Heat regenerators are often employed when the use of heat recuperators is either uneconomical, because of the enormous heat-transfer areas required, or impractical, due to the likelihood of surface fouling by particulate-laden gases. The use of regenerators abounds in the metallurgical industry, glass manufacturing, air-separation plants, Fischer-Tropsch synthesis, storage of solar energy, etc. The above areas involve mainly large units processing enormous flow rates of gases. However, in Europe, the use of small, rotary (Ljungstrom) regenerators has been widespread for recovering heat from exhaust gases and preheating inlet air in commercial and residential oil- and coal-fired furnaces. With current consciousness about energy costs, an increased number of chemical engineers will likely be working with heat regenerators.

Unfortunately, most heat-transfer books offer only a passing reference on this subject, with the exception of Jakob [1] and Schack [2], which present a sound, but somewhat outdated, treatment. There are two monographs on this subject, one by Hausen [3] and the other by Schmidt and Willmott [4]. Although the treatment in the above four texts is thorough, the jargon used in this field is somewhat alien to outsiders, especially chemical engineers. Moreover, if the user is searching for a simple and fast, yet accurate evaluation method of regenerator performance, the literature falls short. Design charts are presented [5, 6], but they usually suffer from two major drawbacks; they are model-dependent and their notation requires considerable deciphering.

Here, we will familiarize chemical engineers with heat regenerator concepts, briefly review what these devices are, and discuss which types are frequently used and how they are modeled. Then, based on results of the authors' recent studies, we will present simple formulas for evaluation of regenerator performance. These do not rely on elaborate computer algorithms and can be used with the help of tabulated functions, pocket calculators, etc.

**Principles of Operation**

In a heat regenerator, heat is not exchanged directly between hot and cold fluids across a separating solid wall. Instead, hot and cold gases flow alternately over solids that periodically absorb and release heat. Advantage is taken of high solids heat-capacity per unit volume, compared with that of gases, to make the solids an efficient medium to transfer heat between hot and
cold gases. A regenerator typically operates cyclically. When hot gas flows past the solids, it heats them while being cooled. This heating period is followed by flow of cold gas over the hot solids—the cooling period.

Heat regenerators can be used either continuously to heat a cold gas and recover heat from a hot one, or to store thermal energy for use later on. In the former case, regenerators are usually employed in plant operations; in the latter, they are used for storage of solar energy. For continuous heating and cooling, the main task is to ensure maximum energy recovery (the best thermal efficiency); for the other use, the job is to find the fraction of energy that has been stored (heat storage factor) and predict heat leaks during storage. Here, we will mainly consider continuous regenerator operation, but will also address single-pass operation as part of the storage problem.

For continuous operation, two or more regenerators are used in parallel. While hot gas passes into one, cold gas flows through the other; and gas flows are switched at appropriate times. This is called swing operation. The alternative is to rotate the solids between the hot and cold gas streams, as is done in a rotary (Ljungstrom) regenerator. Since the analysis of the rotary regenerator is analogous to that of swing units, we will concentrate our attention on swing regenerators only. Cocurrent operation takes place when hot and cold gases are introduced successively at the same end of the unit; countercurrent operation when the gases are introduced at opposite ends. Schematics are in Fig. 1.
Thermal Efficiency

The key performance index for a regenerator is its thermal efficiency, $E$:

$$E = \frac{\text{heat actually transferred during time } \theta}{\text{maximum possible heat transfer during time } \theta}$$  \hspace{1cm} (1)

Assume, in continuous operation, that heat losses to surroundings are negligible and that mean physical properties, independent of temperature, can be used. Also, assume constant mass flow rate and gas inlet temperature. Then:

Total heat actually transferred during time $\theta$

$$= \sum_0^\theta \left[ \left( \text{enthalpy of inlet gas} \right) - \left( \text{enthalpy of outlet gas} \right) \left( \text{interval } d\theta \right) \right]$$

$$= (m_g C_{pg}) \int_0^\theta \left[ T_{hi} - T_{he}(\theta) \right] d\theta$$  \hspace{1cm} (2)

Maximum possible heat transfer would be accomplished if the temperature of the hot gas for the duration $\theta$ were reduced to the initial solids temperature, $T_s^0$, which, if everything operated ideally, would be equal to the inlet cold gas temperature, $T_{ci}$, and is taken as such.

Maximum possible heat transferred during time $\theta$

$$= (m_g C_{pg})(T_{hi} - T_{ci}) \theta$$  \hspace{1cm} (3)

Substituting (2) and (3) into Eq. (1), we get the following expression for thermal efficiency during the heating period:

$$E_h = \frac{\int_0^\theta \left( T_{hi} - T_{he}(\theta) \right) d\theta}{(\theta)T_{hi} - T_{ci}} = \frac{1}{\theta} \int_0^\theta \left[ 1 - t_{he}(\theta) \right] d\theta$$  \hspace{1cm} (4)

We define dimensionless exit temperature by

$$t_{he}(\theta) = (T_{he}(\theta) - T_{ci})/(T_{hi} - T_{ci})$$  \hspace{1cm} (5)

The higher $t_{he}(\theta)$, the hotter the exiting gas is during the heating period. This implies that a smaller amount of heat has been exchanged and, hence, efficiency is lower. Analogously, we define thermal efficiency for the cooling period, based on cold-gas exit temperature:

$$E_c = \frac{1}{\theta} \int_0^\theta t_{ce}(\theta) d\theta$$  \hspace{1cm} (6)

where

$$t_{ce}(\theta) = (T_{ce}(\theta) - T_{ci})/(T_{hi} - T_{ci})$$  \hspace{1cm} (7)
Often, the heat storage factor is also used [4]:

\[ q = \frac{\text{thermal energy actually stored in the solids}}{\text{maximum thermal energy storage in the solids}} \quad (8) \]

\[ q = \frac{(\dot{m}_s C_{ps})_h \int_0^\theta (T_{he} - T_{he}(\theta)) \, d\theta}{M_s C_{ps}(T_{hi} - T_{ci})} \quad (9) \]

