REVIEW PROBLEMS No. 9

Textbook: Problems 10.3 and 10.4 parts (a) only, *i.e.,* the derivation of the mean.; Problem 10.7

Problem S9.1 [Sum of Random Number of Random Variables] Assume that $X_1, X_2, X_3, \ldots$ are i.i.d. discrete random variables, all with the same distribution as the random variable X. Assume further that N is a positive, integer-valued discrete random variable independent of X_i for all i. Let

$$S = \sum_{i=1}^{N} X_i$$

Show that $\widehat{S}(z) = \widehat{N}\left(\widehat{X}(z)\right)$ where $\widehat{N}(z)$ and $\widehat{X}(z)$ are the z-transforms of N and X, respectively.

Note: Problem 10.7 relies on z-transforms and Problem S9.1 is meant to have you practice with z-transforms.