CSE464
Coupling Calculations via Odd/Even Modes
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Coupled Lines

Here are two coupled lines

\[ V_D = \frac{(V_1 - V_2)}{2} \quad I_D = \frac{(I_1 - I_2)}{2} \quad \text{(Odd, Difference)} \]

\[ V_C = \frac{(V_1 + V_2)}{2} \quad I_C = \frac{(I_1 + I_2)}{2} \quad \text{(Even, Common)} \]

\[ V_C = V_{Cf} + V_{Cr} \quad V_D = V_{Df} + V_{Dr} \]
Coupled Lines

- **Even-mode impedance,** $Z_{OE}$
  - impedance seen on each line by a common-mode signal
- **Odd-mode impedance,** $Z_{OO}$
  - impedance seen on each line by differential mode signal
- **Differential impedance,** $Z_{OD} = 2Z_{OO}$
  - impedance seen across a pair of lines by differential mode signal
- **Common-mode impedance,** $Z_{OC} = 0.5Z_{OE}$
  - impedance seen between a pair of lines and a common return by a common-mode signal.

\[
Z_{OE} = \left( \frac{L + M}{C - C_d} \right)^{\frac{1}{2}} \quad Z_{OO} = \left( \frac{L - M}{C + C_d} \right)^{\frac{1}{2}}
\]

- $C = \text{total line Capacitance (line to GND + } C_d)\$
- $C_d = \text{line to line capacitance}$
Coupled Lines

Matched termination
(No reflection)

\[ R_1 = Z_{OE} \]

\[ R_2 = 2 \left[ \frac{Z_{OO}Z_{OE}}{Z_{OE} - Z_{OO}} \right] \]

Coupled line equivalent circuit
(T-line only, no external components)

Equivalent circuit at sending end

\[ V_C = V_{C0r} ; \quad V_D = V_{D0r} \]

Equivalent circuit at receiving end

\[ V_C = V_{Clf} ; \quad V_D = V_{Dlf} \]
Coupled Lines

Matched termination (Wye) (No reflection)

\[ R_3 = Z_{oo} \]

\[ R_4 = \left[ \frac{Z_{OE} - Z_{OO}}{2} \right] \]
Coupled Lines

Procedure: (Assuming $V_1 = V_2 = 0 \ t < 0$)

1. Solve for $V_1 \ & \ V_2$ at sending end ($V_{10} \ & \ V_{20}$)

2. Find $V_{D0f} \ & \ V_{C0f}$ at sending end ($V_{D0r} = V_{C0r} = 0$)

3. Find $V_1 \ & \ V_2$ at receiving end

\[
\text{at } t = \frac{l^+}{v} \left( V_{1l} \left( \frac{l^+}{v} \right), V_{2l} \left( \frac{l^+}{v} \right) \right)
\]

4. Find $V_D \ & \ V_C$ at receiving end

5. Find $V_{Dr} \ & \ V_{Cr}$ at receiving end

\[
V_{Dlr} = V_{Dl} - V_{Dlf}; \ V_{Clr} = V_{Cl} - V_{Clf}
\]

6. Find $V_{10} \left( \frac{2l^+}{v} \right), V_{20} \left( \frac{2l^+}{v} \right)$ from EQ circuit

7. Continue until exhaustion or small changes
Backward Crosstalk Next

Boxes represent transmission line impedances, Resistor symbols are physical resistors. $R_t$ is an external termination resistor. $V_{20}$ is the voltage coupled into Line 2 at $x=0$. $V_c$ and $V_d$ are zero in this case since the far end is matched. The voltage on line 2, and the coupling coefficient is given by the voltage divider.

Line 2 at $x = 0$: $V_{20} = V_s \cdot \frac{R_1 // R_t}{R_2 + R_1 // R_t}$

$k_{rx} = \frac{R_1 // R_t}{R_2 + R_1 // R_t}$ (Dally Eq 6-16)
Forward Crosstalk Next

For inhomogeneous media, same procedure applies but now $v_C \neq v_D$. They do not arrive at same time so calculation is a little more tedious.
Forward Crosstalk Next (cont.)

\[ V_{2f} = V_{cl} - V_{Dt} \]

\[ V_{2f} \text{ forward crosstalk} \]

\[ \frac{l}{v_c} \quad \frac{l}{v_c} + T_r \]

\[ \frac{l}{v_D} \quad \frac{l}{v_D} + T_r \]

\[ \text{for } \quad \frac{l}{v_D} - \frac{1}{v_C} \leq T_r \quad \text{and} \quad v_D \leq v_C \]

\[ \text{for } \quad \frac{l}{v_C} - \frac{1}{v_D} \leq T_r \quad \text{and} \quad v_C \leq v_D \]

\[ V_{2f} = l \left( \frac{1}{v_D} - \frac{1}{v_C} \right) \cdot \frac{\Delta V_{Eif}}{T_r} \approx l \frac{1}{v_D} \left( 1 - \frac{v_D}{v_C} \right) \cdot \frac{1}{2} \frac{dV_s}{dt} = \frac{1}{2} \left( 1 - \frac{v_D}{v_C} \right) \cdot t_x \cdot \frac{dV_s}{dt} \]

\[ \text{Dally, Eq 6-18) } \quad k_{fx} = \frac{1}{2} \left( 1 - \frac{v_D}{v_C} \right) \]
Configurations

- \( V_O > V_E \)
  - Microstrip

- \( V_O < V_E \)
  - 300 Ohm TV or Twisted Pair or flat cable (ground alternate conductors, may have gnd plane)

- \( V_O = V_E \)
  - Ansley “Black Magic” flat cable. Note wide ground, narrow signal to reduce backward crosstalk
  - PC board stripline
Multiple Aggressors

- We calculate for single aggressor, use superposition for multiple aggressor lines
- Typical coupling is a little more than twice that for single aggressor line
What Else?

• Simulation (SPICE) is widely used in evaluating/calculating transmission line waveforms
  – Can easily deal with lossy lines, non-linear termination, etc
  – Time consuming to setup and evaluate
  – Has replaced measurement for the most part (still want to get some confirmation from measurement (reality))
  – You must know what questions to ask
  – If you don’t simulate critical cases, don’t learn much

• Parameters are obtained by Field Solver programs
  – Maxwell, Mentor, … ($50K or so)
  – Describe geometry, get L, C, etc
  – 2D (uniform cross-section) and 3D
Crosstalk Wrapup

- Non-Linear terminations: Use Bergeron method (covered later)
- Backward crosstalk lasts for round-trip propagation delay and amplitude is independent of coupled length (for $t_r < \text{round-trip delay}$)
- Forward crosstalk is proportional to length, zero for homogeneous dielectric, width equal to $T_r$ and amplitude inversely proportional to $T_r$. Typically small for moderate length with inhomogeneous dielectric.
- Both forward and backward crosstalk will be reflected from non-matched terminations.
- Separate signals, place signals close to ground plane, use differential signals, …
Need to Mention Limits

- We have not considered radiation or energy loss.
- When distance between conductor and return is small with respect to rise/fall time our approximation is good.
- When rise/fall time becomes comparable to signal-return spacing the system acts as antenna radiating energy.
  - Our model is no longer valid
    - Loss of signal amplitude or energy
    - EMI (electromagnetic interference) which is frowned on by FCC and any one with radio or tv receivers.
  - Not usually a problem of coupling to digital signals, energy is too small.
  - More complicated problem than digital crosstalk and not dealt with in CSE464. However, keep lateral dimensions and spacings small!
  - Electromagnetic fields and antennas