CSE 547T Automata Theory February 13, 2017

Homework 2 Practice Problem Solutions

WARNING: if you haven't at least tried hard to solve the practice problems before reading these solutions, you are missing the point. If you can't make *any* progress, talk to me or to the TAs before reading these solutions. Otherwise, you should come up with a solution of your own that you can compare to the one shown here.

1. (Answers are in the book.)

2. **(a)** Let L be *any* nonregular language (pick your favorite). We claim that, if L is nonregular, so too is $\overline{L}$. Suppose not; i.e. suppose $\overline{L}$ is regular. The regular languages are closed under complement, so the complement of $\overline{L}$ must also be regular. But this complement is exactly L, which contradicts our assumption that L is not regular.

   Now neither L nor $\overline{L}$ are regular. However, their union $L \cup \overline{L}$ is by definition all of Σ^*, which is trivially regular. (For proof, build the one-state DFA accepting Σ^*, or observe that Σ is a finite language, and hence regular, and so Σ^* is regular.)

   (b) Let $L_1 = \{1^n 0^m \mid m > n\}$, and let $L_2 = \{0^m 1^n \mid m > n\}$. Each of these two languages is non-regular. For L_1, we can apply Myhill-Nerode Theorem as follows. Let $x_i = 1^i$, and let $z_i = 0^{i+1}$. For any pair $i < j$, $x_i z_i \in L_1$, but $x_j z_i \notin L_1$. A rather similar argument shows that L_2 is not regular either: set $x_i = 0^i$, and set $z_i = 1^i$ for $i > 0$.

   Now every string in L_1 either has form 0^i or includes a leading 1. Similarly, every string in L_2 either has form 0^i or includes a *trailing* 1. Thus, the intersection $L_1 \cap L_2$ contains only strings of the form 0^i. Moreover, for every $i > 0$, 0^i is in both L_1 and L_2, so their intersection is in fact $\{0^i \mid i > 0\}$. This language can be accepted by a two-state NFA, whose construction I will leave as an exercise.

   (c) Let $L = \{0^p \mid p \text{ is prime}\}$. We showed in class that L is non-regular. But by the proof in Problem 5.54, L^* is regular.

3. We claim that the equivalence classes of L are as follows:

   - For every $k \geq 0$, $[0^k]$ is a distinct equivalence class.
   - For every $m \geq 0$, $[0^{m+1}1]$ is a distinct equivalence class.
   - $[1]$ is a class distinct from any previous class.

   To prove distinctness, observe first that for $j \neq k$, the classes $[0^j]$ and $[0^k]$ are distinguished by the string 1^j, which completes the former but not the latter. Similarly, for $n \neq m$, $[0^{m+1}1]$ and $[0^{n+1}1]$ are distinguished by 1^m, which completes the former but not the latter. The classes $[0^k]$ and $[0^{m+1}1]$ are distinguished by 01^{k+1}, which completes only the former. Finally, observe that *no* string can complete 1, so $[1]$ is distinct from every other class.

   To show that our classes cover all of Σ^*, consider any string $x \in \Sigma^*$.

   - If x does not have the form 0^k or $0^k 1^j$, or has the latter form with $j > k$, then no string can complete it. Hence, $[x] = [1]$.

- If x consists only of zeros (or is ε), it has the form 0^k for some $k \geq 0$ and so belongs in $[0^k]$.
- If x has the form $0^k 1^j$ for $j, k > 0$, $j \leq k$, then let $m = k - j$. Observe that x is completed only by the string 1^m, which means its class is $[0^{m+1}1]$.