REVIEW PROBLEMS No. 18

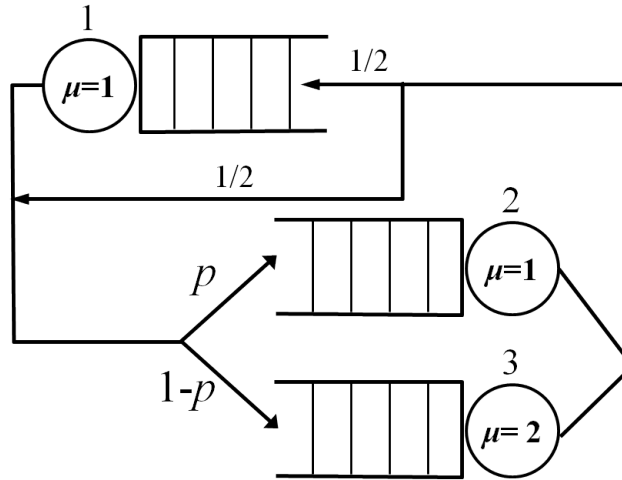**Textbook:** Problems 19.1, 19.2

Figure 1: Closed network.

Problem S18.1 Consider the closed networ of Fig. 1, which consists of three servers. Servers 1 and 2 have a unit processing rate, *i.e.,* $\mu_1 = \mu_2 = 1$, while server 3 is twice as fast, *i.e.,* $\mu_3 = 2$. There are $N = 2$ jobs in the system.

Jobs leaving server 2 or server 3 either visit next server 1 with probability $\frac{1}{2}$ or bypass it with probability $\frac{1}{2}$. Jobs that complete their service at server 1 or that bypass it entirely are then assigned to either server 2 or server 3 with probabilities p and $1 - p$, respectively.

Compute the value of p that minimizes the system response time $E[T]$. Note that the derivation must rely on the *exact* value of $E[T]$ and not bounds.

Hint: Use MVA to first compute $E[T_1^{(2)}]$, $E[T_2^{(2)}]$, and $E[T_3^{(2)}]$ for $N = 2$, and then $E[T]$. The optimal p value can then be computed from this expression.