Given a second-order differential equation with a time-varying stiffness:

\[
d^2x/dt^2 + [K + \varepsilon(e^{-t} + \varepsilon e^{+t})]x = 0
\]

Assume an expansion of \( x \) of three terms and multiple time scales expansion also of three terms:

\[
x(t) = x_0(t_0, t_1, t_2) + \varepsilon x_1(t_0, t_1, t_2) + \varepsilon^2 x_2(t_0, t_1, t_2)
\]

\[
d( )/dt = \partial( )/\partial t_0 + \varepsilon \partial( )/\partial t_1 + \varepsilon^2 \partial( )/\partial t_2
\]

\[
t_0 = t \quad t_1 = \varepsilon t \quad t_2 = \varepsilon^2 t
\]

1.) Write three multiple time scale equations: one each for \( x_0, x_1, \) and \( x_2 \).

2.) For \( K = -1/4 \), solve the \( x_0 \) for \( x_0(t_0) \) and place it into the \( x_1 \) equation. By eliminating secular terms, show whether \( \varepsilon \) causes a frequency shift or a damping shift and, if so, how much.

3.) For \( K = -4 \), solve for \( x_0(t_0) \) and place it into the \( x_1 \) equation. After eliminating secular terms, solve for \( x_1(t_0) \). Hint: you should find that, in order to eliminate secular terms, there is no \( t_1 \) dependence.

4.) For \( K = -4 \), place your \( x_0 \) and \( x_1 \) solutions into the \( x_2 \) equation. By eliminating secular terms, show whether \( \varepsilon \) causes a frequency shift or a damping shift and, if so, how much.