11.3 a) Check for consistency

By material balance: \[ E = \frac{V}{\sigma} = 69\% = 15 \text{ sec} \]

But from the experimental curve:
\[ E = \frac{1}{3} \left( 18 \times 1 + 21 \times 2 \right) = 20 \text{ sec} \]

The tracer comes out too late. Thus, the experiment was done incorrectly. Inconsistent, something is wrong.

11.5 b) From experiment

Mean of the curve: \[ \bar{E} = \frac{\sum EC}{2C} = \frac{30(15.5) + 65(30 + 2)}{105 + \frac{k}{2}} = 60 \text{ days} \]

Area under the curve: \[ A = \frac{(20 - 10) \times 10^6}{2} = 5 \times 10^5 \text{ unit-days} \]

From the material balance:
\[ A = \frac{M}{a} \]

i. \[ M = (\bar{E}) \sigma = \left( \frac{50.5 \times 10^6 \text{ unit-days}}{12000 \text{ m}^3} \right) \left( \frac{3600 \times 24 \times 365 \text{ days}}{\text{unit}} \right) = 7216 \text{ units} \]

b) Also because \[ \bar{E} = \frac{V}{\sigma} \]
\[ V = (\bar{E}) \sigma = 60 \text{ days} \left( \frac{5000 \text{ m}^3}{\text{day}} \right) \left( \frac{3600 \times 24 \times 365 \text{ days}}{\text{unit}} \right) = 2.11 \times 10^{10} \text{ m}^3 \]

11.9 Calculate the conversion for

\[ A \rightarrow R \quad C_{\text{A}0} = 6 \quad -r_A = 2 \text{ mol/} \beta \text{ mol} \cdot \text{min} \]

Zero order reaction

The performance equation is \[ \frac{C_A}{C_{\text{A}0}} = \left( \frac{C_A}{C_{\text{A}0}} \right) E dt \]

From Eq. 8.81 ... \[ \frac{C_A}{C_{\text{A}0}} = 1 - \frac{kt}{C_{\text{A}0}} \quad \text{for} \quad 0 < t < \frac{C_{\text{A}0}}{k} = \frac{6}{2} \quad \text{replace into above} \]

\[ \frac{C_A}{C_{\text{A}0}} = 0 \quad \text{for} \quad t > \frac{C_{\text{A}0}}{k} \]

\[ \frac{C_A}{C_{\text{A}0}} = \int_0^{2} \left( 1 - \frac{kt}{C_{\text{A}0}} \right) \frac{1}{3} dt = \int_0^{2} \left( 1 - \frac{3}{6} t \right) \frac{1}{3} dt = \frac{1}{6} \int_0^{2} (2 - t) dt \]

\[ = \frac{1}{6} \left[ \frac{t^2}{2} - t \right]_0^2 = \frac{1}{6} \left[ \frac{4}{2} - 2 \right] = \frac{1}{3} \]

\[ \Rightarrow \chi_{A} = \frac{2}{3} \]
12.1 This looks like two plug flow units side by side. From Fig 1 with
\[ V_{\text{total}} = 1 \text{ m}^3 \] and \( \delta = 1 \text{ m}^3 \text{ s}^{-1} \) we have
\[ \begin{align*}
    \delta_1 &= \frac{16}{16+24} = 0.4 \text{ m}^3 \text{ min}^{-1} \\
    \delta_2 &= \frac{24}{16+24} = 0.6 \text{ m}^3 \text{ min}^{-1} \\
    V_1 &= 1 \cdot \delta_1 = (0.25)(0.4) = 0.1 \text{ m}^3 \\
    V_2 &= 1 \cdot \delta_2 = (1.5)(0.6) = 0.9 \text{ m}^3
\end{align*} \]

12.5 Again look at Fig 1
\[ \begin{align*}
    \delta &= \text{ from experiment} \left\{ \begin{array}{l}
        E_p = 20 \\
        T_m = 30 \\
        T_{\text{actual}} = 50 \text{ s} \\
        T_{\text{total}} = \frac{V_p}{\delta} = 60 \text{ s} \\
        T_{\text{dead}} = 10 \text{ s}
    \end{array} \right. \\

    \text{Thus our model is}
\end{align*} \]
### 14.5 For plug flow

\[
\ln \frac{C_0}{C_A} = \ln \frac{1000}{1} = 6.9078
\]

For small deviation from plug flow, by the tanks in series model first calculate \( \sigma^2 \) from the tracer curve. From Fig 13-21A

\[
\sigma^2 = \frac{a^2}{N} = \frac{\frac{4}{24}}{24} = \frac{2}{3}
\]

\( \tau = 10 \)

From Eq 3

\[
\frac{1}{N} = \sigma^2 = \frac{\sigma^2}{\tau^2} = \frac{2}{(10)^2} = 0.0671 \times 10^{-2} \quad \text{...} \quad N = 150 \text{ tanks}
\]

For tanks in series

\[
\frac{C_A}{C_{AO}} = \left( 1 + \frac{\sigma^2}{\tau^2} \right)^{-\frac{1}{2}} = \left( 1 + 0.0671 \right)^{-\frac{1}{2}} = 0.866 = 0.0016
\]

\( \therefore C_A = 1.16 \)

---

### 14.9 For \( N \) tanks in series

Fig 25 pg 292 shows that ....

\[
N = 1 + 4 \left( \frac{\Theta_{max}}{\Delta \Theta} \right)^2 \quad \text{location of the maximum}
\]

