Reminder: You may work in groups and use outside sources. But, you must write up solutions in your own words and properly reference your sources for each problem. This includes listing your collaborators and properly citing any sources you use. Solutions to each problem must be electronically typeset and submitted online via Canvas.

Problem 2-1. The Missouri Department of Transportation has decided to plant wildflowers alongside Interstate 64 between St. Louis and Kansas City. MODOT has bid out the job and received \( n \) proposals. Say that St. Louis is at mile 0 of the interstate, while Kansas City is at mile \( D \). Proposal \( p_i \) offers to plant flowers over a (fixed) interval \( [s_i, e_i) \) of the highway, \( 0 \leq s_i \leq e_i \leq D \), for a fee of \( c_i \) dollars (\( c_i \geq 0 \) of course).

For political reasons, MODOT cannot accept only part of a proposal, so the endpoints \( s_i \) and \( e_i \) of each proposal are fixed. However, it can accept two proposals that overlap, since that simply results in some parts of the highway having more flowers.

Give an efficient algorithm to choose a subset of the proposals sufficient to plant flowers along the entire route from St. Louis to Kansas City, while minimizing the total cost.


(a) Give a divide and conquer algorithm that runs in time \( \Theta(n \lg n) \) time. You should prove the correctness of the algorithm and analyze its running time.

(b) Give a more efficient \( \Theta(n) \) running time algorithm.

Hint: This will not be a divide and conquer algorithm. You will have to use your creativity to solve this problem part.