ESE 588. Quantitative Image Processing

R. Martin Arthur


I. LECTURE #1
TAKE THE ROLL

A. Introduction

QUANTITATIVE image processing
→ N-DIMENSIONAL signal processing

1) Objective: To develop quantitative methods to sample, characterize, compress, analyze, recover and interpolate images. It is equally important, however, to apply our judgment and experience in evaluating and interpreting the results of those methods.

2) Image Characteristics: Signals: Continuous f(x,y) and discrete f[n,m]
Sample (S/H) ⇒ Continuous to Discrete Interpolate, Convert (D/A), Low-Pass Filter ⇒ Discrete to Continuous
Quantize (A/D) ⇒ Discrete to Digital for Representation in a computer

Signal-to-noise ratio (SNR)
1) Before we determine SNR, we need to know
⇒ What’s signal?
⇒ What’s noise?
Run → rmsplsf.m in ese588/mlfs
2) For what noise and signals are Gaussian and Poisson distributions appropriate descriptions?

Frequency content (characterize, solve DEs, design systems)
1) Ultrasonic B-scans of scatterers in turkey and simulated scatterers
   Show ⇒ usimagedistr.pdf
2) Dunes and mountains
   Run → imgsptr.m in ese588/mlfs
   What other image characteristics would you like to add?

⇒ Bounded image? Stable system? What’s the difference? NONE for LTI (LSI) systems!

Specific Signals?
⇒ Sinusoids? Exponentials? → e^{j\omega t} e^{j\omega n}
e^{j\omega t} = \cos(\omega t) + j \sin(\omega t)
⇒ Impulse? \delta(t) Unit-Sample? \delta[n]

3) Transfer Functions:
   Input → System → Output
⇒ In general there may be many levels of abstraction for a problem. Specifically, in our context there may be many subsystems, i.e., multiple transfer functions that apply to the same problem. State-space descriptions, however, are usually not used.

4) Website:
1) Catalog description (changes).
2) Images: generic, rma work with mpg
3) rma website
4) Syllabus
5) Handouts
6) Homework will be posted here. Be sure to check for assignments and DUE dates.
7) Office hours, test, final
8) Ethics
9) Matlab: Run → flpmandrill.m

B. Homework #1
Go over Homework assignment #1 DUE 10 September with 2D spectral matches.
Go over the form of the solutions and Matlab figure labeling and program documentation.
⇒ Prepare Detailed, Self-Contained SOLUTIONS. NOT JUST ANSWERS! (Answers only certify the process.)

C. Prerequisite Quiz
Prerequisite quiz
Solve quiz with 1D spectral matches

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II. LECTURE #2

Go over prerequisite quiz and homework assignment with requirement details.
Run \( \rightarrow \) wavespt.m in /tests. Go over Homework assignment #1 DUE 10 September with 2D spectral matches.
Go over the form of the solutions.
Run \( \rightarrow \) flpmandrill.m in /mfiles, which includes figure labeling and program documentation, and demonstrates properties of the 2D Fourier transform.

\( \Rightarrow \) Prepare Detailed, Self-Contained SOLUTIONS.
NOT JUST ANSWERS! (Answers only certify the process.)

- Definition of an Image
- Definition of a Digital Image

A. Vision: The Eye and Perception
B. Image Acquisition: Types of Digitizers
C. Geometric Operations

Spatial Transformation: 1) Pixel transfer method, 2) 2D affine spatial transformation (eq. 2.6-23),
\[
\begin{bmatrix}
  x \\
  y \\
  1
\end{bmatrix} = \begin{bmatrix}
  v \\
  w \\
  1
\end{bmatrix}T = \begin{bmatrix}
  t_{11} & t_{12} & 0 \\
  t_{21} & t_{22} & 0 \\
  t_{31} & t_{32} & 1
\end{bmatrix}
\]
(1)
3) Gray-level (color) interpolation
III. LECTURE #3

Go over Homework Assignment #2.
Analytic transformations and control grids


Show → tbme-00095-2012preprint.pdf in /lecture/docum
Show → mankin5.pdf in /lecture/docum

A. Geometric Operations
Review pixel filling and control grid
Bilinear transformation, equations 2.4-6

\[ v(x', y') = ax' + by' + cx'y' + d \] (2)

Bilinear gray-scale example
Bilinear control-grid example
Run → cntrlgrid.m in /mfls

IV. LECTURE #4

A. Arithmetic Operations
1) Image Averaging
2) Image Subtraction

B. Gray-level transformations
Run → mandrillhist.m in /mfls
1) Negative image → colormap (1-gray) on mandrill
2) Take log to see low-level pixels, eg, in image spectra using dB scale Open → imgsprt.m in /mfls
3) Power Law - exponent is \( \gamma \), (film exposure description)
4) Piece-wise linear
   - Contrast stretching → See histogram operations
   - Gray-level & bit-plane slicing
Run → flower2.m demo to show rgb components and rgb colormaps

C. Histogram-based Operations
Histograms.
- Point operations
- Histogram flattening
Histogram prediction through equation for \( H_B \).

