Recall the MAX-CUT problem: we are given a graph $G$, and we wish to partition the vertex set $V$ of $G$ into two parts $S$ and $V - S$, such that the number of edges crossing between the parts is maximized.

Let’s consider the following randomized algorithm for MAX-CUT. Initially, set $S$ empty. For each vertex $v \in V$, add $v$ to $S$ with probability $1/2$; otherwise, add it to $V - S$.

1. Prove that the expected size of the cut produced by this algorithm is $|E|/2$.

2. Describe how to derandomize this algorithm by the method of conditional expectations. Hint: Suppose you know that $A \subseteq S$ and $B \subseteq V - S$, where $A$ and $B$ are subsets of the vertices in $V$. How do you compute the expected cut size over the possible assignments of the remaining vertices?

3. Try to formulate an ILP that captures the MAX-CUT problem. (You need not build an LP approximation algorithm from this ILP.)

1 Shared Critique

Your TA will organize this part of the exercise. Be prepared to explain and defend your algorithms and proofs.