HOMEWORK No. 5

Textbook: Problems 15.10^*, 17.1, 18.3,
* The most direct solution for Problem 15.10 relies on using z-transforms. A direct approach is possible but involves some computational tricks.

H5.1 Consider the closed queueing system of Fig. 1, and assume that there are M jobs in the system. Every job uses the CPU. After finishing service at the CPU, it then uses the i^{th} I/O sub-system with probability $p_i, i = 1, \ldots, m$, or returns immediately to the CPU with probability $p_0 = 1 - \sum_{i=1}^{m} p_i$. Jobs service times at the CPU are exponentially distributed with mean $\frac{1}{\mu_0}$, and service times at the i^{th} I/O sub-system are also exponentially distributed but with mean $\frac{1}{\mu_i}$. All service times are independent of each other, including services times of successive visits to the CPU or I/O sub-systems.

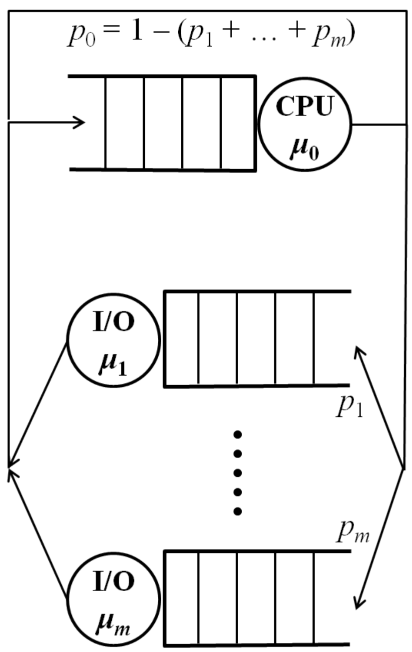

Figure 1: Closed queueing system.

What are (i) the jobs' arrival rate at the CPU, *i.e.,* the system throughput, and (ii) the distribution of the system occupancy, *i.e.,* the probabilities of a given number of jobs in each sub-system?

H5.2 Consider the closed transmission network of Fig. 2. Packets enter the network at A and leave at B. When a packet enters the network, it is first transmitted on one of k transmission lines $L_1, \ldots, L_k$ with an average transmission time of X. It is then transmitted on transmission line L_{k+1} with an average transmission time of Y. Packet transmissions are flow controlled by way of acknowledgments, and a maximum of N packets are allowed to have outstanding acknowledgments, *i.e.,* there can never be more than N packets in the network. Acknowledgments are sent each time a packet is delivered at B, and take Z units of time to reach A, at which time a new packet can be transmitted.

Use Little's Law to find upper and lower bounds for the system throughput under two different sets of assumptions.

1. The routing of packets to one of the transmissions lines $L_1, \ldots, L_k$ when they arrive at A is unspecified.

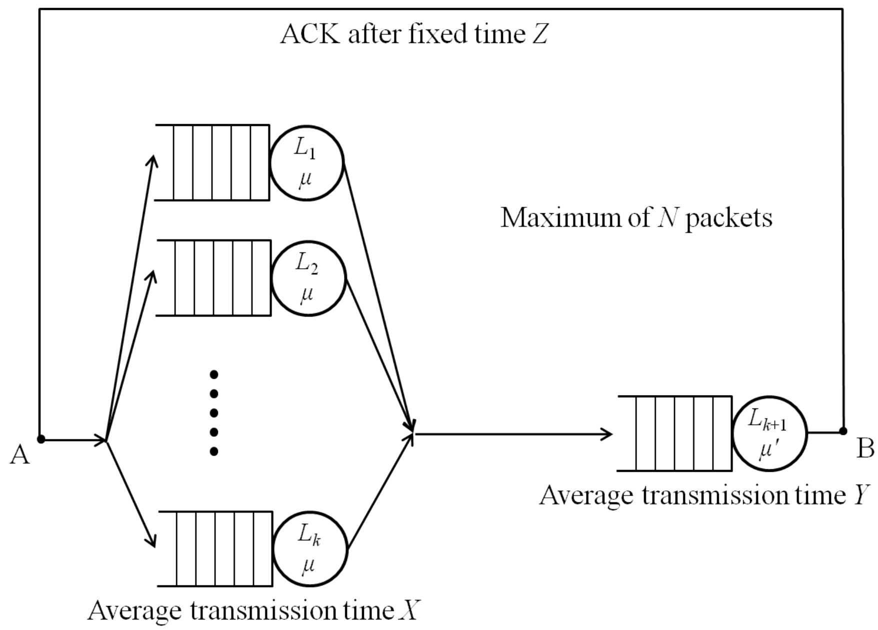

Figure 2: Closed transmission network.

2. The routing of packets entering the network at A is such that whenever one of the lines $L_1, \ldots, L_k$ is idle there are no packets waiting in a queue at any one of the other lines (those lines can, however, be busy transmitting a packet), *i.e.,* the routing realizes an efficient load-balancing.