Here, \( M_s = \rho_s(1 - \epsilon_s)V_R \). Using the formulas for thermal efficiency and dimensionless gas exit-temperature (Eqs. 4 and 5), we can reduce the expression for \( q \) to

\[ q = E \theta / \mu \quad (10) \]

where

\[ \mu = \frac{M_s C_{ps}}{\dot{m}_s C_{ps}} = \frac{\rho_s(1 - \epsilon_s)C_{ps}L}{G_s C_{ps}} \quad (11) \]

In some applications, a measure of exit-gas temperature deviation around the mean may be desired:

\[ D = \frac{1}{\theta} \int_0^\theta (t_{he} - \bar{t}_{he})^2 \, d\theta = \frac{1}{\theta} \int_0^\theta t_{he}^2 \, d\theta - (1 - E)^2 \quad (12) \]

where

\[ \bar{t}_{he} = \frac{1}{\theta} \int_0^\theta t_{he} \, d\theta = 1 - E \quad (13) \]

To relate the thermal efficiency, \( E \), heat storage factor, \( q \), or temperature variance, \( D \), to regenerator design parameters and operating conditions, we must evaluate the dimensionless exit temperatures, \( t_{he}(\theta) \) and \( t_{ei}(\theta) \). Normally, this is done by solving an appropriate mathematical model for the regenerators. The type and form of the model depend on the type of regenerator and on the assumed dominant heat-transfer mechanisms.

**Regenerator Types**

There are two types—packed beds (pebble heaters) and checkerwork structures (plate regenerators). These are schematically shown in Fig. 2. Packed beds have large or small particles (Fig. 2a). The particles are considered small when their conduction resistance is negligible compared with other resistances. Assume that all complex checkerwork structures can be represented by an equivalent setup having parallel plates or hollow cylinders (Fig. 2b).

Modeling of heat transfer in solid structures with gas flow is still based on the assumptions made by Singer and Wilhelm [7] in 1950 that several transport mechanisms are possible (see Ref. 7 for details).
FIG. 2. Heat regenerators are modeled as either packed beds (large or small particles) or checkerworks (plates or cylinders).

Simplifying assumptions are made and only the most dominant heat-transfer mechanisms are preserved in arriving at a model for a particular regenerator type. The most commonly used models for packed beds and checkerwork structures are shown in the next section.

Mathematical solutions for periodic regenerator operation for even the simplest of these models are quite involved and, in general, efficiency evaluations require extensive numerical computations [4]. Here, we will demonstrate how to use a much simpler, yet accurate procedure. The method is based on representing the unit-step thermal response of the regenerator (regenerator breakthrough curve) by a gamma distribution function of the system variance. The variance for each model is presented as a function of system parameters in the following section.

With this information, we can show how to approach the two typical problems in heat regenerators:

1. Given hot-gas flow rate and properties, and inlet hot and cold-gas temperatures, determine the regenerator size for the desired efficiency.
2. Estimate the performance (thermal efficiency, etc.) of an existing regenerator and suggest changes in operating procedures for improvements in efficiency.
Models for Packed-Bed and Checkerboard Regenerators

Note: The term in square brackets in Eqs. (a), (e), (h), and (l) = 1.

A1. Packed Beds with Large Particles (Spheres) (Model of Sagara et al. [14])

Accounts for: bulk transport in gas; eddy transport in gas; gas–solid convection; conduction in solids.

Neglects: gas conduction; solids point of contact transport; all radiation effects; accumulation term in gas phase; heat losses from walls or ends.

Dimensionless variance of the impulse response:

\[
\sigma_D^2 = \frac{2}{S_{h,\text{r}}^2} + \frac{2B_i}{5S_{h,\text{r}}^2} + \frac{2}{P_{\text{e,m}}} \left[ 1 - \frac{1}{P_{\text{e,m}}} (1 - e^{-P_{\text{e,m}}}) \right]
\]  

Gas–solid film resistance \hspace{1cm} \text{Particle conduction resistance} \hspace{1cm} \text{Eddy transport resistance}

For gas-particle heat-transfer coefficient [15]:

\[
\varepsilon_k J_k = \frac{2.876}{\text{Re}_p} \left( \frac{0.3023}{\text{Re}_p^{0.55}} \right)
\]

\[
J_k = \frac{h_p}{C_{p,m} G_p} \left( \frac{\mu_r C_{W,\text{r}}}{k_r} \right)^{2/3}
\]

Suggested for “eddy” coefficient [12, 16]:

\[
P_{\text{e,m}} \approx P_{\text{e,m}}
\]

and

\[
P_{\text{e,m}} = Bo \frac{L}{d_p}
\]

\[
\frac{1}{Bo} = \frac{0.3}{Sc \text{Re}_p} + \frac{0.5}{1 + \frac{3.8}{\text{Re}_p Sc}}
\]

For \( \text{Re}_p > 10 \), \( Bo \approx 2 \)

A2. Packed Beds with Small Particles (Pseudohomogeneous model [18])

Accounts for: bulk flow transport; gas–solid convective transport; conduction in solids (continuous phase); conduction at solids point of contact.

Neglects: conduction in gas; eddy transport in gas; conduction in solid particles; radiation effects; accumulation term in gas; heat losses from walls or ends.

