HOMEWORK No. 3

Textbook: Problems 11.5, 11.9, 11.11

Problem H3.1   Consider a single server system with an infinite waiting room. In each time slot, a new job arrives with probability p at the *end* of the time slot. Conversely, a job in service completes its service time with probability q at the end of each time slot (just before any arrival). Furthermore, a job that just finished its service is immediately resubmitted (to the end of the queue) with probability r, or leaves with probability $1 - r$.

1. What is the total average processing time of a job, *i.e.,* including all its visits to the processor?
2. Give a condition for the system to be stable.
3. Assuming a stable system, obtain an expression, function of p, q, r for the probability π_0 that the system is empty. Obtain this expression directly from first principles without solving the balance equations.

Problem H3.2   Consider again a single server system with an infinite waiting room. In each time slot, a new job arrives with probability p at the *end* of the time slot. Given that a job arrived in a slot, it is a "red" job with probability r and a "green" job with probability $g = 1 - r$. Both red and green jobs have the same service time distribution, namely, a job in service completes its service time with probability q at the end of each time slot (just before any arrival). The main difference between red and green jobs is that red jobs are blocked (dropped) if there are N or more jobs in the system when they arrive, while green jobs are always accepted.

1. Formulate a Markov chain representation for the system and use it to solve for the stationary probabilities under the assumption that the system is stable, *i.e.,* $p(1 - r) < q$.
2. What is the probability that a red job is blocked?