REVIEW PROBLEMS No. 27

Textbook: Problems 32.1, 32.2

Note: In question (d) of problem 32.2, the last equation should read

\[ E[W_i^\pi] = E[N_i^\pi] \cdot \frac{1}{\mu_i} \quad \text{(why?)} \]

i.e., \( \mu \) should be replaced by \( \mu_i \).

Problem S27.1 Consider a system identical to the M/G/1 queue except that whenever the system empties out, service does not restart until after there are \( k \) jobs in the system, where \( k \) is a fixed parameter. Once service starts, jobs are served as in the M/G/1 queue. Establish that

1. The following steady-state probabilities verify

\[ P\{\text{system empty}\} = \frac{1 - \rho}{k} \]
\[ P\{\text{system non-empty and waiting}\} = \frac{(k-1)(1-\rho)}{k} \]
\[ P\{\text{system non-empty and serving}\} = \rho \]

2. The average length of a busy period (non-empty system) is given by

\[ E[B_{(k)}] = \frac{\rho + k - 1}{\lambda(1 - \rho)} \]

Verify that \( E[B_{(k)}] \) is equal to the sum of the time between the arrival of the first job to an empty system and the start of its service time and the duration of \( k \) average busy periods in a regular M/G/1 system \((k = 1)\).

3. Assume that we are dividing the system’s busy period into busy/waiting and busy/serving periods. Show that the average number of customers in the system in a busy/waiting period is \( k/2 \) and that it is given by the expression of Eq. (1) for the busy/serving period.

\[ \frac{N_{\text{M/G/1}}}{\rho} + \frac{k - 1}{2} \quad \text{(1)} \]

where \( N_{\text{M/G/1}} \) is the average number in the system in the busy period of a regular M/G/1 queue \((k = 1)\).

[Hint: Relate the busy/serving portion of a busy period to \( k \) independent busy periods of the corresponding regular M/G/1 queue \((k = 1)\).]

4. The average number in the system is

\[ N_{\text{M/G/1}} + \frac{k - 1}{2} \]

Note: Problem S27.1 is a review of concepts seen earlier, and not about the material of Chapter 32.