Quiz No. 1

Problem 1 [5 points] Consider a processing system with a single, *unit rate* server handling two types of jobs, green (G) and red (R) jobs. Green jobs arrive at a rate of λ_G jobs/sec, and red jobs arrive at a rate of λ_R jobs/sec. Green jobs have an average service time of $E[S_G] = 1$, while red jobs have an average service time of $E[S_R] = 2$. The system has an infinite waiting facility were jobs can wait to access the server.

a) **[2 points]** Give a condition, function of λ_G and λ_R for the system to be stable (ergodic).
Answer: In order for the system to be stable, the amount of work arriving per unit of time needs to be less than the server capacity, *i.e.,*

$$\lambda_G \cdot 1 + \lambda_R \cdot 2 < 1$$

b) **[2 points]** Assuming a stable system, obtain an expression for the probability that the server is busy serving a red job?
Answer: The probability of serving a red job is equal to the expected number of red jobs in the server. Applying Little's Law to the server and focusing on red jobs, we have

$$E[N_R^{server}] = \lambda_R \cdot E[S_R] = 2\lambda_R$$

c) **[1 point]** Assuming that the average system time of a red job is $E[T_R] = 10$, what is the average *waiting* time (in the queue) of a red job.
Answer: The system time, T_R, of a red job is of the form $T_R = W_R + S_R$, so that $W_R = T_R - S_R$, which gives $W_R = 10 - 2 = 8$.

Problem 2 [5 points] Consider the closed interactive system depicted in Fig. 1, which consists of two servers of rates 1 and 2, respectively. There are $N = 10$ jobs in the system. When a job is submitted, it is assigned to the rate 1 server with probability $1/3$ and to the rate 2 server with probability $2/3$. When a job completes service it is immediately resubmitted to the service facility with probability $1/2$, or return to the job pool also with probability $1/2$. Once in the job pool, the job waits on average $Z = 2$ (its think time) before being resubmitted.

Give an expression for an upper bound for the system throughput X.

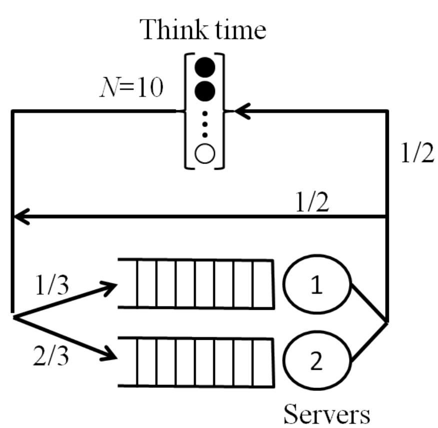

Figure 1: Closed server system.

Answser: We have $E[S_1] = 1$ and $E[S_2] = \frac{1}{2}$. Additionally, the average number of visits to the server facility is $E[V_{server}] = 2$, so that we have for individual servers $E[V_{server1}] = 2 \cdot \frac{1}{3} = \frac{2}{3}$ and $E[V_{server2}] = 2 \cdot \frac{2}{3} = \frac{4}{3}$. This gives $E[D_1] = \frac{2}{3} \cdot 1 = \frac{2}{3}$ and $E[D_2] = \frac{4}{3} \cdot \frac{1}{2} = \frac{2}{3}$. Hence $D = E[D_1] + E[D_2] = \frac{4}{3}$ and $D_{\max} = \frac{2}{3}$. This implies

$$N^* = \frac{D + Z}{D_{\max}} = \frac{\frac{4}{3} + \frac{6}{3}}{\frac{2}{3}} = 5$$

Since $N = 10 > 5$ we are in large N regime and the throughput is upper-bounded by

$$X \leq \frac{1}{D_{\max}} = \frac{3}{2} = 1.5 \text{ jobs/sec}$$