CSE 547T Automata Theory Spring Semester 2017

Homework 0 (Optional): Practice Your Proofs!

Assigned: 1/18/2017 *Due Date: 1/25/2017*

This homework must be completed and submitted electronically. Formatting standards, submission procedures, and (optional) document templates for homeworks may be found at

`http://classes.engineering.wustl.edu/cse541/ehomework/ehomework-guide.html`

Advice on how to compose homeworks electronically, with links to relevant documentation for several different composition tools, may be found at

`http://classes.engineering.wustl.edu/cse541/ehomework/composing-tips.html`

Please remember to

- **create a separate PDF file (typeset or scanned) for each problem;**
- **include a header with your name, WUSTL key, and the homework number at the top of each page of each solution;**
- **include any figures (typeset or hand-drawn) inline or as floats;**
- **upload and submit your PDFs to Blackboard before class time on the due date.**

Always show your work.

An important prerequisite for this course is some ability to create and write correct proofs of simple propositions and to realize when you have failed to properly complete a proof. The proof techniques that are used most in this course are proof by contradiction (or indirect proof), induction (both weak and strong), and proof by cases.

To help each of you evaluate your readiness to use these proof techniques, I am providing this *optional* homework. This homework will in no way be part of your grade, so it would be pointless to look up solutions in notes or books that you have. Also, remember that it is much easier to understand a proof than create one; hence, this homework will be of little value if a friend tells you how he/she did the proof – even if you understand it completely once it is explained to you. The goal here is to see if you can solve these problems and provide convincing proofs *on your own*.

Some of these proofs are non-trivial and require some thought, but so do the proofs you will be doing throughout this course. If you need some guidance, that is fine – just come by office hours or make an appointment. If you aren't sure about whether or not your proofs for these problems are valid proofs, then I recommend that you submit your solutions. The TAs and I will read them and return them with comments.

1. Let $P(n)$ be the proposition that any n lines, where no two are parallel and no three pass through the same point, divide the plane into $n^2 + 1$ regions. What is wrong with the following inductive proof? It is not sufficient to give a counterexample to the theorem. Rather, you must find and describe the flaw in the proof.

   Theorem: $\forall n \geq 1,\ P(n)$

   Proof: By induction on n.
   Basis Step: 1 line divides the plane into 2 regions and $1^2 + 1 = 2$. Hence $P(1)$ is true.

   Inductive Step: We must show that $\forall n \geq 1\ P(n) \rightarrow P(n+1)$. By the inductive hypothesis there are n^2+1 regions formed with n lines. Note that $(n+1)^2+1 = n^2+1+2n+1$. So adding the $(n+1)$st line creates $2n+1$ new regions. Hence the number of regions with $n+1$ lines is $n^2+1+2n+1 = (n+1)^2+1$

   Since $P(1)$ is true and $\forall n \geq 1,\ (P(n) \rightarrow P(n+1))$, by the principle of mathematical induction we have that $\forall n \geq 1,\ P(n)$. ∎

2. Prove that n lines separate the plane into $(n^2 + n + 2)/2$ regions if no two of these lines are parallel and no three pass through a common point.

3. A *perfect number* is an integer which is equal to the sum of all its divisors except the number itself. Thus 6 is a perfect number, since 6 = 1+2+3, and so is 28. By definition, 1 is not considered to be a prime number.

   Prove that no perfect number is prime.

4. Prove that for all configurations of four points in the plane, it is possible to color each point either red or blue, so that *no* line can be drawn so as to place all red points on one side of the line and all blue points on the other.

   Hint: Use a proof by cases.