60}$$

**FIGURE 3:** Sketch of $A_2$ impeller




The <u>power</u> consumption for an impeller of fixed height, b, and diameter, d, increases with increase in width, w, of the impeller, until a point is reached when w = $r_I$-$r_c$. There the power peaks and equals the power of the paddle. Only the part of the paddle $r_I \geq r \geq r_c$ <u>accelerates</u> the liquid and contributes to power consumption.

Power = force x velocity of paddle
= density x (velocity)$^2$ x cross sectional area x paddle velocity

$$P_{paddle} = 2 \int_{o}^{w=r_I - r_c} K \rho u_r^2 \left(\omega r\right) b \, dw$$

$$u_r = \omega \left( r - \frac{r_c^2}{r} \right) \qquad r = r_I - w$$

$$P_{paddle} = 16\pi K \rho\, N^3 r_I^4 \left[ \frac{1}{4}\left( 1 - \left(\frac{r_c}{r_I}\right)^4 - \left(\frac{r_c}{r_I}\right)^2 \left( 1 - \left(\frac{r_c}{r_I}\right)^2 \right) \right) + \left(\frac{r_c}{r_I}\right)^4 \ell n \frac{r_I}{r_c} \right]$$

$$\frac{P_w}{P_{paddle}} \leq 1; \qquad \frac{P_w}{P_{paddle}} \approx 1 \quad \begin{cases} \dfrac{w}{R_I} > \ 0.3 \ \ \dfrac{r_c}{R_I} > 0.6 \\ \dfrac{w}{R_I} > 0.8 \ \ \dfrac{r_c}{r_I} = 0 \end{cases}$$

Data show

$$\frac{r_c}{r_I} = \frac{\text{Re}}{10^3 + 1.6\,\text{Re}} = 0.6 \quad to \quad 0.65 \; for \; \text{Re} > 4x10^4$$

$$\text{Re} = \frac{d^2 N}{\nu}$$

To evaluate the power required one should use the available correlations for the relationship among the dimensionless numbers.

$N_p = f\,(\text{Re},\text{Fr})$

$$N_p = \frac{\rho g_c}{\rho N^3 d^5} \qquad \text{- Power number}$$

$$\text{Re} = \frac{d^2 N}{\nu} \qquad \text{- Reynolds}$$

$$Fr = \frac{dN^2}{g} \qquad \text{- Froude}$$

Froude number correction for <u>well baffled</u> vessels is insignificant. Correction is important for vessels with free shaped surface.

$$N_p = \frac{\overbrace{A}^{laminar}}{\text{Re}} + B\left(\overbrace{\frac{10^3 + 0.6 f\ \text{Re}^{\alpha}}{10^3 + 1.6 f\ \text{Re}^{\alpha}}}^{turbulent}\right)^p$$

$$f \approx 2 \qquad \alpha \approx 0.66$$

A, B, p depend on impeller type, etc. baffled or unbaffled tanks.

See:

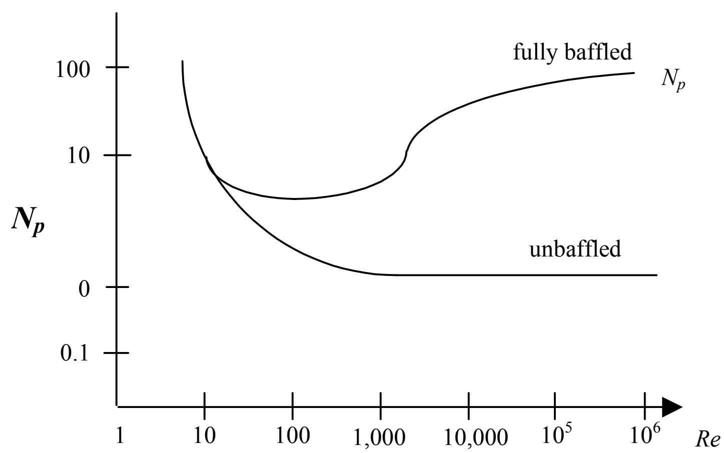
S. Nagata, "Mixing: Principles and Applications", Halsted Press, 1975.
J.Y. Oldshue, "Fluid Mixing Technology", McGraw Hill, 1983.

Fully baffled conditions:

$$\left(\frac{b_{baffle}}{D}\right)^{1.2} n_B = 0.35$$

$b_{baffle}$ - width of a baffle, $n_B$ = number of baffles

**FIGURE 4:** Power number as a function of Reynolds number for baffled and unbaffled stirred tanks.




Critical Reynolds number for transition from laminar to turbulent mixing is ill defined and lies in the range $10 < Re_c < 100$

Power at start up $\approx$ 15 power at steady state.

Laminar flow

$$\frac{Pg_c}{\rho N^3 d^5} = A\frac{\mu}{d^2 N\rho}$$

P = $A'\mu N^2 d^3 = A''\mu N^2 D^3$

Power is independent of density but directly proportional to viscosity in laminar mixing.

