

## REVIEW PROBLEMS No. 20

### Problem S20.1 Time sharing system

Consider jobs arrive at a time sharing system where the service discipline is based on processor sharing. Assume that the system has a unit service rate. The job arrival process is assumed to follow a Poisson process with rate of  $\lambda$  jobs/sec and job sizes are distributed according to a two phase Coxian with parameters  $\mu_1, \mu_2$ , and  $p$ , *i.e.*, jobs require on average  $\frac{1}{\mu_1}$  service in the first phase, move to the second phase with probability  $p$ , and require on average  $\frac{1}{\mu_2}$  service in that second phase.

1. What is the mean number of jobs in the system?
2. What is the average system response time for a job?
3. What is the average number of jobs in phase 1 in the system?
4. What is the average number of jobs in phase 2 in the system?
5. Assuming that a job's service time only requires phase 1, *i.e.*, it is a short job, what is the average system response time for such a job?
6. Assuming that a job's service time requires both phase 1 and phase 2, *i.e.*, it is a long job, what is the average system response time for such a job?

### Problem S20.2 Classed time sharing system

Assume that jobs arriving to a time sharing system are classified into  $n$  classes, with class  $i, i = 1, 2, \dots, n$ , submitting jobs according to a Poisson process or rate  $\lambda_i$ . Class  $i, i = 1, 2, \dots, n$ , jobs have a mean service time (assuming they are alone in the system) equal to  $h_i$ . The service rate of the time sharing system is  $\mu$ .

1. What is the mean system time of a job?
2. What is the mean system time of a job that requires a service time of  $x$ ?
3. What is the mean number of jobs in the system?
4. What is the mean number of class  $i$  jobs in the system?
5. Verify that your expressions for the mean number of class  $i$  jobs in the system are such that they add up to the mean number of jobs in the system.

**Hint:** You will need to use the result that the response time  $T(x)$  of a job that requires a service time of  $s$  (if alone in the system) is of the form  $T(x) = \alpha x$ , where  $\alpha$  is a constant, *i.e.*, the job's response time is proportional to its service time. Taking the average will allow you to find the value of the constant  $\alpha$ .