Dimensionless variance of the impulse response:

\[
\sigma_D^2 = \frac{2}{P_{\text{e,m}}^2} \left[ 1 - \frac{1}{P_{\text{e,m}}^2} (1 - e^{-P_{\text{e,m}}}) \right]
\]
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$$k_{s, eff} = k_{s, eff}^0 + \frac{G_s^2 C_p^2 \rho_s d_s}{\delta h_p (1 - \varepsilon_s)}$$

(f)

Effective axial transport coefficient of the bed with no gas flow

Contribution to effective axial conduction by gas particle heat transfer

$$k_{s, eff}^0$$ can be estimated from [18]:

$$\frac{k_{s, eff}^0}{k_s} = \left( \frac{k_s}{k_s} \right)^{10.28 - 0.759 \log \frac{r_s}{r_p} - 0.017 \log \left( \frac{\rho_p}{\rho_s} \right)}$$

(g)

and \( h_p \) is found from Eq. (b). Other variables as defined in Model A1.

B1. Parallel-Plate Regenerators

Accounts for: bulk transport; eddy transport; conduction in solids perpendicular to flow; and gas–solid convective transport.

Neglects: conduction in gas; conduction in solids in direction of flow; radiation effects; accumulation term in gas phase; and heat losses from walls or ends.

Dimensionless variance of the impulse response:

$$\sigma^2 = \frac{2}{St} \left( \frac{r_s}{L} \right) + \frac{2Bi}{3St} \left( \frac{r_s}{L} \right) + \frac{2}{Pe_{go}} \left[ 1 - \frac{1}{Pe_{go}} (1 - e^{-Pe_{go}}) \right]$$

(h)

Evaluation of parameters:

$$St' = 0.023 Re^{-0.33 Pr^{-0.7}}$$, for \( Re > 10^4 \) [4]

(i)

$$Pe_{go} \approx Pe_{m} \frac{L}{D_s}$$ (by analogy)

(j)

$$Pe_{m} = \frac{G_s D_s}{\rho_s k_{m, eff}}$$

$$\frac{1}{Pe_{m}} = \frac{3 \times 10^7}{Re^{2.7}} + \frac{1.35}{Re^{7.8}}$$, for \( Re > 2000 \) [16]

(k)

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Accounts for: bulk transport; eddy transport; conduction in solids perpendicular to flow; and gas-solid convection transport.
Neglects: conduction in gas; conduction in solids in direction of flow; radiation effects; accumulation term in gas phase; and heat losses from walls and ends.

Dimensionless variance of the impulse response:

\[ \sigma_d^2 = \frac{2}{St} \left( \frac{r_e}{2L} \right) + \frac{Pe_re}{l(1 - \rho^2)} \left[ \frac{\rho^4 - 1}{4} - \frac{(1 - \rho^2)^2}{2} - \ln \rho \right] + \frac{2}{Pe_p} \left[ 1 - \frac{1}{\frac{1}{Pe_p}(1 - e^{-Pe_p})} \right] \]  (1)

Single-Pass Efficiency

The regenerator breakthrough curve can be represented with good accuracy by a gamma distribution function of the system variance as follows (see Fig. 3):

\[ t_{keq} = \mu(\tau) = \frac{1}{\Gamma\left(\frac{1}{\sigma_d^2}\right)} \int_0^{\tau/\sigma_d^2} x^{\left(1/\sigma_d^2 - 1\right)} e^{-x} dx = P\left(\frac{2\tau}{\sigma_d^2}, \frac{2}{\sigma_d^2}\right) \]  (14)

![Graph showing the regenerator breakthrough curve as a gamma function of the system variance.](image)

**FIG. 3.** For single-pass operation, the regenerator breakthrough curve is a gamma function of the system variance.
This chi-square distribution function, $P$, is tabulated in standard handbooks [8–10].

If system parameters are known, we can calculate $\mu$ and $\sigma^2_D$ for the appropriate model (see the preceding section on models) and, at a given dimensionless time, $\tau = \theta/\mu$, obtain the dimensionless exit temperature from Eq. (14). The appropriate value of $P(2\tau/\sigma^2_D)$ can be obtained either by interpolation of tabulated values [8–10] or by interpolation on Fig. 3. Both methods provide sufficient accuracy, considering the uncertainty in the values of heat-transfer parameters.

With $M = 1/\sigma^2_D$, single-pass efficiency up to switching time $\theta_s$, $\tau_s = \theta_s/\mu$ is obtained from Eq. (4); after some algebra it can be represented by

$$E_{\omega}(\tau_s) = 1 - \left( 1 - \frac{1}{\tau_s} \right) P(2M \tau_s; 2M) - \frac{(M \tau_s)^{M+1} e^{-M \tau_s}}{\Gamma(M)} \quad (15)$$

Values of single-pass efficiency are plotted as a function of $M$, with $\tau_s$ as parameter in Fig. 4. Once single-pass efficiency is known, $q$ is found from Eq. (10).

$$q = E_{\omega} \tau_s \quad (16)$$
If we select the thermal mean residence time for the switchoff time, \( \beta = \mu \), and hence \( \tau = 1 \) (which we will later show leads to optimal performance for periodic, concurrent, symmetric operation), the expression for single-pass efficiency can be considerably simplified for \( M \geq 1.6 \).

\[
E_{sp}(1) \sim 1 - \frac{1}{\sqrt{2\pi M}}
\]  
(17)

**Example 1.** A thermal storage unit is composed of 24 parallel rectangular-flow channels 10 m long, 1 m wide, and 2 cm high. The thickness of the Feolite storage material between channels is 10 cm, while the top and bottom channels have a 5-cm Feolite thickness between them and a material of negligible heat capacity that insulates the whole unit from the outside air. Hot air at a flow rate of 30,000 kg/h at an inlet temperature of 120°C is available periodically for 3 h. The storage unit is at 20°C initially.

a. Determine thermal efficiency and the heat-storage factor for this single-pass operation of 3 h and the exit gas temperature at switchoff time.

b. Calculate the regenerator length to increase single-pass efficiency to at least 0.9 and evaluate heat-storage factor and exit gas temperature at switching time.

