Quiz No. 2

Your Name:

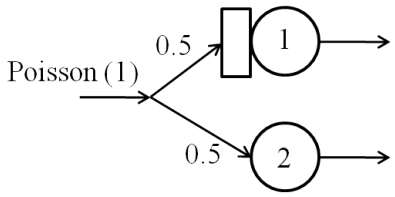

Figure 1: Two-Server System.

Problem 1 Consider the two-server system of Fig. 1. The first (top) server has a service rate of $\mu_1 = 1$ job/sec. The second (bottom) server has a rate of $\mu_2 = 2$ jobs/sec. Job sizes are exponentially distributed and jobs arrive according to a Poisson process of rate $\lambda = 1$ job/sec. The first (top) server sub-system can process one job at the time, and have at most one job waiting to be processed, *i.e.,* for a maximum of two jobs in that subsystem. Conversely, the second (bottom) subsystem has no waiting facility and can, therefore, only accommodate at most one job. When jobs arrive, they are randomly assigned to one of the two subsystems. Jobs assigned to the first (top) subsystem and that find the waiting space occupied are dropped. Jobs assigned to the second (bottom) subsystem and that find the server busy are dropped.

What is the probability P_B that a job is dropped?

Write your answer in the space below

The two subsystems can be represented through finite CTMCs. The first (top) one has three states, $0, 1, 2$, while the second (bottom) one has only two states, $0, 1$. The balance equations for the two subsystems yield

$$\begin{aligned} \pi_1^{(1)} &= \frac{1}{2}\pi_0^{(1)} \\ \pi_2^{(1)} &= \frac{1}{4}\pi_0^{(1)} \\ 1 &= \pi_0^{(1)} + \pi_1^{(1)} + \pi_2^{(1)} \\ \Rightarrow \pi_0^{(1)} &= \frac{4}{7}, \pi_1^{(1)} = \frac{2}{7}, \pi_2^{(1)} = \frac{1}{7} \end{aligned}$$

and

$$\begin{aligned} \pi_1^{(2)} &= \frac{1}{4}\pi_0^{(2)} \\ 1 &= \pi_1^{(2)} + \pi_0^{(2)} \\ \Rightarrow \pi_0^{(2)} &= \frac{4}{5}, \pi_1^{(2)} = \frac{1}{5} \end{aligned}$$

The blocking probability P_B is then of the form

$$P_B = 0.5\pi_2^{(1)} + 0.5\pi_1^{(2)} = \frac{6}{35} \approx 0.1715$$

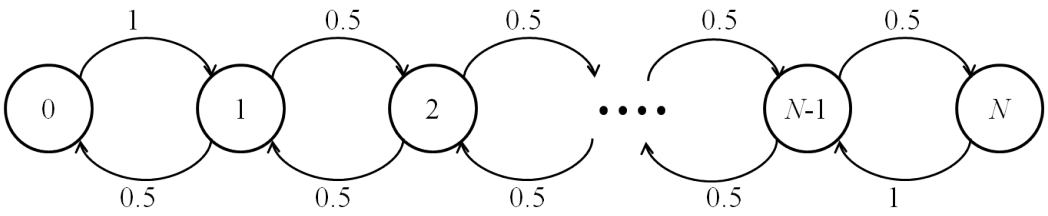

Figure 2: Discrete Time Markov Chain.

Problem 2 Consider the Discrete Time Markov Chain (DTMC) of Fig. 2. Does the chain admit a limiting distribution. If yes, what is the value of π_0. Justify your answer.

Write your answer in the space below

The chain does not admit a limiting distribution as it is periodic with period 2. This is readily seen since the return to any state always requires an even number of steps.