Turbulent flow

$$\frac{Pg_c}{\rho N^3 d^5} = K$$

P = $k'\rho N^3 D^5$

Power is independent of viscosity but proportional to density in turbulent mixing.

There is a forced vortex zone $(u = \omega r)$ in a range of 70% of $d_I$ and a quasi-free vortex zone $\left(u = \frac{\omega r_c^2}{r}\right)$ in the outer part, for case of water. With increase in $\mu$, $r_c$ is reduced and becomes zero at the transitional viscosity from turbulent to laminar case. The transition is a function of the power characteristics, discharge flow and mixing time.

**FIGURE 4a:** Turbine power correlation for baffled vessels.

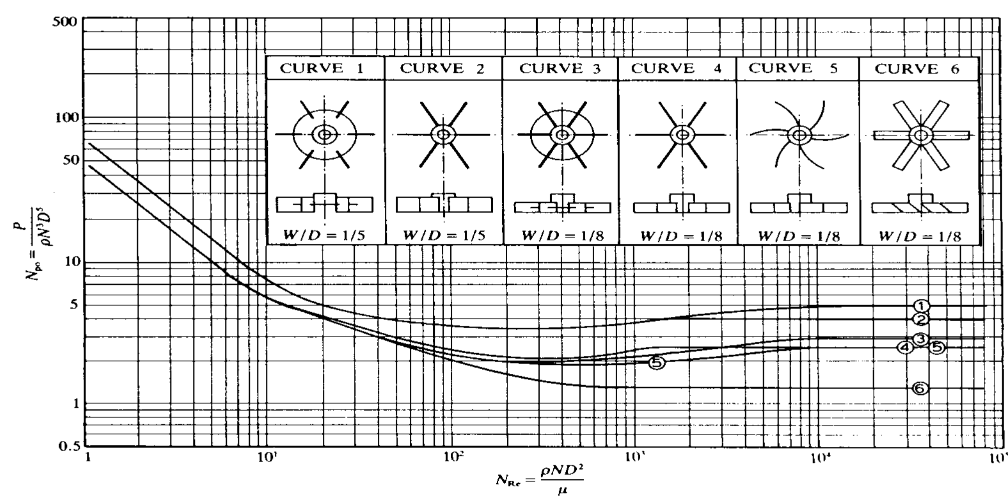
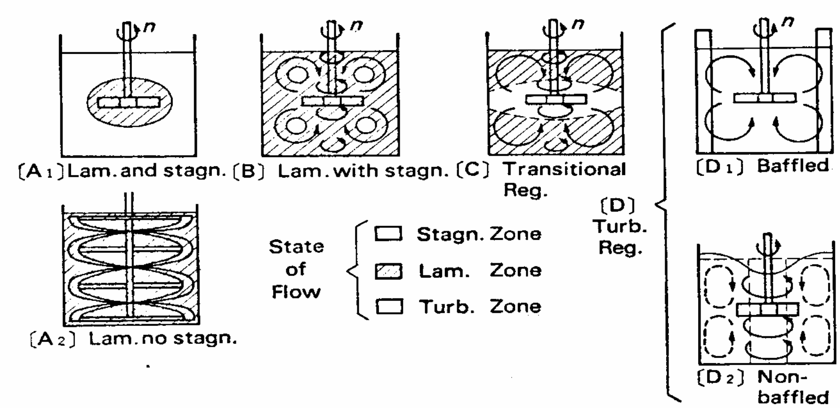



**FIGURE 5:** Various mixing conditions in stirred tanks.



Flow Pattern in Mixed Vessels

The observed flow pattern, as a function of Reynolds number, is illustrated schematically in Figure 5. The flow is a function of the dimensionless quantities listed below and the observed flow patterns *A* to *D* are described below.

$$N_P = \frac{Pg_c}{\rho n^3 d^5} \qquad \text{power number}$$

$$N_{qd} = \frac{qd}{nd^3} = \text{discharge flow number for the impeller}$$

$$N_m = \theta_m \cdot n = \text{dimensionless mixing time}$$

$q_d$ = discharge flow rate in ($cm^3/s$)

$\theta_m$ = mixing time

A. Re ≤ 10. Observed by tracers that the liquid in the impeller region moves with the impeller, the rest is stagnant. Resistance to impeller rotation mainly viscous, hence $N_P$ = A/Re. Centrifugal effect is negligible and discharge flow weak. To prevent stagnant zone one needs helical (spiral) large impeller.

B. 10 < Re < $Re_c$. Centrifugal force is felt and discharge flow develops and contributes to transfer of angular momentum to distant part of the liquid. Mixing is improved but stagnant domains of doughnut shape appear near the upper and lower parts of the impeller.

