CSE 547T Automata Theory Spring Semester 2017

Homework 1: Languages, DFAs, and NFAs

Assigned: 1/25/2017 *Due Date: 2/13/2017*

This homework must be completed and submitted electronically. Formatting standards, submission procedures, and (optional) document templates for homeworks may be found at

`http://classes.engineering.wustl.edu/cse547/ehomework/ehomework-guide.html`

Advice on how to compose homeworks electronically, with links to relevant documentation for several different composition tools, may be found at

`http://classes.engineering.wustl.edu/cse547/ehomework/composing-tips.html`

Please remember to

- **create a separate PDF file (typeset or scanned) for each problem;**
- **include a header with your name, WUSTL key, and the homework number at the top of each page of each solution;**
- **include any figures (typeset or hand-drawn) inline or as floats;**
- **upload and submit your PDFs to Blackboard before class time on the due date.**

Always show your work and prove your constructions correct.

1. Let L be the language of all *duplicated* bit strings, that is, all strings of the form yy, for $y \in \{0,1\}^*$. Can L be constructed as $L_1 L_2$ for any two languages L_1, L_2, neither of which is equal to L? Prove your assertion.

2. Do Sipser's Problem 1.38 (1.43i).

3. For any string x, let x^R be the reverse of x. For example, the reverse of "abcde" is "edcba". Let L be a regular language, accepted by a DFA M. Prove that the following language L^R is also regular:

$$L^R = \{x \mid x^R \in L\}.$$

   I expect you to construct a finite automaton M^R (not necessarily a DFA) that accepts L^R. In this problem (and other problems asking you to construct one automaton from another), you should define M^R formally (i.e. as a 5-tuple) in terms of the 5-tuple for M.

4. (a) Let L be a regular language, accepted by a DFA M. Construct a finite automaton M_c, over the same alphabet as M, that accepts the language

$$L_c = \{x \mid \exists y \in L \text{ such that } |x| = |y|\}.$$

   (b) Let L be a regular language, accepted by a DFA M. Define the language

$$L^{1/2} = \{x \mid \exists y \text{ such that } |x| = |y| \text{ and } xy \in L\}.$$

   In other words, $L^{1/2}$ is the language of all first halves of (even-length) strings from L. Using the ideas from your constructions in (a), construct a finite automaton that accepts $L^{1/2}$. (*Hint*: you might want to use the idea described at the bottom of page 46 of your book about how to use Cartesian product to construct a DFA that accepts the intersection of two regular languages.)

5. Let L be an infinite language that is accepted by a DFA M. Say that a state q of M is *live* if, for some $x \in L$, there is a prefix y of x such that $\delta^*(q_0, y) = q$. In other words, some string of L causes M to pass through state q on its way to acceptance.

   Prove that there exists another DFA M' accepting L, such that M' contains strictly more live states than M. (*Hint*: you can use cross-product constructions as in the previous problem for intersection and union on DFAs.) Does your argument hold if L is finite? Why or why not?