Data: \( \rho_s = 3,900 \text{ kg/m}^3; \ C_p = 0.95 \text{ kJ/kg} \cdot ^\circ C; \ h_s = 2.1 \text{ W/m} \cdot ^\circ C; \ \rho_g = 1.1 \text{ kg/m}^3; \ C_{pg} = 1.0 \text{ kJ/kg} \cdot ^\circ C; \ \dot{m}_g = 30,000 \text{ kg/h} = 8.33 \text{ kg/s}; \ \mu_g = 2 \times 10^{-5} \text{ kg/m} \cdot \text{s}; \ \kappa_g = 0.028 \text{ W/m} \cdot ^\circ C.

**Solution:** Part a. Select model BI from the section on models:

\[
A = 1 \times 0.02 = 0.02 \text{ m}^2; \ A_{TOT} = 24 \times 0.02 = 0.48 \text{ m}^2
\]

\[
G_s' = \dot{m}_s/A_{TOT} = 8.33/0.48 = 17.4 \text{ kg/m}^2 \cdot \text{s}
\]

\[
D_h = \frac{4A}{2(\text{height} + \text{width})} = \frac{4 \times 0.02}{2(1 + 0.02)} = 0.039 \text{ m}
\]

\[
Re = \frac{G_s'D_h}{\mu_g} = \frac{17.4 \times 0.039}{2 \times 10^{-5}} = 33,900
\]

\[
Pr = \frac{C_{pg} \mu_g}{\kappa_g} = \frac{1000 \times 2 \times 10^{-5}}{0.028} = 0.71
\]

\[
St' = 0.023Re^{0.2}Pr^{1/3}
\]

\[
= 0.023 \times (33,900)^{0.2} \times (0.71)^{1/3}
\]

\[
= 0.0036
\]

\[
h = St'G_s'C_{pg} = 0.0036 \times 17.4 \times 1000
\]

\[
= 62.6 \text{ J/m}^2 \cdot \text{s} \cdot ^\circ \text{C}
\]

\[
Bi' = \frac{R_g h}{\kappa_g} = \frac{0.05 \times 62.6}{2.1} = 1.49
\]

\[
1/Pe_{m'} = \frac{3 \times 10^7}{33,900^{1/4}} + \frac{1.35}{33,900^{1/8}} = 0.0092 + 0.366 = 0.376
\]

\[
Pe_{m'} = 2.66
\]

(k)
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\[
P_{ep} \approx P_{euc} \frac{L}{D_h} = 2.66 \times \frac{10}{0.039} = 683
\]  \hspace{1cm} (j)

\[
\sigma_D^2 \approx \frac{2}{St'} \left( \frac{\tau_c}{L} \right) + \frac{2B_i'}{3St'} \left( \frac{\tau_c}{L} \right) + \frac{2}{P_{ep}}
\]

\[
= \frac{2}{0.0036} \left( \frac{0.01}{10} \right) + \frac{2 \times 1.49}{3 \times 0.0036} \left( \frac{0.01}{10} \right) + \frac{2}{683}
\]

\[
= 0.556 + 0.276 + 0.0029 = 0.834
\]  \hspace{1cm} (h)

\[
M = 1/\sigma_D^2 = 1/0.834 = 1.20
\]

\[
\mu = \frac{M \mu_{\mu}}{\dot{n}_x C_{\mu \mu}} = \frac{24 \times 10 \times 1 \times 0.1 \times 3900 \times 950}{30,000 \times 1000} = 2.96 \text{ h}
\]

\[
\tau_r = \theta_d/\mu = 3/2.96 = 1.01 \approx 1
\]

\[
E_{\mu}(l) = 0.65 \quad \text{Fig. 4 or Eq. (15)}
\]

\[
q = E_{\mu} \tau_r = 0.65 \quad \text{(16)}
\]

\[
\dot{m}_{w,c}(l) = u(l) = 0.626(\tau_r \approx 1) \quad \text{Fig. 3 or Eq. (14)}
\]

\[
T_{\text{w}} = T_{\text{d}} + (T_{\text{w}} - T_{\text{d}})u = 20 + (120 - 20) \times 0.626 = 83^\circ C
\]

Part b. From the above, it is clear that the new \( L \) changes the thermal mean residence time and variance:

\[
\mu_\text{new} = 0.296L; \sigma_{\mu_\text{new}}^2 = 8.34/L
\]

This means that:

\[
\tau_\text{new} = 10.1/L; M_\text{new} = L/8.34
\]

Trial and error is now required. From Fig. 4 we see that if we choose \( M = 5 \), then \( L = 41.7 \, \text{m} \), \( \tau_{\text{new}} = 0.242 \) and \( E_{\mu} = 1 \). Lower values of \( M \) are sufficient for \( E_{\mu} \approx 0.9 \). We finally get \( \tau_{\text{new}} = 2.2 \), \( L = 18.3 \, \text{m} \); \( \tau_{\text{new}} = 0.552 \), \( E_{\mu} = 0.9 \), \( q = 0.50 \). From Fig. 3 for \( M = 2.2 \), \( u (0.55) = 0.265 \), hence \( T_{\text{w}} (\theta = 3h) = 46.5^\circ C \).

Example Summary: Clearly, we can improve single-pass efficiency (i.e., the fraction of heat recovered from the hot-gas stream) by increasing regenerator length (capital cost) at the expense of a reduced heat-storage factor. The only true improvement would come by redesigning the unit.