Using a ruler with mm scale and a keen eye we find from Fig P21

- For 1st peak: \( N_{1_{peak}} = 1 + 4 \left( \frac{5.2}{2.0} \right)^2 = 104 \text{ tanks} \quad \ldots \quad \text{or} \quad N = 104 \text{ tanks/pas}\)
- For 2nd peak: \( N_{2_{peak}} = 1 + 4 \left( \frac{52.6}{4.6} \right)^2 = 202 \text{ tanks} \quad \ldots \quad \text{or} \quad N = 101 \text{ tanks/pas}\)
- For 3rd peak: \( N_{3_{peak}} = 1 + 4 \left( \frac{48.7}{5.0} \right)^2 = 204 \text{ tanks} \quad \ldots \quad \text{or} \quad N = 101 \text{ tanks/pas}\)
- For 4th peak: \( N_{4_{peak}} = 1 + 4 \left( \frac{45}{4.6} \right)^2 = 389 \text{ tanks} \quad \ldots \quad \text{or} \quad N = 97 \text{ tanks/pas}\)

These values average to \( N \approx 100 \text{ tanks/pas} \)

---

![Physical setup](image) ![Measurement point](image) ![Model](image)
14.11 To find the non ideal characteristics of the experimental reactor determine the proper $D/uL$ to use for the dispersion model, or the proper $N$ value to use for the tanks-in-series model.

This is done in one of two ways — by matching the experimental tracer curve with the family of curves shown in Fig. 13.9 (for the dispersion model) or with Fig. 12 (for the tanks-in-series model), or by calculating $\sigma^2$, and from that $D/uL$ or $N$.

Let us use the latter procedure. So first calculate $\bar{E}$ and $\sigma^2$ from the table of data with Eqs. 13.1 and 13.3. This gives

\[
\bar{E} = 2.13 \quad \bar{t} = 2.199 \quad \bar{D} = 10.09 \quad \bar{E} = 2.149
\]

\[
\bar{t}^2 = 2.762 \quad \bar{E}^2 = 0.7668 \quad \bar{D}^2 = 10.08 \quad \bar{t}^2 = 2.762
\]

Next determine the behavior in an ideal plug flow reactor

\[
k = 0.456 \quad k = (0.456)(10.09) = 4.6
\]

So for plug flow

\[
Y_t = 1 - e^{-kt} = 1 - e^{-4.6} = 0.99
\]

Now we are ready to proceed with our problem.

(a) Use the dispersion model

Here Eq. 13.15 relates $\sigma^2$ with $D/uL$. So

\[
\sigma^2 = 0.7878 = 2(D/uL) - 2(D/uL)\left[1 - e^{-4.6}\right]
\]

Solve by trial and error. This gives $D/uL = 1$

Then from Fig. 13.19

\[
D/uL = 1 \quad \text{from this figure}
\]

\[
Y_t = 0.99 \quad \text{disp.}
\]
(b) Use the tanks-in-series model (continued) from Eq. 3 we find

\[ N = \frac{1}{\epsilon_0} = \frac{1}{0.1288} = 7.864 \text{ tanks} \]

So from Fig. 6.5 we find \( X_{\text{add}} = 68\% \)

(c) Use the tracer data directly.

From Eq. 11.13 \( \frac{C}{C_0} = \frac{E}{E_0} \text{ batch} \)

To find the \( E \) curve make the area under the \( C \) curve unity, or as shown in Example 11.4

\[ E = \frac{\text{area}}{E_0} = \frac{C}{E_0} \text{ batch} \quad \text{or} \quad E \Delta t = \frac{C}{E_0} \text{ batch} \]

\begin{tabular}{|c|c|c|c|}
\hline
\( t_{\text{min}} \) & \( C \) & \( e^{x \cdot 0.456 \cdot t} \cdot C_{\text{batch}} \) & \text{area} \\
\hline
0 & 0 & 0 & 0 \\
1 & 17 & 0.0506 & 17 \times 0.0506 \\
3 & 33 & 0.0454 & 33 \times 0.0454 \\
5 & 53 & 0.0158 & 53 \times 0.0158 \\
7 & 26 & 0.0050 & 26 \times 0.0050 \\
9 & 20 & 0.0015 & 20 \times 0.0015 \\
11 & 16 & 0.0005 & 16 \times 0.0005 \\
13 & 13 & 0.0022 & 13 \times 0.0022 \\
15 & 10 & 0.0001 & 10 \times 0.0001 \\
17 & 8 & 1.8 \times 10^{-4} & 8 \times 1.8 \times 10^{-4} \\
19 & 6 & 5 \times 10^{-5} & 6 \times 5 \times 10^{-5} \\
21 & 5 & 2 \times 10^{-5} & 5 \times 2 \times 10^{-5} \\
23 & 4 & 1 \times 10^{-6} & 4 \times 1 \times 10^{-6} \\
25-27 & 3 & - & - \\
29-31 & 2 & - & - \\
33-45 & 1 & - & 1 \times 0.1191 \\
\hline
\end{tabular}

\[ x_{\text{from curve}} = 0.88 \]

(d) Which answer is most reliable

Naturally the direct use of the tracer curve gives the most reliable answer. In this problem the given RTD came from the dispersion model with \( D_h/L = 1 \). Thus we'd expect that the answers to parts a) and c) should agree. They do.