C. Re ≈ 0 (100). Stagnant regions vanish. Turbulent zone around the impeller spreads to the wall. Laminar and turbulent zones co-exist. Discharge flow increases and reaches maximum at transition Reynolds number (Re ≈ 90). Velocity of the liquid away from the impeller is in the laminar regime and small; velocity of impeller is large, hence, large discharge flow delivery.

Discharge flow rate is however the largest and grows asymptotically with Re in baffled fully turbulent vessels.

Remember in regions A, B, C baffles can only add to stagnancy and are of no help.

$r_c$ goes towards zero in transition flow.

D. Re > $10^3$. In fully turbulent unbaffled tank tangential flow dominates. There is a weak secondary circulation flow also superimposed on it. No stagnancy. $r_c$ increases now and reaches a constant value at large Re. Now the discharge flow sucks the liquid into the impeller region from the outside, strikes the wall at the impeller level with strong radial flow, turns upward and downward generating upward and downward flow by the wall and inverts to radial flow again and flows back axially down or up to the impeller region.

The discharge flow of the impeller is

$$q_d = 4\pi r_s \int_o^{z_p - \text{height of paddle}} v_r dz$$

The flow up the vessel walls is the circulation flow $q_c = 4\mu\pi \int_{ro}^{D/2} v_z r\, dr$

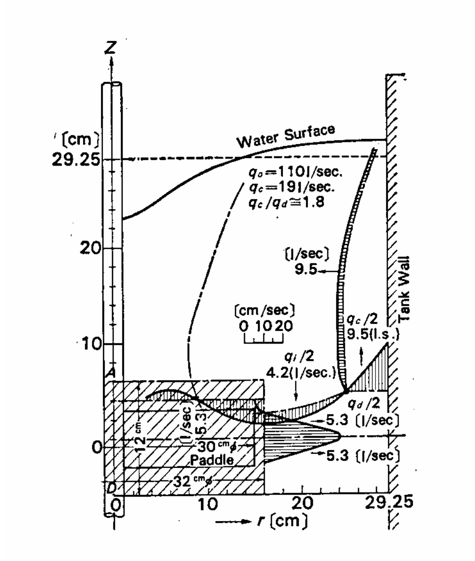
The flow moving toward the impeller (induced flow) :

$$q_t = 4\pi r_s \int_{z_p}^{z_s} v_r dz + 4\pi \int_{o}^{r_s} r v_z dr = q_d \text{ by mass balance}$$

Finally

$q_c = q_d$

**FIGURE 6:** Schematic diagram of secondary circulation flow (16-flat-blade paddle, 72 r.p.m.). (Fran Nagata, Mixing, 1975).




The point $r_o$, $z_o$ is the circulation eye where both radial and axial components are zero. $q_H$ - horizontal circulation flow is obtained from tangential velocity distribution.

Consider a vessel D = 58.5 cm with original stagnant liquid water height $H_o$ = 29.25 cm and a volume of $V_{water} = 78.6\ liters$

$d_I = 30\ cm \qquad \frac{d}{D} = 0.512$

$b = 6\ cm \qquad \frac{6}{d} = 0.2$

$n_p$ = 16 impeller blades

N = 72 rpm

$q_d$ = 10.6 lit/s

$q_i$ = 8.4 lit/s - induced into discharge from flow up and down side of the impeller

$q_c = q_d + q_i$ = 19 lit/s

$q_H$ = 110 lit/s $\qquad q_H >> q_c$

Baffles are needed to impede horizontal circulation and convert it to vertical flow.

D. At Re > $10^3$ in a baffled vessel the flow is fully turbulent. Baffles decrease the tangential velocity considerably and enhance radial and axial velocity.

Now $q_c$ = 74 l/s is 4 times larger than the flow in unbaffed vessel. Radial velocity is now equivalent to tangential as flow is discharged at 45˚ to the circumferential direction.

At high Re, discharge flow of the impeller generates vertical circulation and gives good mixing action.

Impellers by type:

1) radial flow - paddles and turbines
2) axial flow - propellers, and pitched blade turbines

Volumetric flow rate is proportional to $nd^3$ in completely turbulent range.

Discharge efficiency $\alpha \left( \frac{N_P}{Nq_d} \right)^{-1}$

$$1.3 < \frac{N_P}{Nq_d} < 3.6$$

but there are values up to 36 and propeller in a draft tube is the best at $\frac{N_P}{Nq_d}$ = 0.54.

Shear type impellers - large $\frac{N_P}{Nq_d}$

Circulation type impellers - low $N_P/Nq_d$

For best discharge efficiency:

- retreated blades
- large width
- pitched blades
- small $d_I/D$

Baffles increase circulation flow but also increase power consumption. With baffles $N_P/Nq_d$ goes up and efficiency goes down. The ratio $\frac{q_c}{q_d}$ is not much affected by baffles. Impeller superior to other when used without baffles is also superior when used with baffles.