Ideal Regenerator

Now consider the operation of an ideal regenerator, i.e., one in which heat is transported by bulk flow of gas at a finite rate, while all other heat-transfer resistances perpendicular to flow direction are zero. The breakthrough curve of an ideal regenerator would be a step-up curve at \( \tau = 1 \), in Fig. 3. Clearly, this sharp rise is approached as \( M \to \infty \) or \( \sigma_D^2 \to 0 \). This, in a regenerator with finite heat-transfer resistances, can be approached by increasing its length, since \( \sigma_D^2 \propto 1/L \).
Heat Regenerators, Design and Evaluation

Since finite heat-transfer resistances define the length of the zone in which transport takes place (the height of the transfer unit), the longer the regenerator, the smaller the ratio of the height of the transfer unit to regenerator length and the higher the efficiency. For ideal regenerators:

\[
[E_p(\tau_i)]_{ideal} = 1 - \left(1 - \frac{1}{\tau_i}\right)H(\tau_0 - 1) = \begin{cases} 
1, & \tau_i \leq 1 \\
1/\tau_0, & \tau_i > 1 
\end{cases}
\]  
(18)

where

\[
H(x - a) = \begin{cases} 
0, & x < a \\
1, & x > a 
\end{cases}
\]  
(19)

Continuous Periodic Operation

Swing regenerators are always on-stream and can be operated cocurrently or countercurrently (Fig. 1). While countercurrent flow always yields higher efficiency, there may be other considerations that favor cocurrent operation. The efficiency for this case can be estimated rapidly by the method presented below.

We also must distinguish between balanced (symmetric) and unbalanced operation. Balanced operation is when the product of the gas mass flow rate and specific heat is the same for the hot and cold gas (the heating and cooling periods). Switching times are the same for both periods and so are efficiencies, provided that heat-transfer parameters are approximately the same.

Unbalanced operation is when the product of flow rate and specific heat are unmatched for heating and cooling, and efficiencies of the two periods are unequal, but real switching times are kept the same. Overall efficiency for both periods is:

\[
E_o = \frac{2E_h}{1 + (\tau_c/\tau_h)} = \frac{2E_h}{1 + (\mu_0/\mu_c)}
\]  
(20)

Cocurrent Symmetric Operation

Consider the operation of an ideal regenerator. As can be seen from Fig. 5, such a device has an efficiency = 1 if switching time is equal to the mean residence time (\(\tau_i = 1\)). For dimensionless switching times >1, efficiency is always <1. Dimensionless switching times <1 yield efficiencies <1. However, even if the efficiency = 1, this implies that the same operation could have been achieved with a smaller regenerator of reduced thermal mean residence time so that the efficiency of unity is accomplished at \(\tau_i = 1\). Thus, the considera-
The operation of ideal regenerators teaches us that the optimal switching time for cocurrent operation is the thermal mean residence time $\theta_r = \mu \tau_s = 1$. This finding holds for nonideal regenerators, too. The dimensionless exit gas temperature for cocurrent operation is given by Ref. 11:

$$t_{ge}(\tau) = \sum_{n=0}^{\infty} \left[ \mu(\tau - 2n\tau_s)H(\tau - 2n\tau_s) - u(\tau - (2n + 1)\tau_s) - H(\tau - (2n + 1)\tau_s) \right]$$

where the breakthrough curve $u(\tau)$ is given by Eq. (14) and the Heaviside unit step function $H(\tau)$ is defined by Eq. (19). The exit gas temperature can then be calculated during any heating period $2n\tau_s < \tau < (2n + 1)\tau_s$ by superposition. After a sufficiently large number of cycles, steady periodic state is reached. The infinite series can be replaced by the finite sum and summation terminated when $n = N$ with

$$N = 1 + \int \left[ \frac{1}{2} \left( 1 + \frac{X_r^2}{2M\tau_s} \right) \right]$$

where $X_r^2$ is such a value of the argument that $P(X_r^2; 2M) \approx 0.995$, which can
readily be assessed from Fig. 3. A few examples of the effluent gas temperature in cocurrent operation are sketched in Fig. 6. Ideal regenerator operation is approached only at \( \tau_s = 1 \) and at high values of \( M \).

The formula for thermal efficiency for cocurrent, periodic, symmetric operation is [11]

\[
E(\tau_i) = E_{sp}(\tau_s) + \sum_{n=1}^{N} [(2n - 1) E_{sp}(2n - 1) \tau_i - 4n E_{sp}(2n \tau_i) + (2n + 1) E_{sp}(2n + 1)]
\]

(23)

Equation (23) can readily be evaluated using either Eq. (15) or Fig. 4 for single-pass efficiency, \( E_{sp}(\tau) \). Figure 7 shows the thermal efficiency of cocurrent, symmetric operation as a function of \( M \), with \( \tau_i \) as parameter. Clearly, optimal operation is achieved at \( \tau_i = 1 \), but the behavior with respect to \( \tau_i \) is not monotonic. For optimal operation (\( \tau_i = 1 \)), the efficiency can be calculated by an approximate formula, accurate within 5% for \( M > 2.5 \) [12]:

\[
E(1) = 2E_{sp}(1) - 1
\]

(24)

Evaluation of thermal efficiency for cocurrent, symmetric operation now becomes simple. For a given system, we calculate all the physical properties over the temperature range and find their average values. Next, we select the appropriate regenerator and its model (see the section on models) and calculate the heat-transfer parameter from appropriate correlations. Then we evaluate \( \sigma_0^2 \) and \( M = 1/\sigma_0^2 \). We should select optimal operation (\( \tau_i = 1 \) and
calculate single-pass efficiency, $E_w$, from Eq. (17) and optimal thermal efficiency for concurrent operation from Eq. (24). If, for some reason, we are forced to use nonoptimal operation with $\tau_s = 1$, we can either calculate single-pass efficiency from Eq. (15) or estimate it from Fig. 4. The efficiency for concurrent operation is then obtained either from Eq. (23) or by interpolation, using a proper value of $\tau_s$, from Fig. 7.

**Example 2.** Two packed-bed regenerators are used in periodic, concurrent swing-operation to recover heat from hot exhaust gases at 800°C and preheat inlet air at 100°C. Each regenerator is 30 m long and 4 m diameter. Solid pebbles used are approximately spherical, with an average diameter of 6 cm.

Other data: $\rho = 3900$ kg/m$^3$; $C_p = 1.0$ kJ/kg·°C; $k_s = 0.5$ W/m·°C; and $\epsilon_s = 0.4$.