V. LECTURE #5

Initial Topics (the big picture)
- Characteristics of human vision
- Geometric manipulation of images
- Arithmetic (point) operations on image pixels
  1) Addition
  2) Subtraction
  3) Enhancement using histograms
- Operations on collections of pixels (convolution)

Finish histogram flattening.
⇒ Compare mandrillhist.m (V) & manhist.m (M) results
Perform histogram matching by equating flattened histograms.

A. Spatial Filtering

The text makes a distinction between spatial and frequency-domain filtering. There is no difference in the result, just in the implementation.

\[ f(x,y) \rightarrow \text{System: } h(x,y) \rightarrow g(x,y) \]

For an \( M \times N \) image \( f(x,y) \) and an \( m \times n \) filter \( h(x,y) \), convolution can be written,

\[ g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} h(s,t) f(x+s,y+t) \] (3)

where \( a = (m-1)/2 \) and \( b = (n-1)/2 \), for m and n odd
Demonstrate
- Smoothing with → lpflt.m in /mfls
- Sharpening with → graddemo.m in /mfls

VI. LECTURE #6

Linear operations on \( D^2 \), \( D^3 \) and \( D^4 \)
Single-pixel operations
- Geometric: Translate, Rotate, Scale, Shear, Perspective using homogeneous-coordinate matrices & control grids
- Arithmetic: Add, Subtract, Histogram Enhance
Multiple pixel operations
- Interpolation
- Spatial Frequency Characterization
- Smoothing / Sharpening
- Restoration
- Feature Enhancement
- Coding, Data Reduction

A. 2D Fourier Transforms

- Analysis, Design, Restoration
- Eigenfunctions and the Convolution theorem
Run → convtst.m
1) Forms:
   - Continuous
   - Discrete-Space
   - Discrete
2) Zero-phase: Run → tdlin2zr.m
3) Convolution Algorithms:
- Direct
- Row/Column
- Row/Column with FFT

B. 2D Sampling
Run → hexsampl.m

VII. LECTURE #7
Periodic Sampling with Rectangular Geometry

A. Relation of DFT to FT
- Consistency of the discrete transform relations
- 2D to 1D FT with circular symmetry, example of the Hankel transform

B. Recovery
- Bandlimit
- Sampling density

C. Interpolation Expression
- Space
- Spatial Frequency

VIII. LECTURE #8
Periodic Sampling with Arbitrary Geometry
Run → circonv.m in /mfls

A. Fourier Transform Pairs

B. Sampling Matrices in the Fourier Transform
- Rectangular sampling in space
- Hexagonal sampling in spatial frequency

IX. LECTURE #9
Periodic Sampling with Arbitrary Geometry

A. Frequency-domain Design of Sampling Schemes
- Rectangular sampling example
- Hexagonal sampling example
- Other sampling geometries

B. Sampling Matrix in Space
- Rectangular sampling
  \[
  S_{\text{rect}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
  \]
- Hexagonal sampling
  \[
  S_{\text{hex}} = \frac{1}{2} \begin{bmatrix} \sqrt{3} & \sqrt{3} \\ 1 & -1 \end{bmatrix}
  \]

Example
A continuous image has the bandlimited spectrum given in Figure p2fig.gif below. Devise a minimum-sampling-density, rectangular-sampling scheme for this image. Determine (show each step in detail) the sampling matrix which will yield the minimum sampling density. What is the value of that minimum density? How much better (percentage reduction in sampling density) is your scheme compared to rectangular sampling? Account for this improvement by the reduction in unused spectrum in the rectangular sampling scheme which is used by your scheme. Note that a furlong is 1/8 mile, 220 yards, or 201.168 meters.

\[
S_{\text{cross}} = \frac{3}{5} \begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix}
\]

→ Run hexsamplmatr.m in /mfls
X. Lecture #10
Questions about sampling and/or Matlab exercises
Run \( \rightarrow \) flpmandrill.m in /mfls to illustrate DFT properties.
Run \( \rightarrow \) zpfromdftmag.m in /mfls to support problem 4.14.

