REVIEW PROBLEMS No. 11

Textbook: Problems 12.1, 12.2, and 13.5 (questions a, b, c, d, h, i, and j) plus Problem S11.1 below.
Note: Problem 13.5 is an initial foray into a (finite) M/M/1 queue that is an example of a continuous time Markov chain. You will need to apply PASTA (Poisson Arrivals See Time Averages), which we already discussed a couple of times in class and will establish more formally in Chapter 13 (PASTA requires an additional technical assumption, namely, the *lack of anticipation assumption* (LOAA) that requires that inter-arrival times and service times be independent).

Problem S11.1 [Generator Matrix] Consider a single server system with an infinite waiting room, where jobs arrive according to a Poisson process of rate λ and were job service times are expoentially distributed with mean $\frac{1}{\mu}$, *i.e.,* this is a system that can be represented using a continuous time Markov chain (CTMC) and that is essentially the M/M/1 queue that we will analyze shortly.

We know (from Problem 12.2) that the "stationary equations" $\boldsymbol{\pi} = \boldsymbol{\pi} \boldsymbol{P}$ do not make sense for CTMCs. However, CTMCs satisfy another simple matrix equation for their limiting distributions. Use the matrix associated with the balance equations of the M/M/1 queue to find what that matrix equation would be in that case. Can you use this example to determine what this matrix would be in a general CTMC?