Mass flow rate of both the hot and cold streams is $m_x = 72,000$ kg/h (55,600 Nm$^3$/h).

The following mean physical properties can be used for both streams: $\mu_s = 3 \times 10^{-5}$ kg/m·s; $k_s = 0.05$ W/m·°C; $\rho_s = 0.5$ kg/m$^3$; and $C_{ps} = 1.02$ kJ/kg·°C.

a. Currently, a switching time of 7.2 h is used. What is the thermal efficiency?

b. What is the optimal efficiency, and what switching time is needed to achieve it?

**Solution:** First, we realize that, since $(m_x C_{px}) = (m_y C_{py})$, and other physical properties are constant, operation is symmetric and concurrent. For a packed bed with large particles, an appropriate model is A1.
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\[ G_s = \frac{m_s}{A} = \frac{4m_r}{\pi d^2} = \frac{4 \times 72,000}{3600 \times \pi \times 4} = 1.59 \text{ kg/m}^2 \cdot \text{s} \]

\[ \text{Re}_p = \frac{d_p G_s}{\mu} = \frac{0.06 \times 1.59}{3 \times 10^{-5}} = 3180 \]

\[ \text{Pr} = \frac{\mu C_p}{k_s} = \frac{3 \times 10^{-5} \times 1020}{0.05} = 0.61 \]

\[ \epsilon_{pJ_s} = \frac{2.876}{\text{Re}_p} + \frac{0.3023}{\text{Re}_p^{0.65}} \]

\[ 0.4J_s = \frac{2.876 	imes 0.3023}{3180} + \frac{0.0009 + 0.01797}{3180} = 0.0189 \]

\[ J_s = \frac{0.04722 \times h_p}{C_{pr} G_s} \text{Pr}^{2/3} = J_h \]

\[ h_p = \frac{C_{pr} G_s J_h \text{Pr}^{2/3}}{1.020 \times 1.59 \times 0.0472 \times (0.61)^{2/3}} \]

\[ = 106 \text{ W/m}^2 \cdot ^\circ \text{C} \]

\[ \text{St}_p = \frac{6h_p(1 - \epsilon_s)L}{d_p G_s C_{pr}} = \frac{6 \times 106 \times (1 - 0.4) \times 30}{0.06 \times 1.59 \times 1020} = 118 \]

\[ \text{Bi}_p = \frac{h_p d_p}{2k_s} = \frac{106 \times 0.06}{2 \times 0.5} = 6.36 \]

Since \( \text{Re}_p > 10 \), \( \text{Bo} = 2 \).

\[ \text{Pe}_{sh} = \text{Bo} \frac{L}{d_p} = \frac{2 \times 30}{0.06} = 1000 \]

\[ \sigma_D^2 = \frac{2}{\text{St}_p} + \frac{2 \text{Bi}_p}{5 \text{St}_p} + \frac{2}{\text{Pe}_{sh}} = \frac{2}{118} + \frac{2}{5 \times 118} + \frac{2}{1000} \]

\[ = 0.0169 + 0.0216 + 0.002 = 0.0405 \]

\[ M = 1/\sigma_D^2 = 1/0.0405 = 24.7 \approx 25 \]

\[ \mu = \frac{(1 - \epsilon_s) L \rho_s C_p}{G_s C_{pr}} = \frac{(1 - 0.4) \times 30 \times 3900 \times 1000}{1.59 \times 1020} \]

\[ = 4.3285 \times 10^4 \text{ s} = 12.0 \text{ h} \]

**Part a.** Current switching time is \( \theta_s = 7.2 \text{ h} \). Dimensionless switching time is \( \tau_s = \theta_s/\mu = 7.2/12 = 0.6 \). From Fig. 7, for \( \tau_s = 0.6 \), \( M = 25 \), we read \( E = 0.4 \). Very low efficiency is obtained. The regenerators are poorly operated.

**Part b.** For symmetric cocurrent operation, optimal efficiency is obtained at \( \tau_s = 1 \). We need switching time \( \theta_s = \mu = 12 \text{ h} \). From Fig. 7 at \( \tau_s = 1 \), \( M = 25 \), we read \( E = 0.84 \). Without any redesign, just by changing the switching time to the optimal value, we are able to improve the thermal efficiency by \( 100 \times (0.84 - 0.4)/0.4 = 110\% \!\)! If, however, other process requirements demand the switching time to be 7.2 h, we have two options to improve efficiency. We can either use countercurrent operation (to be discussed in Example 3) or reduce the length of the regenerators, so that the new thermal mean residence time equals the switching time \( \mu_{new} = 7.2 \text{ h} \). This requires:
\[ L_{\text{new}} = \frac{L_{\text{new}}^{\text{H}}}{\mu} = 30 \times \frac{7.2}{12} = 18 \, \text{m} \]

\[ \sigma_{D_{\text{new}}} = \sigma_{D} \sqrt{\frac{L}{L_{\text{new}}}} = 0.0405 \times \frac{30}{18} = 0.0675 \]

\[ M_{\text{new}} = \frac{1}{\sigma_{D_{\text{new}}}} = 14.8 \approx 15 \]

Figure 7 gives at \( M = 15 \) and \( \tau_s = 1, E = 0.80 \). We double the efficiency by reducing the regenerator length from 30 to 18 m. Clearly, in cocurrent operation, bigger does not necessarily mean better, and overdesign may penalize process efficiency heavily.

**Cocurrent Unbalanced Operation**

Sometimes process constraints are such that we are unable to balance the product of mass gas flow rate and specific heat for the heating and cooling periods. Thus, \( \mu_h \neq \mu_c \), and operation is unbalanced. If heat-transfer resistances are reasonably close for the heating and cooling periods, \( \sigma_D^2 = \sigma_{D_c}^2 \) and we need to operate at the same switching time for both periods. Optimal switching times are approximated by [12]

\[ \tau_{h_{\text{opt}}} = \frac{1}{2} \left( 1 + \frac{\mu_c}{\mu_h} \right) \]

\[ \tau_{c_{\text{opt}}} = \frac{\mu_h}{\mu_c} \tau_{h_{\text{opt}}} \]

If we operate at such optimal switching times, the maximum attainable overall efficiency is given in Fig. 8 as a function of \( \mu_h/\mu_c \) with \( M \) as the parameter. Optimal efficiency is bounded by the efficiency for an ideal regenerator, operated with the same \( \mu_h/\mu_c \) ratio.

