CSE 547T Automata Theory	January 24, 2017

Homework 1 Practice Problems

Below is a set of practice problems on designing and proving the correctness of greedy algorithms, to help you check your understanding of the approach we've discussed in class. If you want even more practice problems, those problems and exercises marked "A" in the textbook have answers at the end of the chapter.

You can find solutions to these problems at the end of the chapter. We're also willing to listen to you describe your solutions or to look at your writeups during office hours.

Note: If you want to discuss these problems, I expect you to give correctness arguments just as on the actual homework.

Practice Problems

1. Show that the definitions given in class for $\delta(\psi, a)$ and $\delta^*(\psi, x)$ for a set of states ψ in an NFA satisfy the following two properties:

   - $\delta^*(\psi, \varepsilon) = \psi$.
   - If $x = ya$, $\delta^*(\psi, x) = \delta(\delta^*(\psi, y), a)$.

   (*Hint*: use the original definition of $\delta^*(q, a)$ for an NFA in your proof.)

2. Let $L \subset \{0, 1\}^*$ be the language of all bit strings that encode a binary number n (possibly with leading 0's), such that
   $$n \equiv 0 \pmod 3.$$
   For example, the strings "00011", "1001", and "0110" are all in L.

   Construct a DFA that recognizes L. For this problem, assume that the input strings are read *right-to-left* (that is, least significant bit first if interpreted as numbers). *Hint*: it may help to start by proving something about the value of 2^k mod 3 for arbitrary k.

3. Let M be a DFA accepting a language $L \subseteq \Sigma^*$. Show how to construct a DFA $\overline{M}$ accepting the language
   $$\overline{L} = \{x \in \Sigma^* \mid x \notin L\}.$$
   You should define $\overline{M}$ formally in terms of the 5-tuple for M and prove that it indeed accepts $\overline{L}$.

4. Consider the family of NFAs $\{M_k\}$, illustrated in Figure 1, over the alphabet $\Sigma = \{0, 1\}$. This family is parameterized by a positive integer k. All members of the family include the states labeled q_0, s_0, and q_a in the figure. The machine M_k also contains k intermediate states $s_1 \ldots s_k$ with transitions $s_{i-1} \to s_i$ on both 0 and 1 for $1 \le i \le k$.

   (a) Show that the NFA M_k accepts a language L_k defined as follows:
   $$L_k = \{x \in \{0, 1\}^* \mid x = yz, \text{ where } z \text{ has form } 1\{0, 1\}^k 1\}.$$

   (b) Show that for every $k \ge 1$, applying the lazy subset construction to M_k yields a DFA with $\Theta(2^k)$ states. Hence, the family M_k exhibits asymptotically worst-case expansion. *Hint*: find $\Theta(2^k)$ strings, each of which leaves M_k in a different set of states.

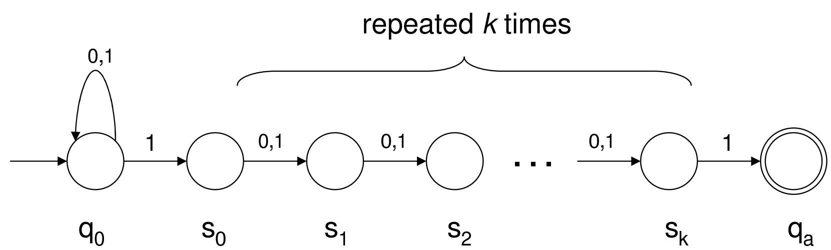

Figure 1: the NFA M_k, for $k \geq 1$.