CSE 538 – Fall 2016 Midterm
4 Problems – 65 points total

Your Name:

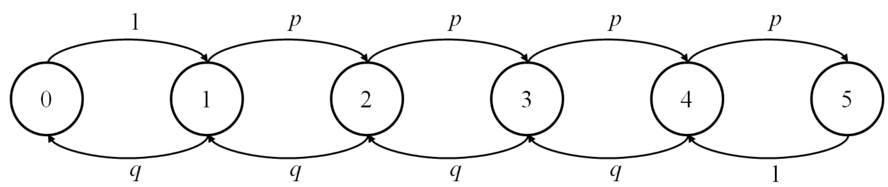

Figure 1: Discrete Time Markov Chain.

Problem 1 [10 points] Consider the Discrete Time Markov Chain (DTMC) of Fig. 1, where the parameters p and q satisfy $p + q = 1$.

1. **[5 points]** Does the chain admit a limiting distribution? If yes, does it for all combinations of p and q and why? If no, why not?

Write your answer in the space below

2. **[5 points]** Can you find values for p and q such that $\pi_0 = \pi_1 = \pi_2 = \pi_3 = 0$ and $\pi_4 = \pi_5 = 1/2$. **Hint**: What does the balance equation for state 5 tells you?

Write your answer in the space below

Problem 2 [25 points] Consider the closed system of Fig. 2 that consists of a dual CPU sub-system (CPU1+CPU2), where CPU1 can process 2 jobs per unit of time and CPU2 can process 4 jobs per unit of time, followed by a dual Disk sub-systems (Disk1+Disk2), where Disk1 can handle 1 job R/W operations per unit of time and Disk2 can handle 5 job R/W operations per unit of time. New jobs are assigned to CPU1 with probability p and to CPU2 with probability $1-p$. Similarly, when leaving the CPU sub-system, jobs are assigned to Disk1 with probability q and to Disk2 with probability $1-q$. The multi-programming level for the system is $N = 50$.

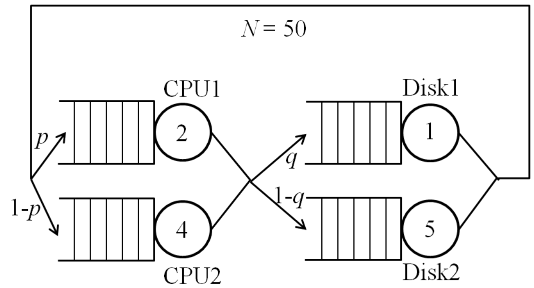

Figure 2: Closed system.

1. **[15 points]** Derive a tight upper bound for the number of jobs in the system, N^*, that corresponds to the threshold between low and high loads (multi-programming level or N value as per Theo. 7.1).
   Hint: Consider the CPU and Disk sub-systems separately and determine their individual thresholds.

Write your answer in the space below

2. **[5 points]** What values of p and q minimize the system response time $E[R]$? Provide an explicit upper bound for $E[R]$.

Write your answer in the space below

3. **[5 points]** You are now considering either replacing the two CPUs with a single faster CPU of speed 10 or alternatively the two disks with a single faster (and bigger) disk also of speed 10. Does either of these two options meaningfully improve the system response time. Justify your answer.

Write your answer in the space below

Problem 3 [20 points] Consider a system where jobs arrive according to a Poisson process of rate λ, and have service times whose duration is exponentially distributed with mean $1/\mu$. Jobs are, however, impatient, and, as illustrated in Fig. 3, each job that waits in the queue leaves after an exponentially distributed time also of mean $1/\mu$. In other words, jobs can leave the queue before they reach the server.

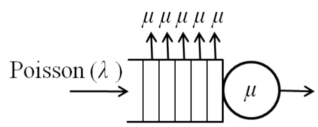

Figure 3: System with "impatient" customers.

1. **[5 points]** Give a Markov chain representation for the system, where the state is the number of jobs.

Write your answer in the space below

2. **[10 points]** Assuming that the chain is ergodic, provide an expression, function of λ and μ, for the probability π_i that there are i jobs in the system.

Write your answer in the space below

3. **[5 points]** Based on the expression of π_i, propose a *simple* condition that ensure that the chain is ergodic.

Write your answer in the space below

Problem 4 [10 points] Consider a single server queueing system with an infinite waiting room, and two types of jobs, where jobs of type $i, i = 1, 2$, arrive according to a Poisson process of rate λ_i. The service times of both types of jobs are exponentially distributed with mean $1/\mu$, where $\mu > \lambda_1 + \lambda_2$. What is the expected number of type i jobs in the system as a function of λ_1, λ_2 and μ?

Write your answer in the space below