REVIEW PROBLEMS No. 20

Problem S20.1 Time sharing system

Consider jobs arrive at a time sharing system where the service discipline is based on processor sharing. Assume that the system has a unit service rate. The job arrival process is assumed to follow a Poisson process with rate of λ jobs/sec and job sizes are distributed according to a two phase Coxian with parameters μ_1, μ_2, and p, *i.e.,* jobs require on average $\frac{1}{\mu_1}$ service in the first phase, move to the second phase with probability p, and require on average $\frac{1}{\mu_2}$ service in that second phase.

1. What is the mean number of jobs in the system?
2. What is the average system response time for a job?
3. What is the average number of jobs in phase 1 in the system?
4. What is the average number of jobs in phase 2 in the system?
5. Assuming that a job's service time only requires phase 1, *i.e.,* it is a short job, what is the average system response time for such a job?
6. Assuming that a job's service time requires both phase 1 and phase 2, *i.e.,* it is a long job, what is the average system response time for such a job?

Problem S20.2 Classed time sharing system

Assume that jobs arriving to a time sharing system are classified into n classes, with class $i, i = 1, 2, \ldots, n$, submitting jobs according to a Poisson process or rate λ_i. Class $i, i = 1, 2, \ldots, n$, jobs have a mean service time (assuming they are alone in the system) equal to h_i. The service rate of the time sharing system is μ.

1. What is the mean system time of a job?
2. What is the mean system time of a job that requires a service time of s?
3. What is the mean number of jobs in the system?
4. What is the mean number of class i jobs in the system?
5. Verify that your expressions for the mean number of class i jobs in the system are such that they add up to the mean number of jobs in the system.

Hint: You will need to use the result that the response time $T(x)$ of a job that requires a service time of s (if alone in the system) is of the form $T(x) = \alpha x$, where α is a constant, *i.e.,* the job's response time is proportional to its service time. Taking the average will allow you to find the value of the constant α.