CSE 464
Homework 6, Solutions

Note: Please list at top of first sheet of homework submission anyone or anything from which you obtained any help for this homework assignment other than the text and class notes/discussion. Please give a word or two as to the nature of the help (e.g.: discussed problems, copied verbatim, whatever). Acknowledging source of help is a requirement for this assignment, and for all assignments in this course. It has no effect on your grade.

1. Assume a PC board that is 10" by 10" and has Vdd and GND 1-oz copper planes separated by 4 mils of FR4 dielectric with $\varepsilon_r=4$. To simplify grading please arrange answers in a 4x3 table, with parameter (a through d) in rows, and board type in columns.

   a) What is the plane to plane capacitance?
   b) What is the inductance between two 20 mil vias in the center of the board, one connected to GND and one to Vdd, to the circumference of the board if the circumference is approximated by a circle with a radius of 5 inches?
   c) What is the resistance (sum of both planes) from the 20 mil vias to the board circumference, approximated as above (5 inch radius circle)? 1 oz copper has resistance per square of 0.5 mΩ.
   d) What is the propagation delay from the vias to a point 5 inches away
   e) What are the new values for above parameters if the board is manufactured with 20 mil separation between the planes?
   f) What are the new values for above parameters if the board is manufactured with a new process that gives plane to plane separation of 2 mils, and has a relative dielectric constant of 20?

Of course, a spreadsheet:

\[
\begin{align*}
C &= \frac{\varepsilon_r^{22.5}\text{fF}/\text{in}^2}{h} \times \text{area} \\
L &= h(\text{mils}) \times \frac{5\text{pH} \times \ln(r_2/r_1)}{\text{area}} \\
R &= \frac{0.5\text{mOhms}}{2\pi} \times \ln(r_2/r_1) \\
\text{tpd} &= \frac{5''}{\nu \text{ ns}} \\
V &= \frac{1\text{ft}/\text{ns}}{\sqrt\varepsilon_r}
\end{align*}
\]

note: $5\text{pH} = \text{Lsquare}/2\pi = (\nu^2/c^2\pi)$

note: Rsquare = 0.5 mOhms for 1 oz copper (from text page 41)

<table>
<thead>
<tr>
<th>h (mils)</th>
<th>4</th>
<th>20</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Er</td>
<td>4</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>a) C plane-plane (nf)</td>
<td>22.5</td>
<td>4.5</td>
<td>225</td>
</tr>
<tr>
<td>b) L r1=10 mils, r2=5 in (pH)</td>
<td>124</td>
<td>62</td>
<td>62</td>
</tr>
<tr>
<td>c) R (both plane) (mOhms)</td>
<td>0.989</td>
<td>0.989</td>
<td>0.989</td>
</tr>
<tr>
<td>d) tpd for 5&quot; (ps)</td>
<td>833</td>
<td>833</td>
<td>1863</td>
</tr>
</tbody>
</table>

Other notes: Optionally, $L = \frac{\nu \ln(r_2/r_1)}{2\pi}$
2. 48V is supplied to a (large) piece of telecommunications equipment by two 100 foot bus bars (48V and 48V return), each 0.5 inch by 1 inch. They are separated on the 1 inch dimension by an insulator with relative dielectric constant of 4, and a thickness of 0.05 inch.

a) Calculate the characteristic impedance of this as a transmission line (use parallel plate approximation, ignore edge effects).

\[ Z_0 = \frac{377 \Omega}{\sqrt{\varepsilon_r}} \cdot \frac{0.05}{1} = 9.425 \Omega \]

b) Calculate the voltage loss from DC resistance (copper conductors) with a 200A load.

\[ R_{dc} = \frac{1.7 \times 10^{-8}}{0.0254} \times \frac{1200''}{.5''} \approx 1.6 \text{ m}\Omega \text{ for one conductor, 3.2 m}\Omega \text{ for both} \]

\[ V_{dc\text{ loss}} = 200A \times 3.2 \text{ m}\Omega = 640 \text{ mV} \]

c) If the source power supply is perfect (48V, 0 \Omega), the load is 200A connected by a circuit breaker located adjacent to the load, and the breaker opens in zero time, what is voltage on the 48V bus at the breaker just after the breaker opens?

