REVIEW PROBLEMS No. 27

Textbook: Problems 32.1, 32.2
Note: In question (d) of problem 32.2, the last equation should read

$$E[W_i^\pi] = E[N_i^\pi] \cdot \frac{1}{\mu_i} \qquad \text{(why??)}$$

i.e., μ should be replaced by μ_i.

Problem S27.1 Consider a system identical to the M/G/1 queue except that whenever the system empties out, service does not restart until after there are k jobs in the system, where k is a fixed parameter. Once service starts. jobs are served as in the M/G/1 queue. Establish that

1. The following steady-state probabilities verify

$$\begin{aligned} P\{\text{system empty}\} &= \frac{1-\rho}{k} \\ P\{\text{system non-empty and waiting}\} &= \frac{(k-1)(1-\rho)}{k} \\ P\{\text{system non-empty and serving}\} &= \rho \end{aligned}$$

2. The average length of a busy period (non-empty system) is given by

$$E[B_{(k)}] = \frac{\rho + k - 1}{\lambda(1-\rho)}$$

   Verify that $E[B_{(k)}]$ is equal to the sum of the time between the arrival of the first job to an empty system and the start of its service time and the duration of k average busy periods in a regular M/G/1 system $(k = 1)$.
3. Assume that we are dividing the system's busy period into busy/waiting and busy/serving periods. Show that the average number of customers in the system in a busy/waiting period is $k/2$ and that it is given by the expression of Eq. (1) for the busy/serving period.

$$\frac{N_{\text{M/G/1}}}{\rho} + \frac{k-1}{2} \tag{1}$$

   where $N_{\text{M/G/1}}$ is the average number in the system in the busy period of a regular M/G/1 queue $(k = 1)$. [**Hint**: Relate the busy/serving portion of a busy period to k independent busy periods of the corresponding regular M/G/1 queue $(k = 1)$].
4. The average number in the system is

$$N_{\text{M/G/1}} + \frac{k-1}{2}$$

Note: Problem S27.1 is a review of concepts seen earlier, and not about the material of Chapter 32.