CSE 347 Analysis of Algorithms September 6, 2017

Homework 1

Instructor: Jeremy Buhler/Brendan Juba *Due: September 13, 2017, 11:59 PM*

Reminder: You may work in groups and use outside sources. But, you must write up solutions in your own words and properly reference your sources for each problem. This includes listing your collaborators and properly citing any sources you use. Solutions to each problem must be electronically typeset and submitted online via Blackboard. Instructions appear in the E-Homework Guide: `http://classes.engineering.wustl.edu/cse347/ehomework/`

1. *Kleinberg & Tardos* Chapter 5, question 3

2. *Dimensionality reduction* is a widely-used technique for accelerating computations on high-dimensional data, by replacing them with a lower-dimensional "summary" that preserves the key properties of the original data, typically the approximate distances of points. The key step in *fast* dimensionality reduction is multiplying a vector by a matrix from the following family of special matrices, $H_0, H_1, \ldots, H_k, \ldots$:

   - H_0 is the 1×1 matrix $[1]$.
   - For $k > 0$, H_k is the $2^k \times 2^k$ matrix

   $$H_k = \left[\begin{array}{c|c} H_{k-1} & H_{k-1} \\ \hline H_{k-1} & -H_{k-1} \end{array}\right]$$

   What makes the method so fast is that, for a vector $\mathbf{v}$ of length $n = 2^k$, the matrix-vector product $H_k\mathbf{v}$ can be computed in time $O(n \log n)$. Show how to do this (and as usual, prove the correctness and time bound of your algorithm).

3. *Kleinberg & Tardos* Chapter 4, question 4