Problem 7-1. We can consider optimization problems based on subset sum. Here is one we will call MAX-SUBSET-SUM: you are given a set of \( n \) non-negative integers \( X \) and a target \( t \), find a subset \( X' \) of \( X \) with the largest sum \( \leq t \).

(a) Give a dynamic program for calculating the exact answer and analyze it. The dynamic program will have running time pseudopolynomial — that is it will depend on the target \( t \).

(b) Here is a heuristic for this problem. First, let \( X_2 \subseteq X \) be the subset of elements of \( X \) that are \( > t/2 \). Let \( S \) be the set consisting of the largest element of \( X_2 \) if it is nonempty, or the empty set otherwise. Now sort the remaining elements of \( X - X_2 \) in non-increasing order. For each element in this list, add it to \( S \) if doing so would not cause \( S \)’s sum to exceed \( t \). Show that the above heuristic is a \( \frac{1}{2} \)-approximation for MAX-SUBSET-SUM.

(b) Show how to extend this heuristic into a \( \frac{k}{k+1} \)-approximation for any \( k \geq 2 \). What is the running time of your method for a given \( k \)?

Problem 7-2. K&T Chapter 11, Problem 10.