If we are forced to nonoptimal, unbalanced operation, although this should be avoided if possible, the thermal efficiency can be calculated from formulas given in Ref. 11. These are somewhat lengthy and are not reproduced here. They are, however, simple enough to be programmed on a personal computer. The interested reader is referred to Ref. 11.

**Periodic Countercurrent Symmetric Operation**

This is a difficult case to calculate exactly [4]. Several simplified approaches have been suggested [11, 12]. Consider the operation of an ideal regenerator (Fig. 9), from which it becomes clear that efficiency stays at 1 for \( \tau_s \leq 1 \), but diminishes for higher switching times, \( \tau_s > 1 \). Theoretical considerations of the nonideal regenerator [1] show that optimal operation is achieved at an infinitesimally short switching time, \( \tau_s \rightarrow 0 \), when the solids temperature-
For cocurrent, unbalanced, periodic operation, optimal overall thermal efficiency is bounded by efficiency of ideal unit.

profile approaches the diagonal line \( T_s = T_{hi} - (T_{hi} - T_{co}) \frac{z}{L} \), where \( z \) is the axial distance measured from the hot-gas inlet. The maximal efficiency for countercurrent operation reached at \( \tau_s = 0 \) can be expressed as

\[
E_{\text{heat}} = E_h(0) = M_h / (M_h + 1)
\]

(27)

This value is never more than 15% higher than the efficiency at \( \tau_s = 1 \). This is important since, fortunately, not much loss in efficiency results by operating at more practical switching times, since zero switching time is impossible to attain.

Departure from ideal regenerator performance increases as \( M \) decreases. We can show [11] that the exit temperature for countercurrent operation during the heating period at dimensionless time \( \tau_s \), \( T_e(\tau_s) \) can be roughly related to the breakthrough temperature at the same time, \( u(\tau_s) \), by

\[
t_e(\tau_s) = \begin{cases} 
(1.872 - 0.964 \log u(\tau_s))^{-1} & ; u(\tau_s) < 0.06 \\
[1 - 1.675 \log u(\tau_s)]^{-1} & ; u(\tau_s) \geq 0.06 
\end{cases}
\]

(28a)

Values of \( u(\tau_s) \) are available for a given \( M \) from either Eq. (14) or Fig. 3.
The exit-gas temperature during the heating part of the cycle at stationary state can be approximated by either a linear or parabolic function:

\[ t_{he} = \begin{cases} 
  t_{he}(\tau_s) \frac{\tau}{\tau_s} & \text{for } 2 \leq M < 5 \text{ and all } \tau_s; \text{ or for } \tau_s > 2 \text{ and } M > 5 \\
  t_{he}(\tau_s) \left( \frac{\tau}{\tau_s} \right)^2 & \text{for } M > 5 \text{ and } \tau_s < 2 
\end{cases} \] (29a)

Comparison of predictions by these formulas and the exact results computed numerically would indicate that the two are in reasonable agreement, especially for the mean temperature. Remember that mean temperature is of interest in efficiency calculations. Using Eq. (4), we get

\[ E_h = \begin{cases} 
  1 - \frac{t_{he}(\tau_s)}{2} & M \leq 5; \text{ or } \tau_s > 2 \text{ and } M > 5 \\
  1 - \frac{t_{he}(\tau_s)}{3} & M > 5 \text{ and } \tau_s < 2 
\end{cases} \] (30a)

Efficiency for countercurrent symmetric operation is presented in Fig. 10. Unbalanced countercurrent operation is more complex and the reader
FIG. 10. For periodic, countercurrent, symmetric operation, thermal efficiency as a function of inverse variance.

should consult the literature for a complete treatment of it [2, 4, 11, 13]. An approximate answer for efficiency can be obtained by using the above approach to calculate $E_s$ and $E_r$ each with its own $\mu$, $M$, and $\tau_s$. The overall efficiency is then calculated from Eq. (20).

Example 3. For the conditions of Example 2, including the switching time of $\theta_s = 7.2$ h, find the efficiency if the system operates with countercurrent flow. Also calculate the maximum attainable efficiency. Take all the bed and gas data as given in Example 2.

Solution: Method 1. The following have been already calculated: $\sigma_\theta = 0.0405$, $M = 25$, $\mu = 12$ h; and $\tau_s = 0.6$.

From Fig. 10, at $M = 25$, $\tau_s = 0.6$, we find $E = 0.94$. Maximum efficiency is at $\tau_s = 0$ and is given by Eq. (27):

$$E_{\text{max}} = \frac{M_s}{M_s + 1} = \frac{25}{26} = 0.96$$

Method 2. Evaluate $u(0.6)$ at $M = 25$ from Fig. 3; $u(0.6) = 0.02$. Evaluate $t_n(0.6)$ from Eq. (28a):

$$t_n(0.6) = [1.872 - 0.964 \log 0.02]^{-1} = 0.285$$
Calculate efficiency from Eq. (30b):

\[ E = 1 - \frac{t_{nd}(0.6)}{3} = 1 - \frac{0.285}{3} = 0.91 \]

The difference between the two methods is:

\[ \frac{(0.91 - 0.94)/0.94}{0.94} \times 100 = 3.2\% \]