When breaker opens, current goes to zero, but this is a transmission line (part a) so \( V_i \) remains the same, instantaneously.

\[ \Delta V = \Delta I \times Z_0 = 200 \times 9.425 = 1885 \text{ V} \]

\[ \text{so } V = 48 + 1885 = 1933 \text{ V} \]

d) If it is desired to keep the transient voltage (increase over DC value) to less than 20V, what impedance would be required for the bus?

requires \( \Delta I \times Z_0 \leq 20 \text{V} \) so \( Z_0 < \frac{20}{200} = 0.1 \text{ \Omega} \)

e) To achieve the impedance calculated in d), what capacitance would have to be added to each foot of the 48V bus? (This may not be a practical solution, but makes a good homework question.)

\[ Z_0 = \sqrt{\frac{L}{C}} \text{ and } Z_{0\text{ new}}/Z_{0\text{ old}} = \frac{.1}{9.425} = \sqrt{\frac{C_{old}}{C_{new}}} \]

\[ C_{old-ft} = (4 \times 225 \text{fF/ft}) \times 12 = 216 \text{ pF/ft} \]

\[ C_{new-ft} = 216 \times (9.425)^2 / 1^2 = 1.92 \text{ pF/ft} \]
3. My brand, new, super-performing CPU I just bought had a maximum current of 20 A. The maximum $\Delta V$ is 5% of its nominal 0.9V supply. The $k_r$ for this device is 0.5. The clock period is 500 ps. Design a decoupling network for the chip given that the power line coming to you has a parasitic $L = 100$ nH.

a) What is the limit on the parasitic inductance for the “small” high-frequency capacitor? (things that are maybe a few hundred nF)

b) Determine how many stages of capacitors you need given a parasitic $L$ as shown and specify what each stage will have (number and value of the capacitors):

<table>
<thead>
<tr>
<th>C</th>
<th>Parasitic L</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 uF</td>
<td>10 pH</td>
</tr>
<tr>
<td>2.2 uF</td>
<td>13 pH</td>
</tr>
<tr>
<td>4.7 uF</td>
<td>15 pH</td>
</tr>
<tr>
<td>10 uF</td>
<td>23 pH</td>
</tr>
<tr>
<td>47 uF</td>
<td>37 pH</td>
</tr>
<tr>
<td>100 uF</td>
<td>75 pH</td>
</tr>
<tr>
<td>470 uF</td>
<td>125 pH</td>
</tr>
<tr>
<td>1000 uF</td>
<td>500 pH</td>
</tr>
</tbody>
</table>
\[ C \geq 100_n \mu F \left( \frac{I}{n} \right)^2 = 10 \text{ mF} \]

\[ C_1 > \frac{I \cdot k \cdot R}{0.045} = 5.5 + 20 + 500 \text{ mF} \]

\[ = 310 \mu F \Rightarrow \text{ use } 250 \mu F \text{ nickel } \]

\[ \text{Need } \Delta V < \frac{\text{d}V}{\text{d}t} = \frac{0.045}{80} \cdot 10^{-3} \cdot 10^{-2} = 0.0007 \text{ pH} \]

\[ \text{for } C_2: \quad L_2 \leq C_1 (\frac{\Delta V}{I})^2 = 1.26 \text{ pH} \]

\[ \text{Choose } 1.0 \text{ mF } \approx 0.1 \text{ pH and } \approx 1 \text{ pH} \]

\[ \text{for } C_3: \quad L_3 \leq C_2 (\frac{\Delta V}{I})^2 = 50 \text{ pH} \]

\[ \text{Choose } 2.0 \text{ mF } \]

\[ \text{Note: } 2.0 \text{ mF} \text{ so we are done.} \]