2D System Design
1) Show high-pass filter without a specification goal.
   Run \( \rightarrow \) graddemo.m in /mfls
2) 2D design based on 1D filter design with circular rotation. Check response to see that specifications are met. Discuss circular rotation.
3) Run \( \rightarrow \) firpm.m Optimal linear-phase 1D design
   Run \( \rightarrow \) optflt.m in /mfls to show design issues with transition region and filter length. Show on log scale for specification test.

XI. Lecture #11
Questions about homework or presentations?

2D System Design
1) Sampling the ideal is not ideal because the interpolated function is not ideal.
   Run \( \rightarrow \) samfilt.m in /mfls
2) Matlab routines
   • freqz2 & fsamp2 & ftrans2
   • fwind1 & fwind2
3) Windowing
4) Optimal FIR Design
5) Chebyshev polynomials
   Run \( \rightarrow \) maclp.m and machp.m in /mfls to demonstrate Maclellan design

XII. Lecture #12
Go over Homework #5
Maclellan transform from 1D zero-phase \( h[n] \) (firpm) to 2D zero-phase \( h[n,m] \) with \( R = \{-N \leq n \leq N \} \)
\[
H[f] = \sum_{n=0}^{N} a[n]cos(2\pi fn) ; \quad \hat{H}[f] = \sum_{n=0}^{N} b[n]cos^n(2\pi f)
\]
To produce \( H(u, v) \) replace \( \cos(2\pi f) \) using Chebyshev polynomials \( T_n \) with \( \hat{F}(u,v) \)
\[
F(u,v) \rightarrow \cos(2\pi f) ,
\]
where \( F(u,v) \) is a zero-phase transformation system
\[
H(u,v) = \sum_{n=0}^{N} a[n]T_n[F(u,v)]
\]
Truncated \( a^nu[n] \) example

2D DFT
• Definition
• 3D example
• Properties

XIII. Lecture #13
Complete Maclellan design example with evaluation of \( T_{ij} \)

2D DFT
• Properties
• Radix 2 fast transform

XIV. Lecture #14

Image Restoration
• Discrete Image Representation
• Matrix manipulations
• Deconvolution \( \rightarrow \) Noise Amplifier
• Weiner filter
• Restoration with aliasing

XV. Lecture #15

Image Restoration
• Matrix implementation of convolution
• Matrix diagonalization with the Fourier transfrom
• System design with a Cost/Objective function
  – Least-squares-error constraint
  – Additional linear constraints
• Wiener filter

XVI. Lecture #16

Image Restoration
Run \( \rightarrow \) blurmandrill.m in /mfls: 2D system design and use
Run \( \rightarrow \) mandrillhilo.m in /mfls: convolution in frequency
Restoration variants for the Wiener filter
Noise Power Ratio to partition an image
Blurring function

- Run → rdtkefs.m in c:/usthrm/teknr
- Show psftekn000.gif to psftekn002.gif in lecture/docum

XVII. HELP SESSION
Sunday 28Oct, Bryan 305, 2-3PM

Closed-book, closed-notes, closed-aid-memoir Test on
- Material covered in class, including, but not limited to
  - Geometric operations & gray-scale interpolation
  - Arithmetic operations, image addition & subtraction
  - Histogram flattening
  - Spatial & spatial-frequency filtering
  - nD Fourier series & transform
  - Arbitrary periodic sampling
  - 2D system design, windows, Maclellan transformation, etc.
- Homework assignments #1 through #4
- Assigned text material through chapter 4, section 9

XVIII. LECTURE #17

Problems for the test
- Sampling the DTFT
- Histogram Equalization → Problem 3.11
- Image smoothing → Problem 4.27
- Other?

Restoration variants to the Wiener filter
- Parametric
- Iterative Restoration

XIX. SESSION #18 - TEST 1

XX. LECTURE #19

Go over homework assignment #6
Review syllabus, last assignment
Go over Presentation

The blurring function

Restoration in Non-random "noise"
Run → sinnoise.m in /mfls

XXI. LECTURE #20

Go over Test
Run → tst1588fb2.m in /tests/
Run → dftzerophase.m in /tests/

XXII. LECTURE #21

Review syllabus, last assignment, presentation

Multi-Resolution Analysis
- Approximation
- Residual (detail)
- Applications
  1) Registration & Segmentation
  2) Restoration & enhancement
  3) Noise reduction & Compression

XXIII. LECTURE #22

Introduction to Compression
1) Redundancy
2) Lossless versus Lossy
3) Color Vision: Show → chomaticity.pdf in lecture/figs/
XXIV. LECTURE #23