**Symbols**

- \( A \): cross-sectional area of one channel (m²)
- \( a_s \): external particle surface area per unit volume of the bed (m⁻¹)
- \( Bi \): \( Rh/k_p \), Biot number for hollow-cylinder model (Model B2)
- \( Bi_p \): \( d_p h_p / 2k_p \), Biot number for particle
- \( Bi' \): \( R_j h / k \), Biot number for parallel-plate model (Model B1)
- \( Bo \): \( d_p \mu / \varepsilon_{sk_{m,eff}} \), Bodenstein number for mass transfer (Model B2)
- \( C_p, C_p' \): mean specific heat for gas \( (g) \) or solids \( (s) \) (kJ/kg · °C)
- \( D \): temperature variance
- \( D_h \): hydraulic diameter (m)
- \( d_p \): particle diameter (m)
- \( E \): thermal efficiency
- \( E_o \): overall thermal efficiency
- \( G'_{st} \): superficial gas mass velocity based on total cross-sectional area of regenerator (kg/m² · s)
- \( G'_{i} \): \( G'_{i}/\varepsilon_s N \), interstitial gas mass velocity (kg/m² · s)
- \( H(x) \): Heaviside unit step-function
- \( h \): gas-solid heat-transfer coefficient (W/m² · °C)
- \( h_p \): gas-particle heat-transfer coefficient (W/m² · °C)
- \( J_{k} \): heat-transfer factor
- \( k_{g} \): gas conductivity (W/m · °C)
- \( k_{vr} \): axial heat effective transport coefficient in channel flow (Models B1, B2) (W/m · °C)
- \( k_{x,eff} \): axial heat effective transport coefficient (Model A2) (W/m · °C)
- \( k_{xu} \): axial heat effective transport coefficient in packed beds (Model A1) (W/m · °C)
- \( k_{m,eff} \): mass-transfer dispersion coefficient (m²/s)
- \( k_s \): solid conductivity (W/m · °C)
- \( k_{r,eff} \): conductivity of packed bed with no gas flow (W/m · °C)
- \( L \): regenerator length (m)
- \( l \): \( 2L/d_p \)
- \( L_{(r+c+R_p)} \)
- \( M \): \( \sqrt{\frac{1}{\sigma_0^2}} \)
- \( M_s \): total mass of solids in the regenerator (kg)
- \( m_g \): gas mass flow rate (kg/s)
- \( N \): number of flow channels for parallel-plate or hollow-cylinder arrangement; also, number of heating periods
- \( P(\chi^2; v) \): chi-square probability distribution
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\[ \text{Pe}_{gh} = \frac{G_sC_{p_s}L}{k_{ht}}, \text{ gas-phase Peclet number (Model A1)} \]
\[ \text{Pe}_{\text{m}} = \frac{uL}{k_{m,ef}}, \text{ Peclet number for mass transfer} \]
\[ \text{Pe}_{p} = \frac{G_sC_{p_s}L}{k_p}, \text{ gas-phase Peclet number (Model B1)} \]
\[ \text{Pe}_{\text{m,ef}} = \frac{G_sC_{p_s}L}{k_{m,ef}}, \text{ gas-phase Peclet number (Model A2)} \]
\[ \text{Pe}_p = \frac{d_pG_sC_{p_s}2k_s(1 - \epsilon_s)}{k_p}, \text{ particle Peclet number (Model A1)} \]
\[ \text{Pe}_R = \frac{RG_sC_{p_s}2k_s}{k}, \text{ solid-phase Peclet number} \]
\[ \text{Pe}_{\text{mx}} = \frac{R_pG_sC_{p_s}k_s}{k_{mx}}, \text{ solid-phase radial Peclet number (Model B1)} \]
\[ \text{Pr} = \frac{C_p\mu_s}{k_p}, \text{ Prandtl number} \]
\[ q \]
heat storage factor
\[ R \]
total radius of cylinder (m)
\[ R_p \]
half thickness of plates around rectangular or cylindrical flow channel (m)
\[ \text{Re} = \frac{D_hG_s}{\mu_p}, \text{ Reynolds number} \]
\[ \text{Re}_p = \frac{d_pG_s}{\mu_p}, \text{ particle Reynolds number} \]
\[ r_c \]
half height of flow channel (m)
\[ Sc \]
Schmidt number
\[ S_{st} \]
\[ \frac{h_{pr}L}{C \rho_s G_s}, \text{ Stanton number (Model A1)} \]
\[ S't' \]
\[ \frac{h}{G_sC_{p_s}}, \text{ Stanton number (Model B1, B2)} \]
\[ T \]
temperature—symbols have one or two subscripts (°C) (see listing of subscripts)
\[ t \]
dimensionless temperature—symbols have one or two subscripts (see listing of subscripts)
\[ u \]
unit-step response (normalized breakthrough curve)
\[ \mu_s \]
superficial gas velocity (m/s)
\[ V_R \]
regenerator volume (m³)

Greek Letters

\[ \Gamma(x) \]
gamma function
\[ \epsilon_R \]
regenerator voidage (porosity)
\[ \theta \]
time (s)
\[ \mu \]
\[ M_sC_{ps}/\rho_sC_{ps} \], thermal mean residence time (s)
\[ \rho \]
r_i/T or r_s/R, dimensionless channelheight (radius)
\[ \rho_s \]
gas density (kg/m³)
\[ \rho_s \]
\[ \text{solids density (kg/m}^3\text{)} \]
\[ \sigma_{i^2} \]
dimensionless variance of the impulse response
\[ \tau \]
\[ \theta/\mu \], (dimensionless time)
\[ \tau_s \]
dimensionless switching time (for single pass)

Subscripts

\[ c \]
cooling period
\[ e \]
exit
\[ g \]
gas
\[ h \]
heating period
\[ i \]
\[ t \]
inlet
\[ opt \]
optimum
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\[ p \quad \text{packed bed} \]
\[ sp \quad \text{single pass} \]
\[ s \quad \text{at switching or solids} \]

Superscripts

\[ ^0 \quad \text{initial} \]
\[ \text{mean} \]


References


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