Compression Examples
1) Color Vision: jpeg Run → jpgcoding.m in /mfls
2) Run → wavedemo (1D)
   - Lossy baseline coding with the DCT (handout)
   - Extended encoding
     - More compression,
     - Greater precision
     - Progressive reconstruction
   - Lossless independent coding (reversible)
3) Constant area coding for text documents:
   - WBS (white block skipping) decoding
     WBS line: 0110010000010000100100000000, where 0 is a black pixel
4) Coding on the fly:

![Image of Lempel-Ziv-Welch vs Huffman Compression](image)

LZW (Lempel-Ziv-Welch), p.551
Run → seeing.m in /images
Show → Fig. 1

5) Singular value decomposition
Run → svdmandrill.m in /mfls with pause on, then off.

![Image of Table 8.28 8x8 n-bit Subimage](image)

XXV. LECTURE #24

Report: Format & Assignments

1) Pseudo random number added (LSBs) to break up contouring → IGS (Improved Gray-Scale) quantization.
   - Sum (initially zero) current pixel value and previous LSBs (typically 1/2 of word length).
   - If MSB sum is all ones, add zero to the sum (last entry of table below).
   - The MSB are the code word.

<table>
<thead>
<tr>
<th>Pixel</th>
<th>Gray Level</th>
<th>Sum</th>
<th>IGS Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>i - 1</td>
<td>N/A</td>
<td>00000000</td>
<td>N/A</td>
</tr>
<tr>
<td>i</td>
<td>01101100</td>
<td>01101100</td>
<td>0110</td>
</tr>
<tr>
<td>i + 1</td>
<td>10001011</td>
<td>10010111</td>
<td>1001</td>
</tr>
<tr>
<td>i + 2</td>
<td>10001011</td>
<td>10011110</td>
<td>1000</td>
</tr>
<tr>
<td>i + 3</td>
<td>11110100</td>
<td>11110100</td>
<td>1111</td>
</tr>
</tbody>
</table>

2) Quantization
   - Decision / Recovery
   - Error pdf
   - Vector quantization

3) Signal Processing Strategies

4) jpg coding → Run jpgcoding
   - shifting
   - DCT
     - Examine dct.m
     - Run → dctdemo.m, based on Matlab DCT demo with added comparison to DFT (from help dct, doc dct)
   - normalization (jnd/isopreference)
   - reconstruction error
XXVI. LECTURE #25

Report topics from website.
Problem 6.13 from G&W 1993 in /hassign.
Hankel transforms

We have covered introductory Image Processing:
- acquisition,
- enhancement,
- restoration and
- compression

This material included parametric representation that could be used for characterization, compression or feature extraction. The next steps often involve ad hoc or non-linear morphologic methods (9), image segmentation (10), and representation and description of images (11).

Among them, however, are techniques that build on the linear-systems framework we have developed, such as:
1) The Method of Moments and
2) Fourier Descriptors

Fourier Descriptors
1) FD notes Run → fdes.m in /mfls
2) Show → und_video.mpg in lecture/docum
3) Run → seerawe.m in vhm/raw with contours, upsampling and resampling, and partial torso model.
4) Show torso movie elpos.mpg in ese588/lecture

XXVII. LECTURE #26

Questions / Issues about Reports
The Fourier transform is important for image generation in solutions of
1) The wave equation in optics, seismic, sonar and ultrasonic imaging and
2) Computed tomography

Method of Moments
1) MOM notes.
3) Show figure 2 from momtof.pdf version of paper.

1) Wave equation
2) Temporal Fourier transform
3) 3D Spatial Frequency transform

XXVIII. LECTURE #27

Questions / Issues about Reports
Image processing Strategies
Computed Tomography
1) Radon transform
2) Fourier Slice theorem

XXIX. HELP SESSION

Sunday 12/9, Bryan 305, 2-3PM
Final with emphasis on material since the test is closed-book, closed-notes, closed-aid-memoir.
- Material covered in class, including, but not limited to
  - Geometric operations & gray-scale interpolation
  - Arithmetic operations, image addition & subtraction
  - Histogram flattening
  - Spatial & spatial-frequency filtering
  - nD Fourier series & transform
  - Arbitrary periodic sampling
  - 2D system design, windows, Maclellan transformation, etc.
  - Restoration
  - Multi-resolution analysis
  - Compression
  - Quantization
- All homework assignments
- Assigned text material

Compression problem solutions
- White-Block Skipping
- CCITT codes

XXX. FINAL

FRIDAY 12/14, CUPPLES I, ROOM 215, 6:00-8:00 PM